Revised Edition

ENGINEERING GRAPHICS

As per Latest Syllabus (2110013) of
Gujarat Technological University, Ahmedabad

(For the Students of B.E., First Year)

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S. CHAND

CHAPTER - 1

Drafting Equipments

Engineering drawing is a language of all persons involved in engineering activity. Engineering ideas are recorded by preparing drawings and execution of work is also carried out on the basis of drawings. Communication in engineering field is done by drawings. Like music drawing is a universal language.

To prepare engineering drawing, special drawing instruments are required. It is advisable to purchase simple and good quality instruments rather than many ordinary instruments. All those who are connected with engineering activity should know the use of drawing instruments, handling of instruments and proper maintenance of drawing instruments. A good draftsman is able to prepare correct, accurate and decent drawings in the least possible time with the proper use of instruments.

The following drawing equipments and materials are required in the preparation of drawing. (See Fig. 1.1)

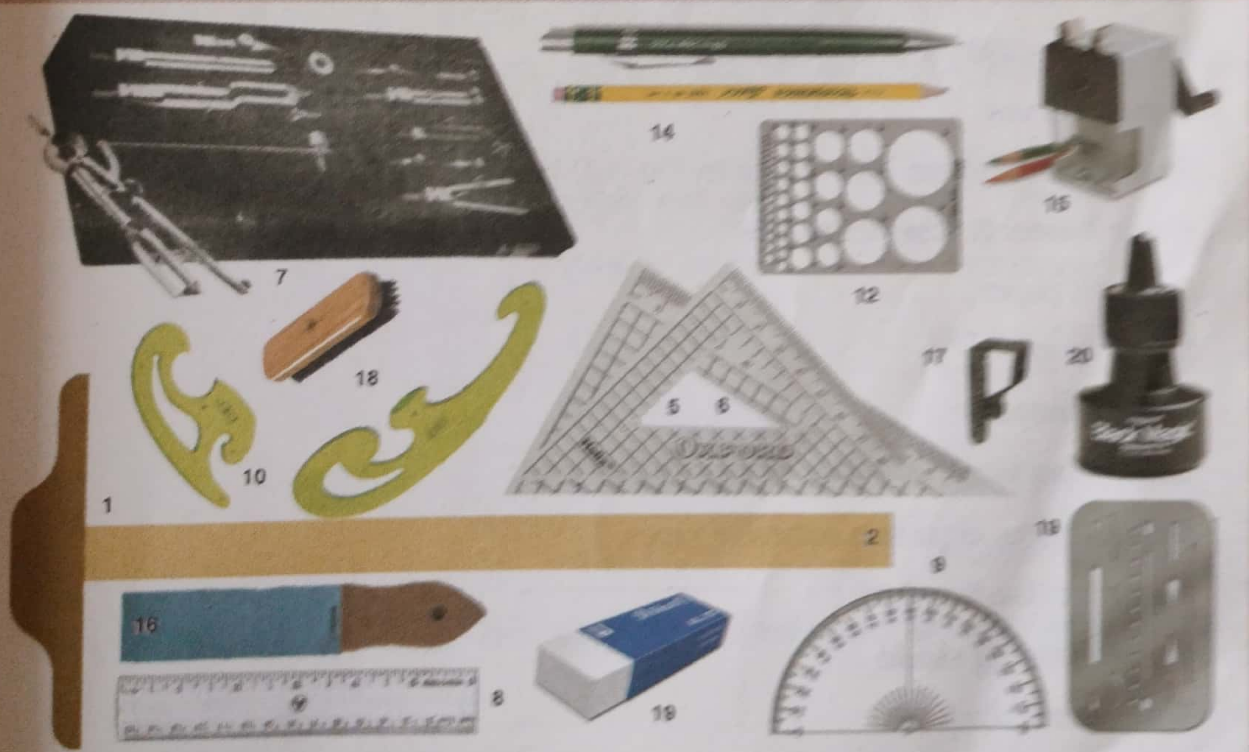


Fig. 1.1. Drawing Equipments

1. Drawing Board
2. Tee-Square
3. Mini-drafter (Not shown in Fig. 1.1) (See Fig. 1.3.)
4. Parallel Straightedge (Not shown in Fig. 1.1) (See Fig. 1.4)
5. Set-Square 45°
6. Set-Square 30° - 60°
7. Instrument Box
   (a) Large compass with interchangeable pencil and pen legs having knee joint
   (b) Large dividers
   (c) Small bow compass
   (d) Small bow divider
   (e) Small bow pen
   (f) Ruling pen
   (g) Interchangeable ruling pen used in compass leg
   (h) Container with spare parts
   (i) Case for lead
   (j) Screw driver
   (k) Drop compass - Rotor
8. Scales-Plastic, boxwood or cardboard scales - 300 mm.
9. Protractor - 180° or 360° - Size 150 mm.
10. French curves
11. Proportional divider. (Not shown in Fig. 1.1) (See Fig. 1.17)
12. Circle Master & other templates
13. Drawing paper
14. Set of pencils
15. Pencil sharpener
16. Sand paper block
17. Drawing pins, clips and adhesive tapes
18. Duster or handkerchief
19. Eraser-Erasing shield
20. Water-proof black ink.

1. **Drawing Board :** (See Fig. 1.2)

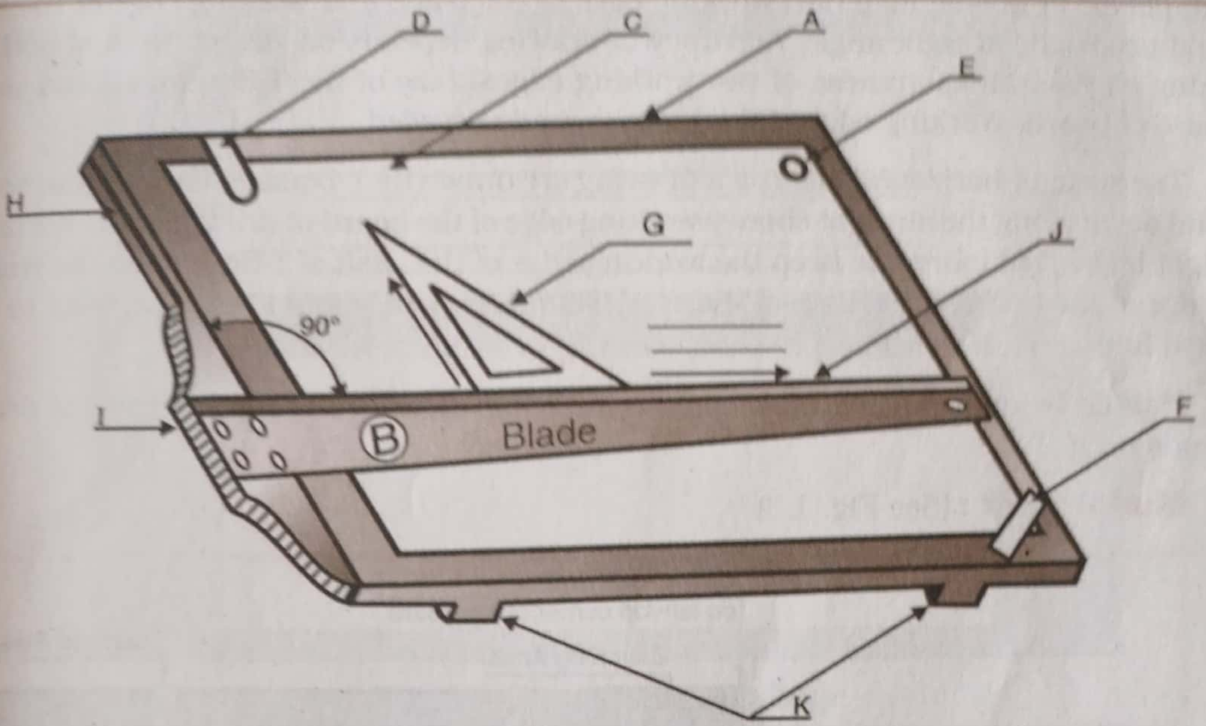


Fig. 1.2. (A) Board, (B) Tee, (C) Paper, (D) S.S. Clip, (E) Pin, (F) Adhesive Tape, (G) Set Square, (H) Eboney Edge of Board, (I) Stock or Head, (J) Working Edge of Tee (K) Battens

In order to prevent warping, the drawing board is made of narrow strips glued edge-to-edge of well seasoned white pine or bass soft wood with minimum of one straight eboney working edge, as a base for the T-Square. Surface of the board should be free from cracks and it should be absolutely flat. On the otherside of board two battens are fixed by screws fitted in the slots to allow for seasonal contraction and expansion.

The standard sizes of drawing boards available are as follows. These sizes are fixed by I.S. 1444.

Sr. No.	Designation	Dimension, mm	Name
1.	B_0	1500 × 1000	Antiquarian
2.	B_1	1000 × 700	Double Elephant
3.	B_2	700 × 500	Imperial
4.	B_3	500 × 350	Half Imperial
5.	B_4	350 × 250	Quarter Imperial

For convenience of working, the board is kept tilted at nearly 15° to 20°.

2. **Tee - Square :** (See Fig. 1.2)

Mostly T - Square is made of two parts

(i) The blade (ii) The stock or The head.

The blade and the head are rigidly fastened together. Tee-Square is made out of hard wood, plastic or acrylic material. Working edge of the blade and working edge of the head should be exactly at right angle. Accuracy of drawing depends on straightness of both the working edges and squareness of two working edges. Size of the T-Square should mat the size of board. Working edge of the blade is made bevelled.

The straight horizontal lines on a drawing are drawn by T-Square. The T-Square slid up and down along the straight eboney working edge of the board to draw parallel horizon straight lines. Don't forget to keep the working edge of the head of T-Square on the work left edge of the board while using T-Square. T-Square alone should never be used to dr vertical lines.

Handle T-Square with atmost care because it is delicate and the accuracy of draw depends on it.

3. Mini Drafter : (See Fig. 1.3)

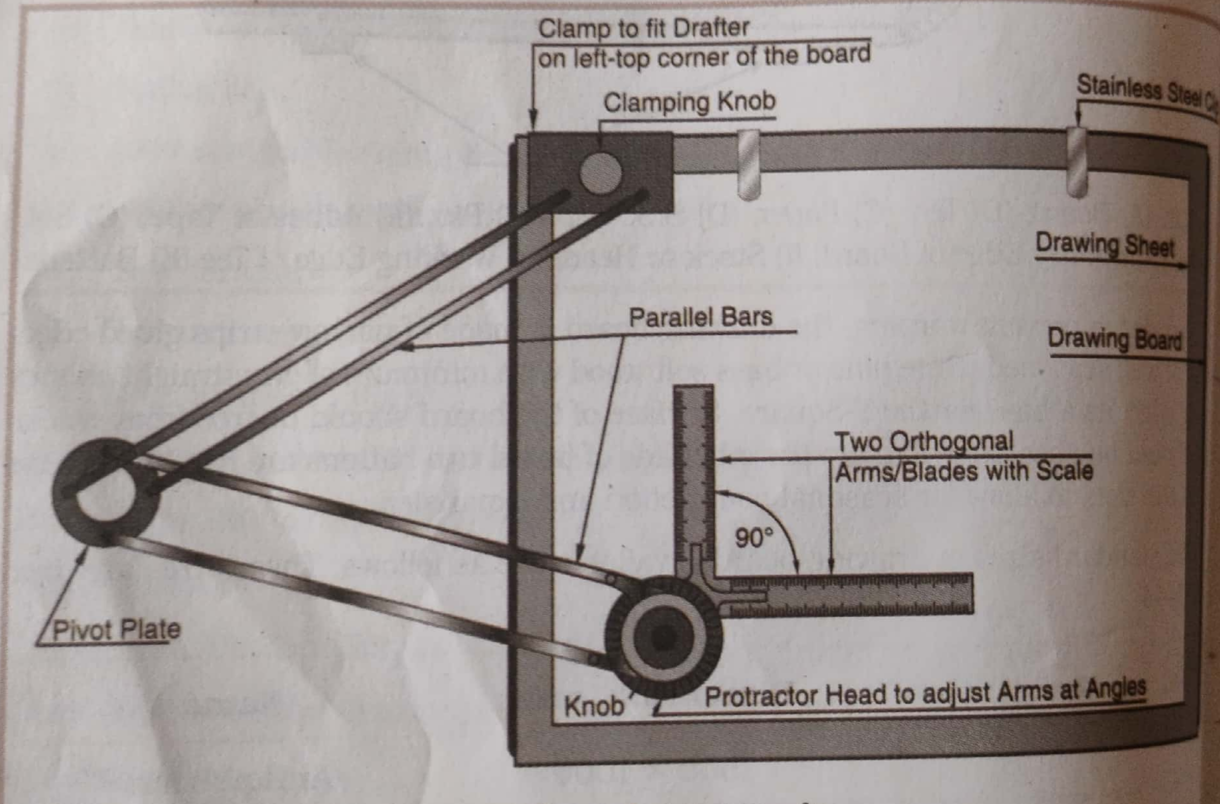


Fig. 1.3. Mini Drafter

Mini drafter is used in many drawing and design offices to do drawing work fas serves the purpose of T - Square, Set - Square, Protractor and Scales. During operatio two blades of the drafter always remain parallel to their set position no matter where are moved on the sheet. Two blades of the drafter are accurately set at right angles to other.

The blades are detachable and hence blades with different scales can be used. T and clamp two blades at desired angle there is a provision of the adjusting head with protractor.

4. **Parallel Straight Edge :** (See Fig. 1.4)

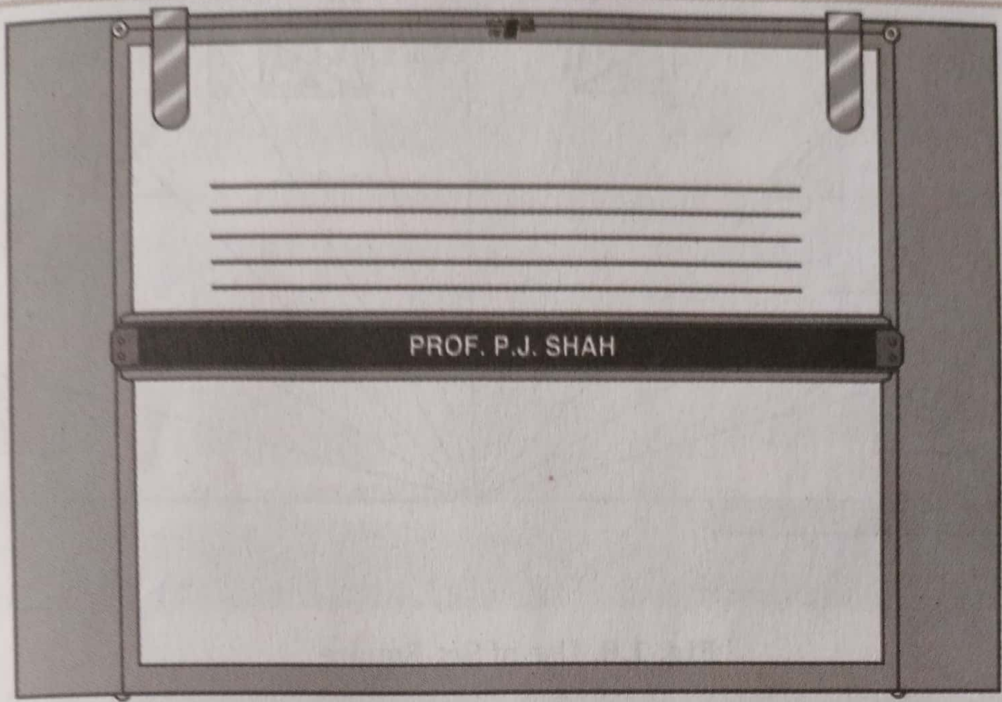


Fig. 1.4. Parallel Straight Edge

It is used in place of T - Square. Mostly it is preferred on large drawing boards. It is simply a straight edge of board size moved up and down on the board by inextensible string - pulley arrangement. It always remain parallel to previously set position.

5. **Set-Square 45° :** (See Fig. 1.5)

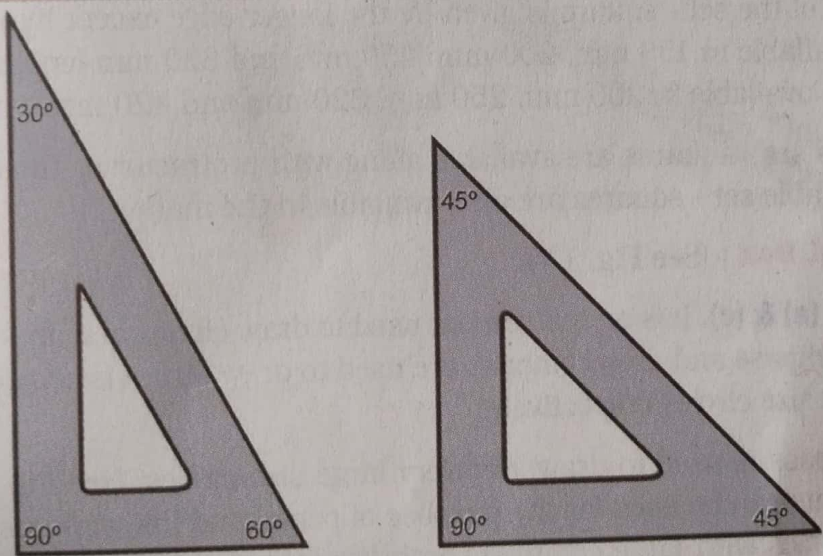


Fig. 1.5. Set-Squares

6. **Set-Square 30° - 60° :** (See Fig. 1.5)

Set - Squares are used with T - Square to draw lines at various angles with the horizontal. The two most common set - squares used by draftsman are 45° Set - Square and 30° - 60° Set - Square. With the combination of T - Square and two Set - Squares angles of 15°, 30°, 45°, 60°, 75°, 90°, 105°, 120°, 135°, 150°, 165°, and 180° are drawn as shown in Fig. 1.6 The above combination can divide the circle into 24 equal divisions.

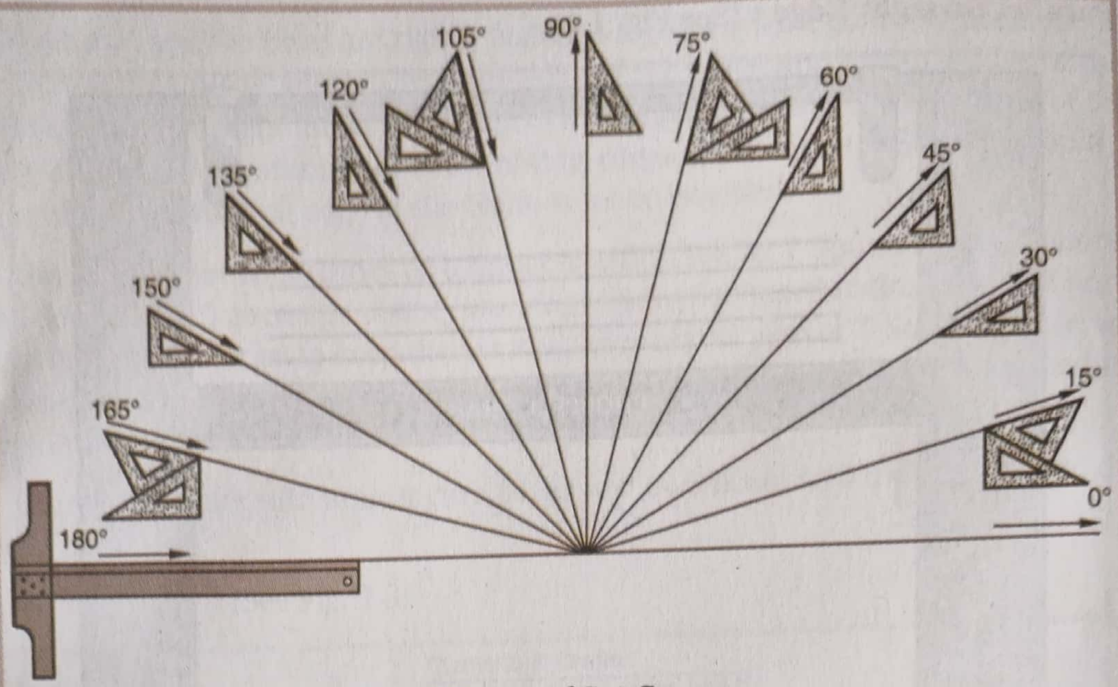


Fig. 1.6. Use of Set-Square

Set - squares are made out of transparent colourless plastic or acrylic material. They are made with straight edge for pencil work and with bevelled edge for ink work. Thickness of set squares varies from 1.5 mm to 2.5 mm.

When laying out lines, set - squares are placed firmly against the upper working edge of the T - Square. Parallel angular lines are drawn by moving the set - square or combination of two set - squares on the working edge of T - Square.

The length of the set - square is given by the longer edge except hypotenuse. 45° set - squares are available in 150 mm, 200 mm, 250 mm and 320 mm length while 30° - 60° set - squares are available in 200 mm, 250 mm, 320 mm and 420 mm length.

Now a days set - squares are available along with protractor or french curve carved with in it. Adjustable set - squares are also available in the market.

7. Instrument Box : (See Fig. 1.7)

Compass : (a) & (c). It is an instrument used to draw circles and arcs of circles; Large compass, bow compass and drop compass are used to draw large size circles, medium size circles and small size circles respectively.

Beam compass is used to draw very very large size circles. (see Fig. 1.9) Pencil and inking attachments can be used for the purpose of pencil and ink work respectively. While drawing large circles with large compass, both the legs should be kept bent at the knee joint so that they become perpendicular to the drawing paper. This situation is advisable for accurate circle.

Compass leads should extend approximately 9 mm. However, the metal needle point of the compass extended about 1 mm more to compensate for the distance it enters the paper, see Fig. 1.10 The flat side of the lead is kept facing outward. The compass is revolved between thumb and index finger, a little downward pressure on the metal needle side is necessary to keep it at the centre.

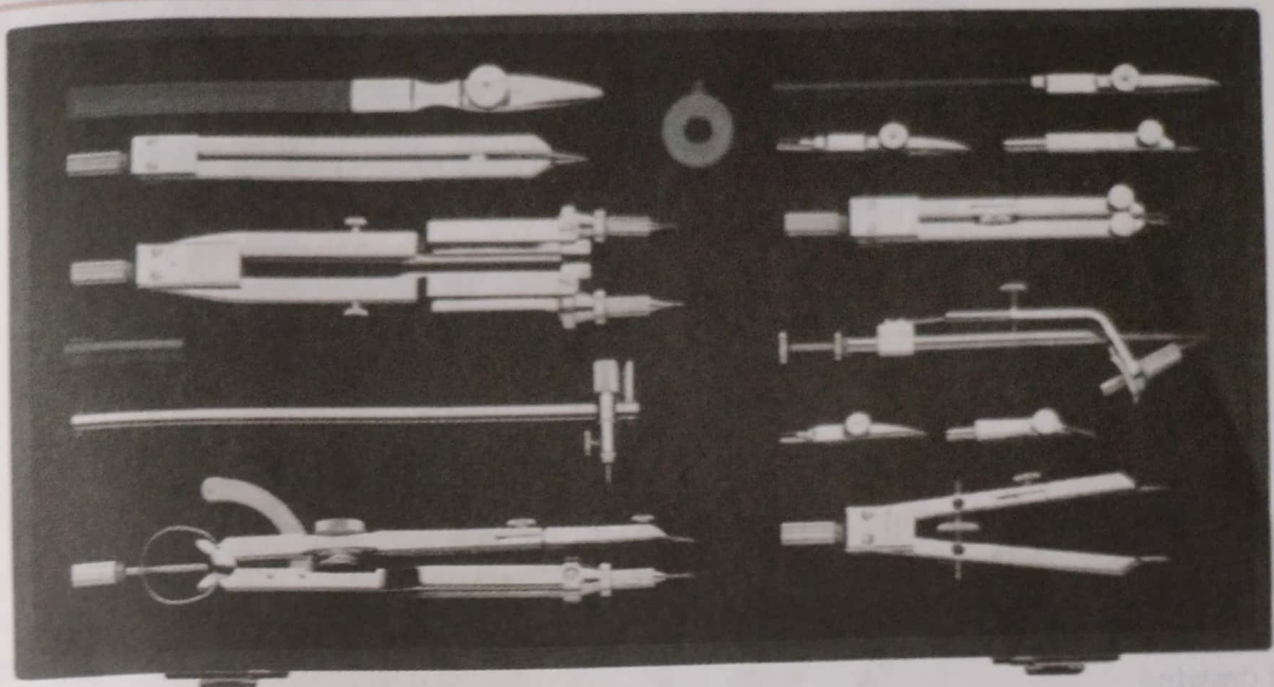


Fig. 1.7. Instrument Box

Lengthening bar can be used to draw very large size circle. (See Fig. 1.8)

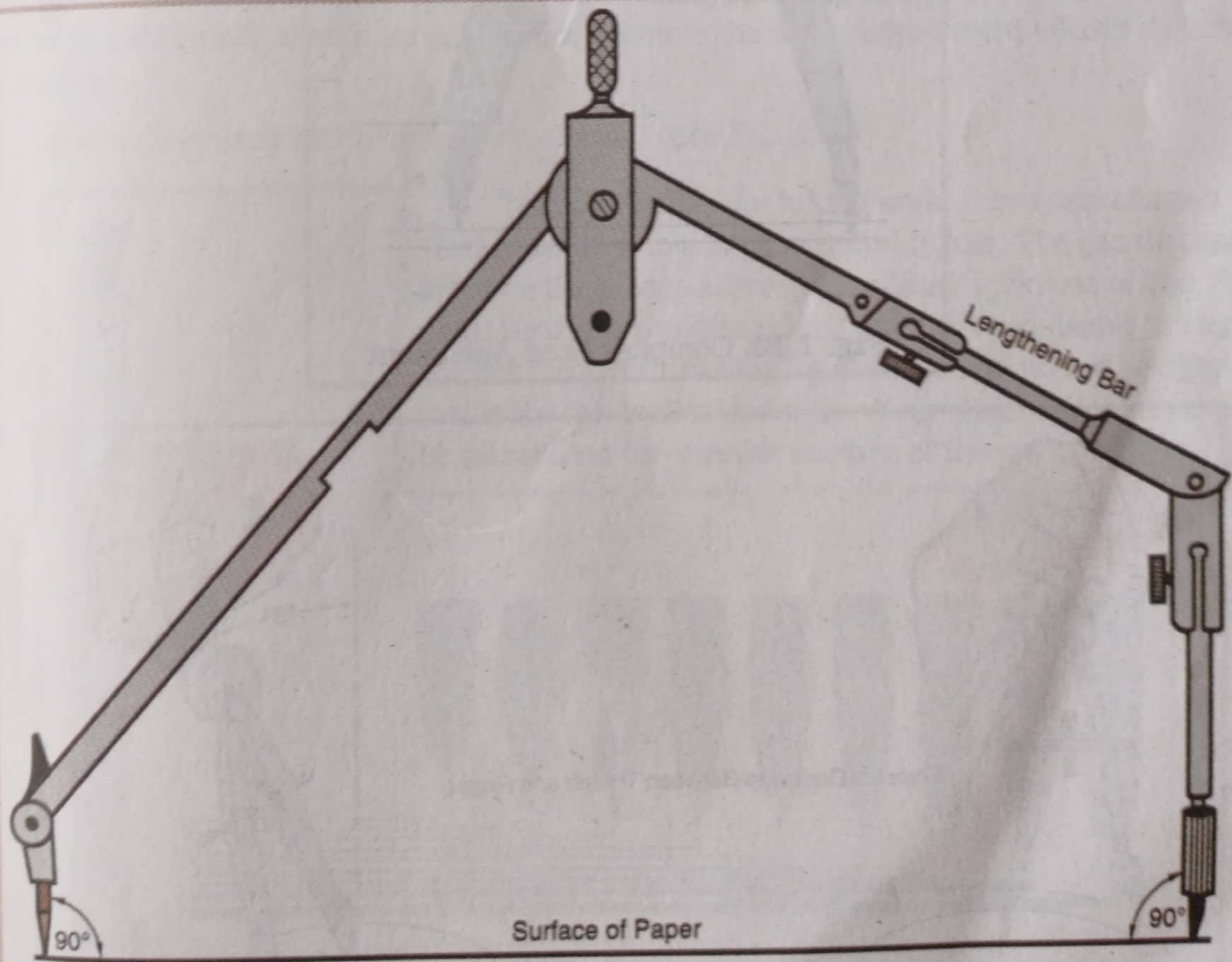


Fig. 1.8. Use of Lengthening Bar

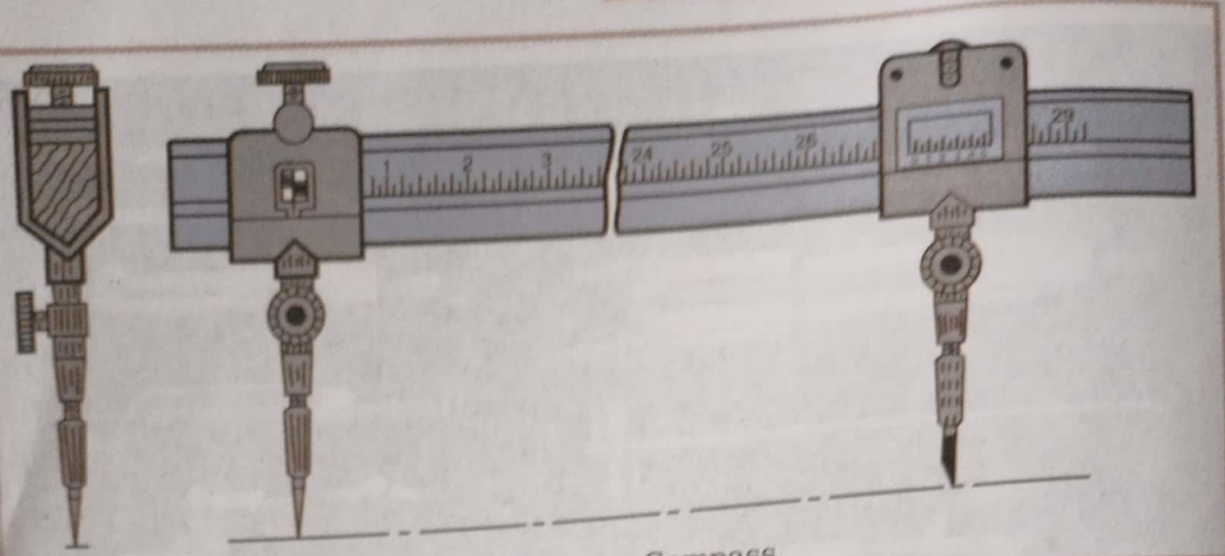


Fig. 1.9. Beam Compass

Smaller circles should be drawn first while drawing concentric circles for better accuracy. One grade lighter lead is used in compass work then usual work for equal darkness of drawing.

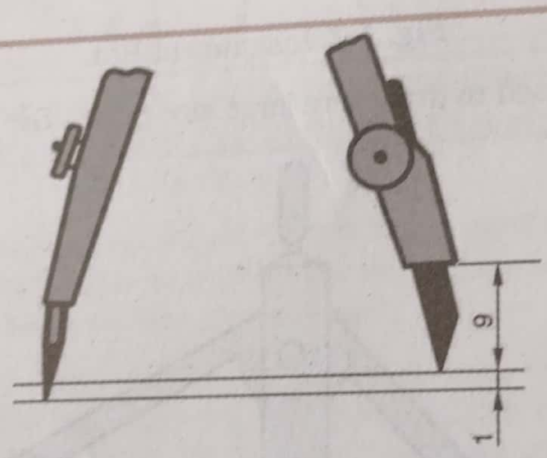


Fig. 1.10. Compass Lead Ajustment

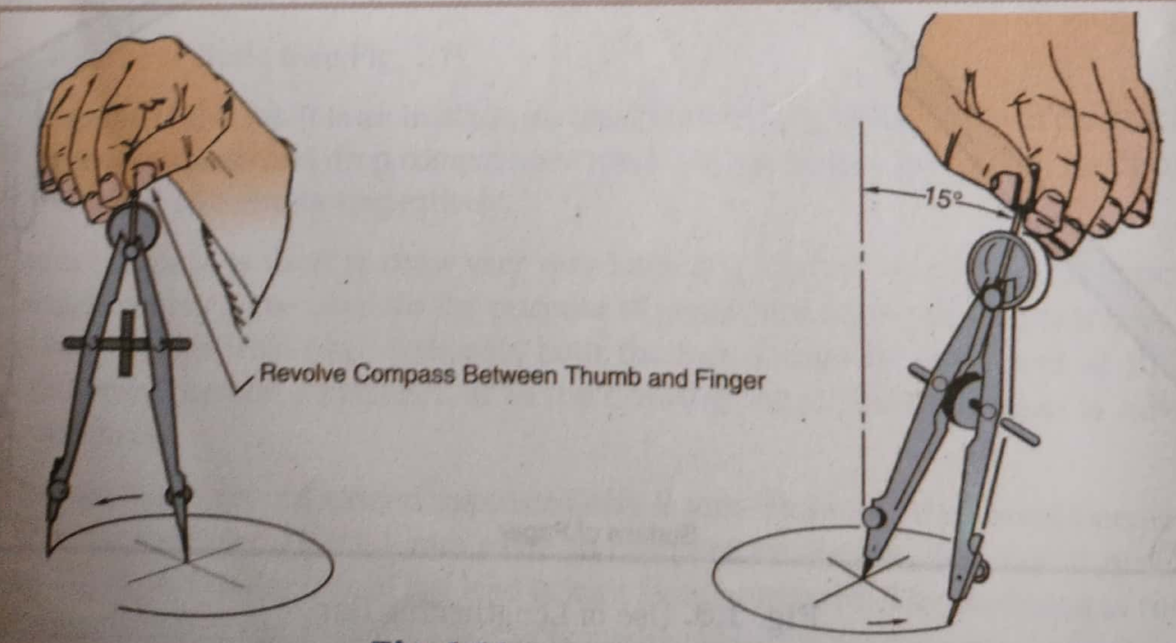


Fig. 1.11. Use of Compass

Divider : (b) and (d) (See Fig. 1.12)

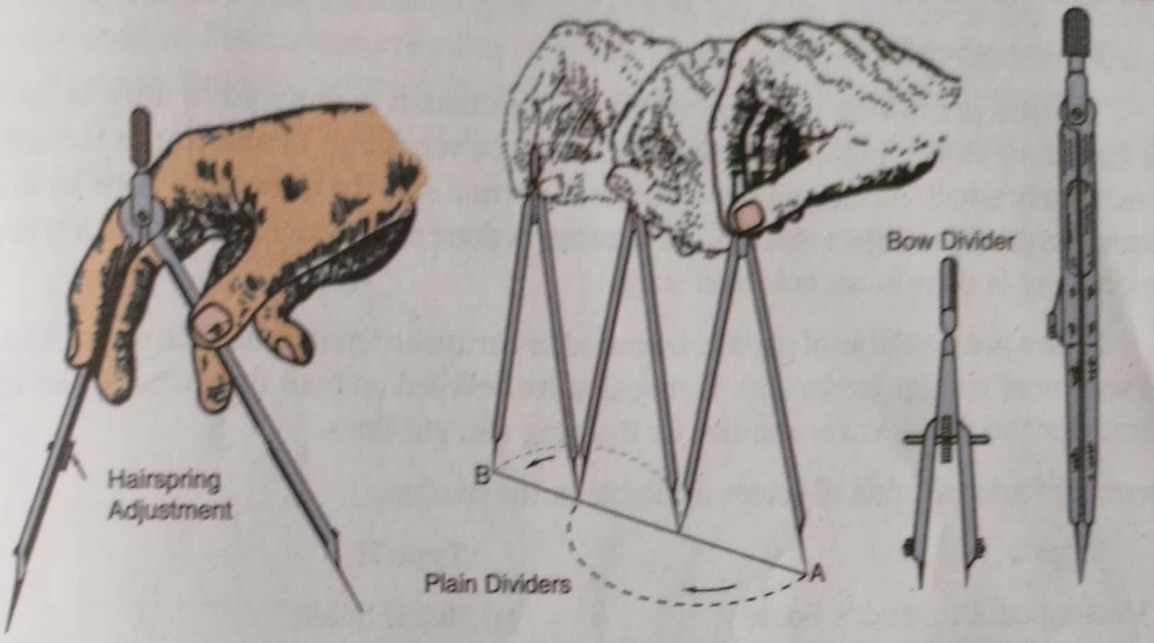


Fig. 1.12. Use of Divider

It is like a compass except it has metal points at legs. Dividers are used to transfer distances or dimensions and sometimes for dividing spaces into equal divisions. The bow divider is particularly useful for exact work because the wheel adjustment effectively holds the setting.

(f) **Ruling Pen and Variant Drawing Pens :** (See Fig. 1.13)

This pen is used for inking work. It consists of a pair of steel nibs fitted to a bone or metal holder. The gap distance between the nibs is adjusted to adjust thickness of line. For clear, neat and uniform inking work, it is advisable to clean the pen frequently during its use. Pen should not be dipped inside the ink bottle. Under no circumstances ink must not be allowed on the outside surface of the nibs.

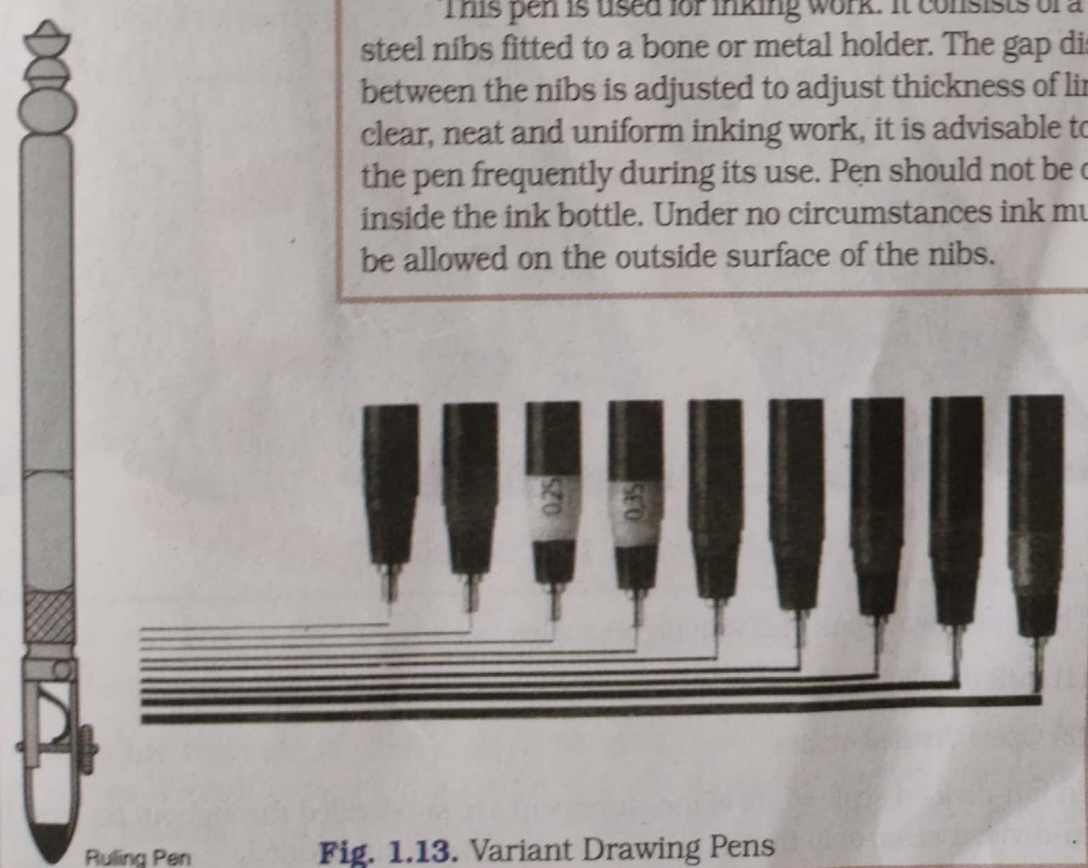


Fig. 1.13. Variant Drawing Pens

Now a days inking pens with different thickness heads are available and they are the only ones which are used. (See variant drawing pens in Fig. 1.13)

8. Scales : (See Fig. 1.14)

The size of drawing paper on which the draftsman is required to draw is limited but the size of the object, building, project etc.., may be very large, or sometimes the object may be extremely small. As long as the object is of normal size the drawing is done by normal or natural scale, If the object is large the drawing is done at a reduced scale and if it is smaller the drawing is done at an enlarged scale.

Scales are available of plastic, boxwood or cardboard materials. The scales have either flat section or triangular section. Flat scales are bevelled on both the sides. Scales are used to transfer the dimensions and not for drawing straight lines.

There are various kinds of scales available in the market.

Type I

(i) Mechanical Engineer's Scale

(ii) Civil Engineer's Scale

(iii) Architect's Scale

Type II

(a) Metric Scale

(b) Inch Scale

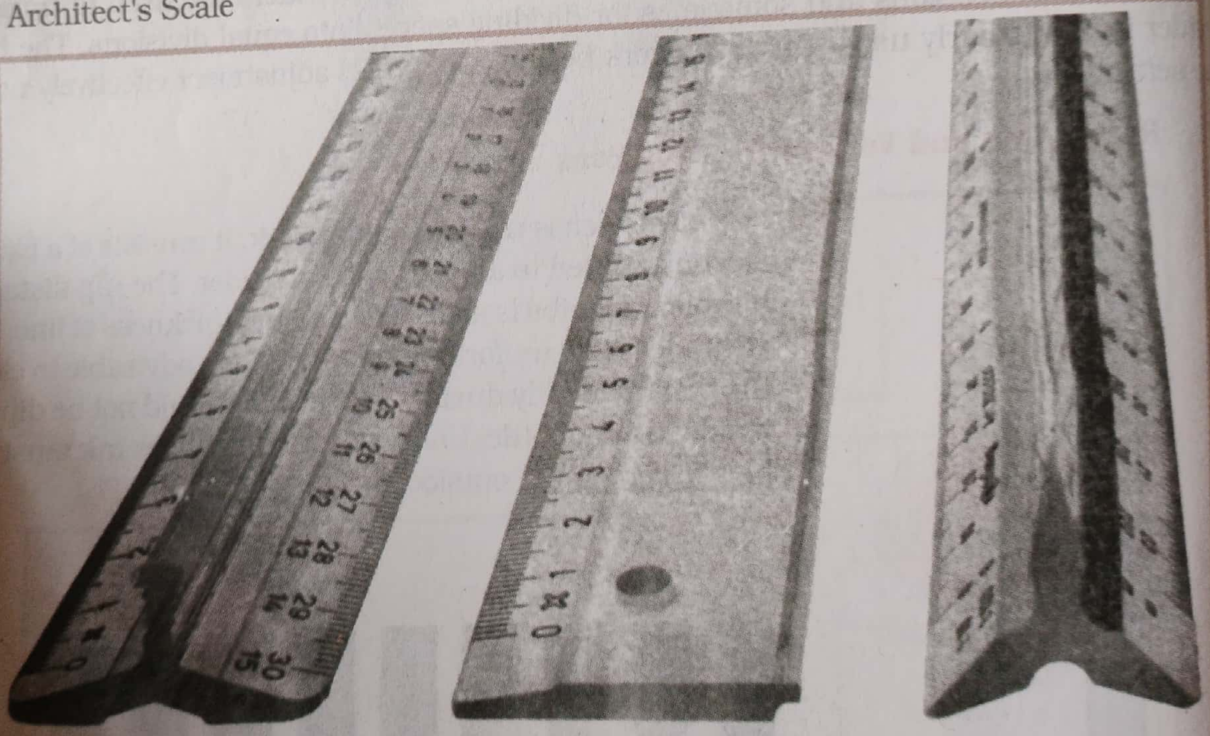


Fig. 1.14. Scales

There are two types of divisions on scales :

(1) Full divided scales

(2) Open divided scales

In full-divided scale units of measurement are subdivided throughout its length while in open-divided scale only first unit of measurement is subdivided.

9. **Protractor :** (See Fig. 1.15)

Protractors are used for the measurement of angles. They are available in semi-circular and circular shapes. Protractors are made of paper, wood, plastic, brass metal or celluloid materials. Transparent protractors with bevelled edge are more often used. Normally it is graduated to ½° or 1° and numbered at an intervals of 10°. For very accurate work protractor with vernier attachment is used.

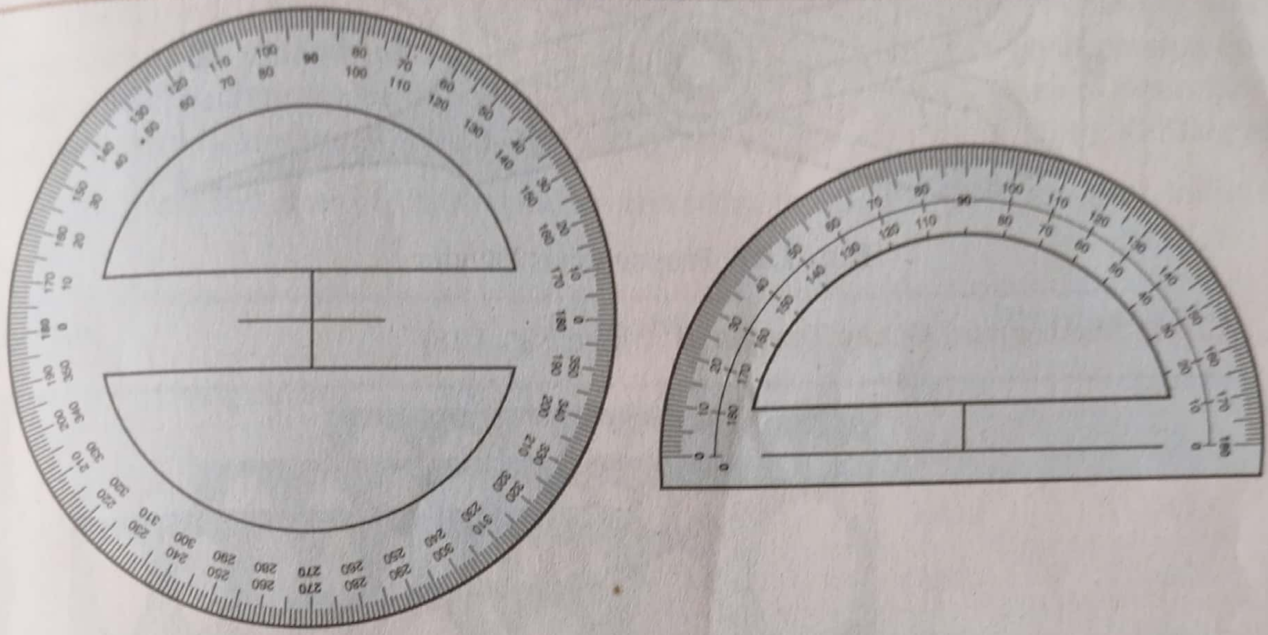


Fig. 1.15. Full and Half Protractors

10. **French Curves :** (See Fig. 1.16)

Fig. 1.16. French Curves

Curves other than arc of circles are drawn accurately and uniformly with the help of french curves. French curves are available in a set of assorted curves. They are made of transparent material. Use of french curves requires sufficient experience, especially in inking work.

11. Proportional Divider : (See Fig. 1.17)

Proportional divider is used to draw drawing at an enlarged or at a reduce scale from the available drawing. It has a sliding adjustable pivot. Pivot position decides the conversion of scale. Scale marks are marked on the proportional divider.

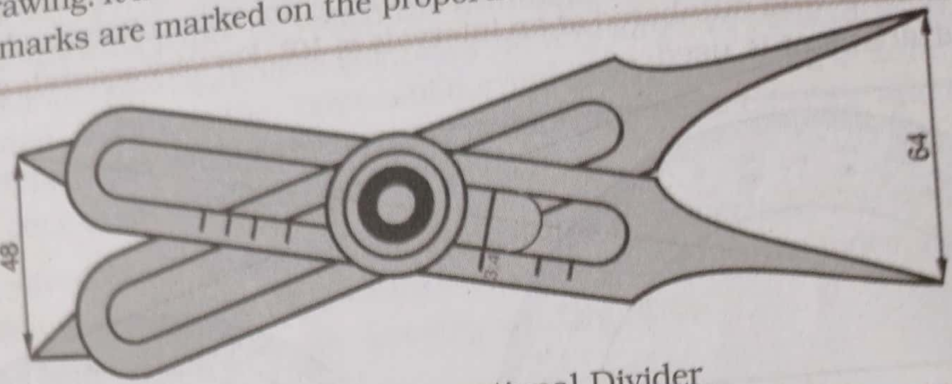


Fig. 1.17. Proportional Divider

12. Circle Master and Other Templates : (See Fig. 1.18)

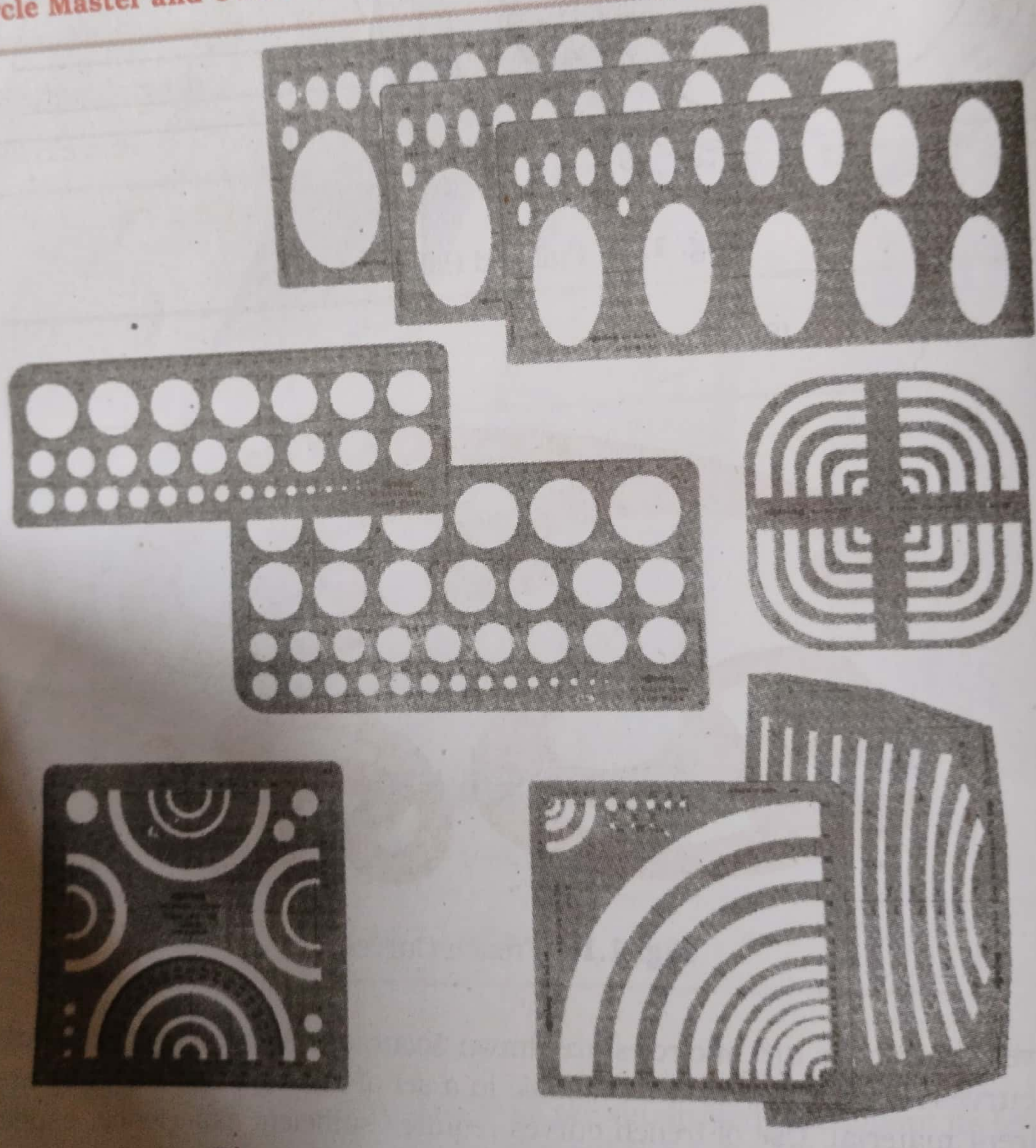


Fig. 1.18. Circle Master and other Templates

Scanned with CamScanner

It is a thin, flat piece of plastic with different sizes of circles cut in it. It is used to speed up the drafting work. Every circle has 4 black line marks on the circumference along two right angle axes. These line marks are used for adjusting circle master on two axes of circle. It is one kind of template. Many such templates are available for ellipses, squares, triangles and other polygons.

13. Drafting Paper :

Hand made paper with smooth surface is used for the purpose of drawing. Drawing paper should be uniform and as white as possible. It should have good erasing quality. A paper with sufficient grains to produce sharp and clean pencil lines is desirable. It is available in rolls or sheets. Weight of drawing paper per rim is about 25 to 30 Kilograms.

The preferred sizes of drawing sheets according to I. S. 696-1972 are given in the table below :

Sr. No.		Trimmed		Untrimmed	
	Designation	Width	Length	Width	Length
1.	A_0	841	1189	880	1230
2.	A_1	594	841	625	880
3.	A_2	420	594	450	625
4.	A_3	297	420	330	450
5.	A_4	210	297	240	330
6.	A_5	148	210	165	240

14. Set of Pencils :

The main two ingredients in the pencil leads are graphite and clay. Pencils are available in 20 grades, 8B softest to 10H hardest. Hardness and softness of the pencils depend upon the proportions of graphite and clay. Soft pencils produce dark black lines while hard pencils produce thin gray lines. Softer pencils have larger size leads while harder pencils have smaller size leads. Even though the hardness number of the pencils may be same, hardness differ with different manufacturer and hence it is advisable to use pencils of the same manufacturer.

Now a days refill pencils or push pencils are used. Lead size in these pencils is generally 0.5 mm. Leads for these pencils are available in different grades. It is more convenient compared to usual wooden pencils. See Fig. 1.19 Grades of pencils and their uses are shown in the Fig. 1.20.

Fig. 1.19. Push Pencil

Different shapes of the pencil points are shown in Fig. 1.21. The conical point is used for general drafting and lettering. The wedge point is used for drawing uniform long straight lines. The bevel shape point is used in compass work.

15. Pencil Sharpener :

Knife edge can be used as a pencil sharpener. Pencil sharpening machines are also available and they are used in drawing design offices.

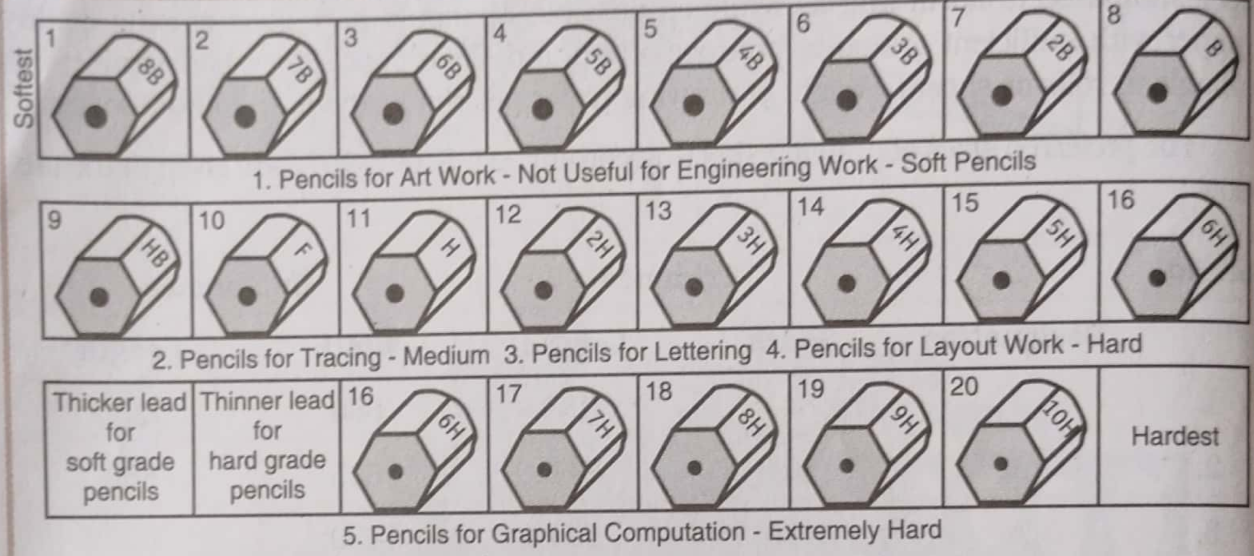


Fig. 1.20. Set of Pencils

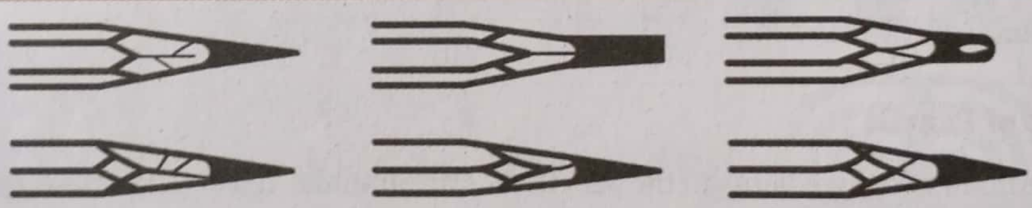


Fig. 1.21. Pencil Points

16. Sand Paper Block : (See Fig. 1.22)

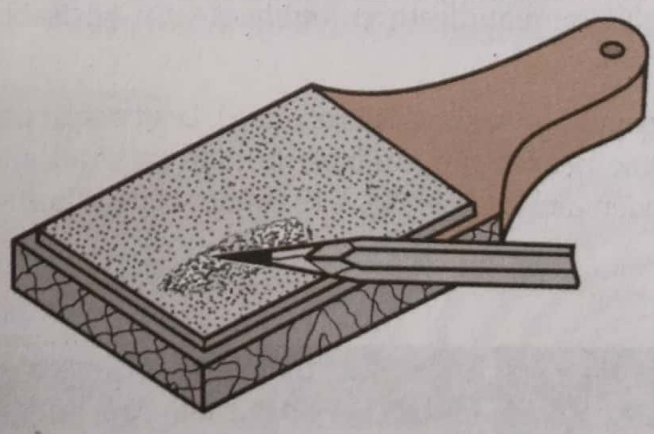


Fig. 1.22. Sand-paper Block

Sand paper block is used for sharpening lead points of desired shapes. It is a wooden block 50 × 50 × 12 mm size with sand paper pasted or nailed over it. Sand paper can be replaced when it warns out. Don't sharpen leads near or over drawing as the graphite powder will smear the drawing surface.

17. Drawing Pins, Clips and Adhesive Tapes : (See Fig. 1.23)

Drawing paper is fixed on the drawing board by pins or clips or adhesive tapes. Steel pins with brass tops are available in the market. Disadvantage of these pins is that they make holes in the drawing paper and spoil the surface of the board.

Adhesive tapes are available in rolls. Adhesive tapes do not make any hole in the paper as well as on the board.

Stainless steel elastic clips are available in the market. It is used extensively because it holds the paper without sticking to it or without making hole on it.

18. Duster or Handkerchief :

It is nothing but a piece of clean cloth used to clean drawing paper and instruments. Don't try to clean a drawing surface by hand as it tends to smudge the drawing. It is used to remove loose graphite and eraser crumbs from the drawing surface.

19. Eraser-Erasing Shield : (See Fig. 1.24)

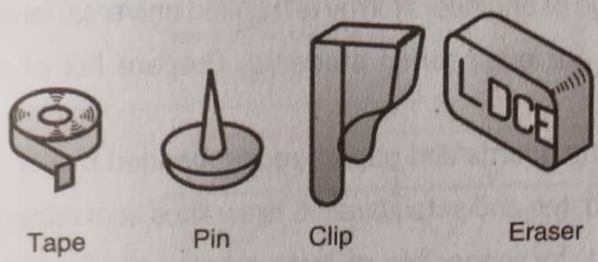


Fig. 1.23

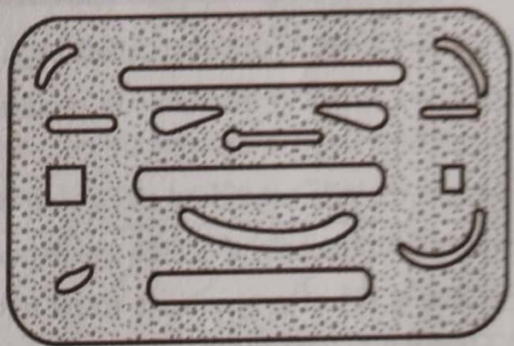


Fig. 1.24. Eraser and Erasing Sheild

Pencil eraser, what we call it as rubber, are available in many varieties. Always use good quality soft rubber so that due to its use the surface of drawing paper does not get damaged. To restrict the erasing area pencil rubbers are used.

The erasing shield is a very convenient device for erasing unwanted line while protecting others. It also prevents paper from possibility of damage.

The electric eraser machine is commonly used in large drawing offices. The machines are available with interchangeable pencil and ink eraser points.

20. Roll-N-Draw Ruler : (See Fig. 1.25)

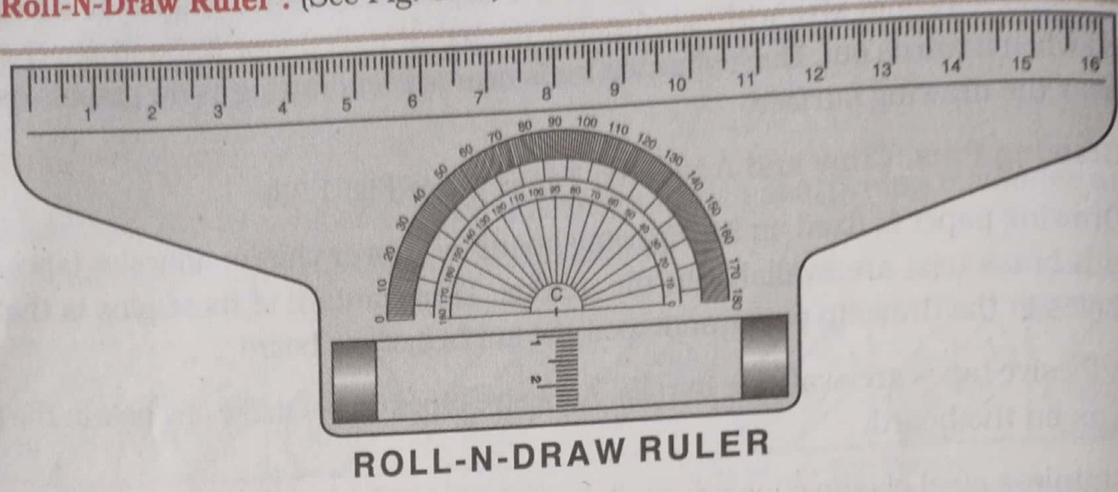
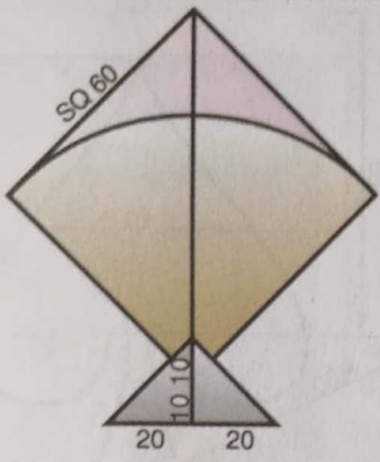


Fig. 1.25. Roll-N-Draw Ruler

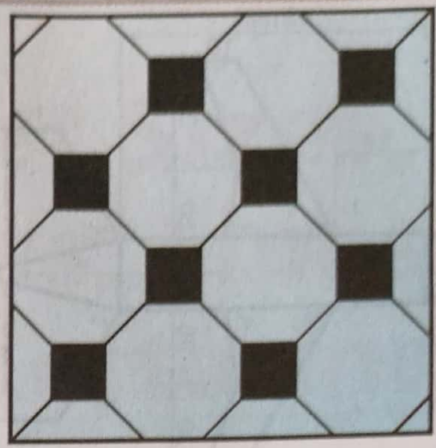
With the help of this ruler parallel lines are drawn quickly and very easily. Straig edge of the ruler is kept on a given / known line and then it is rolled to new required locati and then parallel line to the prior line is drawn. This way parallel horizontal, vertical inclined lines are drawn faster with ease. Used for graphical solutions of problems in differe subjects.

EXERCISE

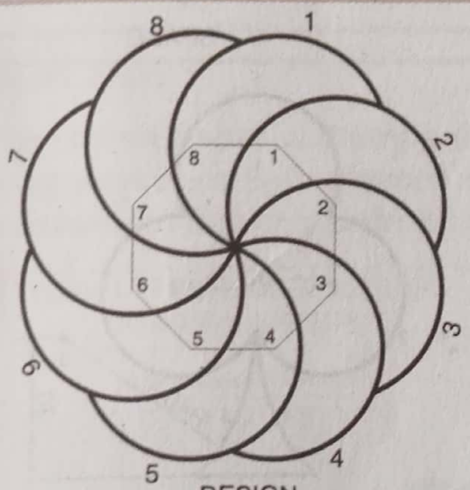
1. Why is engineering drawing called language of engineers? Why is it called universal language
2. Enlist the drawing instruments required for engineering students. Prepare list of speci equipments used in drawing office.
3. Prepare table for standard sizes of drawing boards and papers recommended by I.S.
4. Explain with the help of sketch use of board, tee and setsquares to draw lines in multiple of 1
5. Prepare a list of precautions to be taken, for longer life of instruments, and for prepari accurate and neat drawing.
6. Classify available pencils and specify the uses of different grade pencils.
7. Name different compasses, specifying uses of each one.
8. Draw different types of pencil points. Specify uses of each one.
9. What are french curves and how are they used?
10. What are templates and how they help in preparation of drawing?
11. Why are scales needed?
12. Explain the use of proportional divider.
13. Compare pair of Tee-Set square with parallel straight-edge and drafting machine.
14. Explain the uses of a divider.
15. What is an eraser and what is an eraser shield?
16. Redraw the figures given below to practice proper use of drawing equipments. Use appropria equipment to do the work accurately, quickly and correctly. Assume dimension if missing.



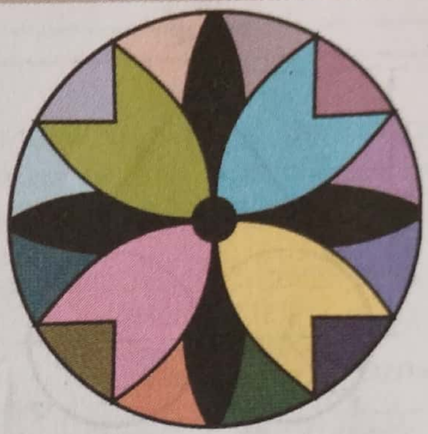
KITE



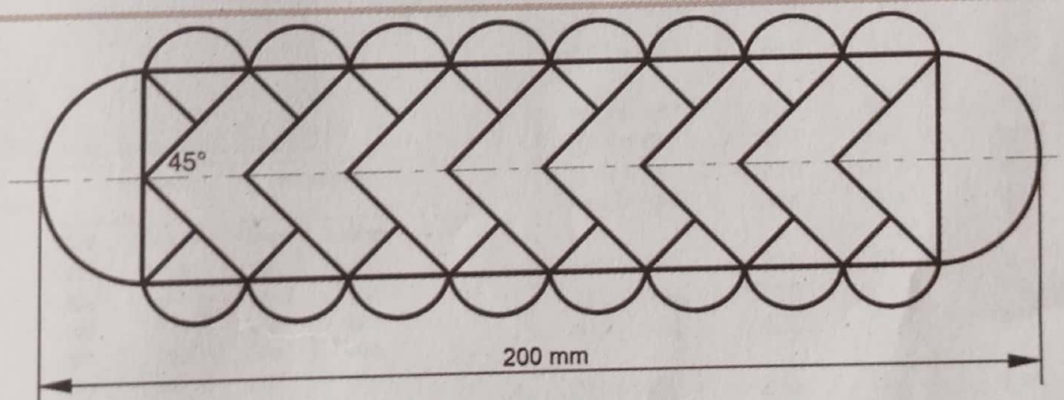
MARBEL DESIGN



DESIGN

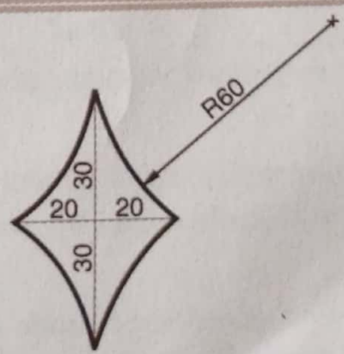


RANGOLI

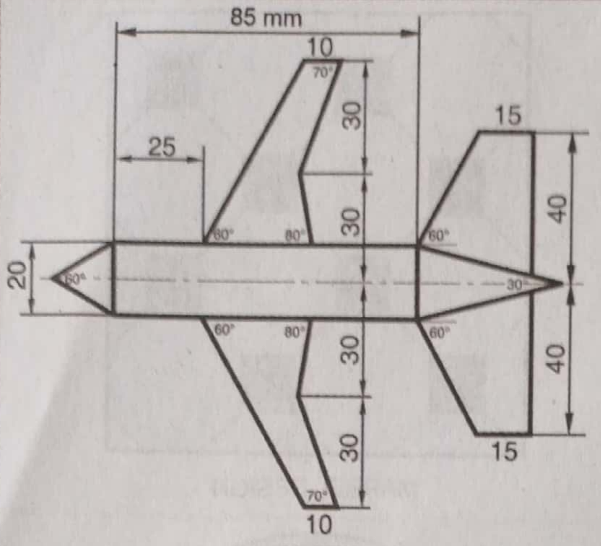


DESIGN

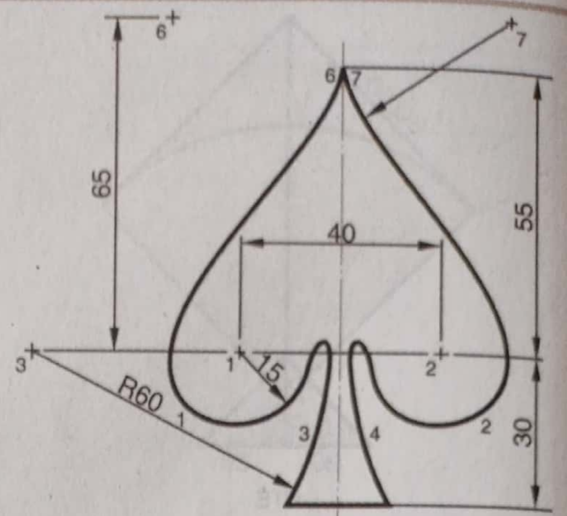
BORDER DESIGN



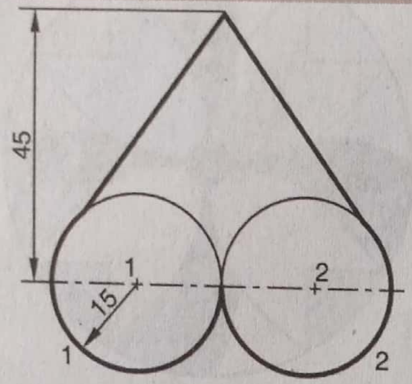
DIAMOND



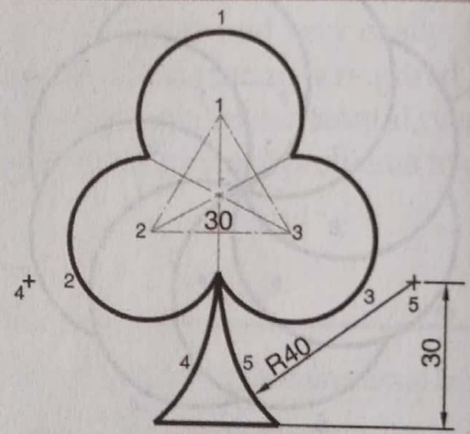
PLANE



SPADE



HEART



CLUB

CHAPTER - 2

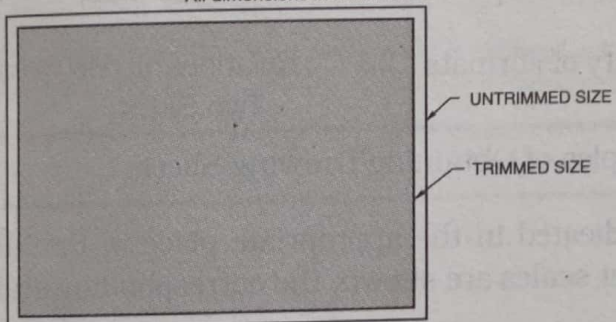
Sheet Sizes, Scales, Lines and Lettering

1. Sheet Sizes :

The preferred sizes of drawing sheets are given in Table-1 below. The second choice of drawing sheet sizes, both trimmed and untrimmed are given in Table-2 below. However, it is recommended that only preferred sizes should be used as far as possible.

Table 1 : PREFERRED SIZES OF DRAWING SHEETS

All dimensions in millimetres.



SHEET DESIGNATION	TRIMMED		UNTRIMMED	
	Width	Length	Width	Length
A0	841	1189	880	1230
A1	594	841	625	880
A2	420	594	450	625
A3	297	420	330	450
A4	210	297	240	330
A5	148	210	165	240

Table 1

Table 2 : SECOND CHOICE OF DRAWING SHEET SIZES

All dimensions in millimetres.

SIZE DESIGNATION	TRIMMED		UNTRIMMED	
	Width	Length	Width	Length
594 × 1189	594	1189	625	1230
420 × 1189	420	1189	450	1230
297 × 1189	297	1189	330	1230
210 × 1189	210	1189	240	1230
841 × 841	841	841	880	880
420 × 841	420	841	450	880
297 × 841	297	841	330	880
210 × 841	210	841	240	880
594 × 594	594	594	625	625
297 × 594	297	594	330	625
210 × 594	210	594	240	625
420 × 420	420	420	450	450
210 × 420	210	420	240	450
297 × 297	297	297	330	330

Table 2

Wherever necessary, sizes of sheet with length 1189 m.m., may be further extended by steps of 210 m.m. on the length only.

In arriving at the trimmed size of drawing sheets, the following basic principles, which have been dealt with in detail in IS : 1064-1961 'Specification for paper sizes' have been taken into consideration.

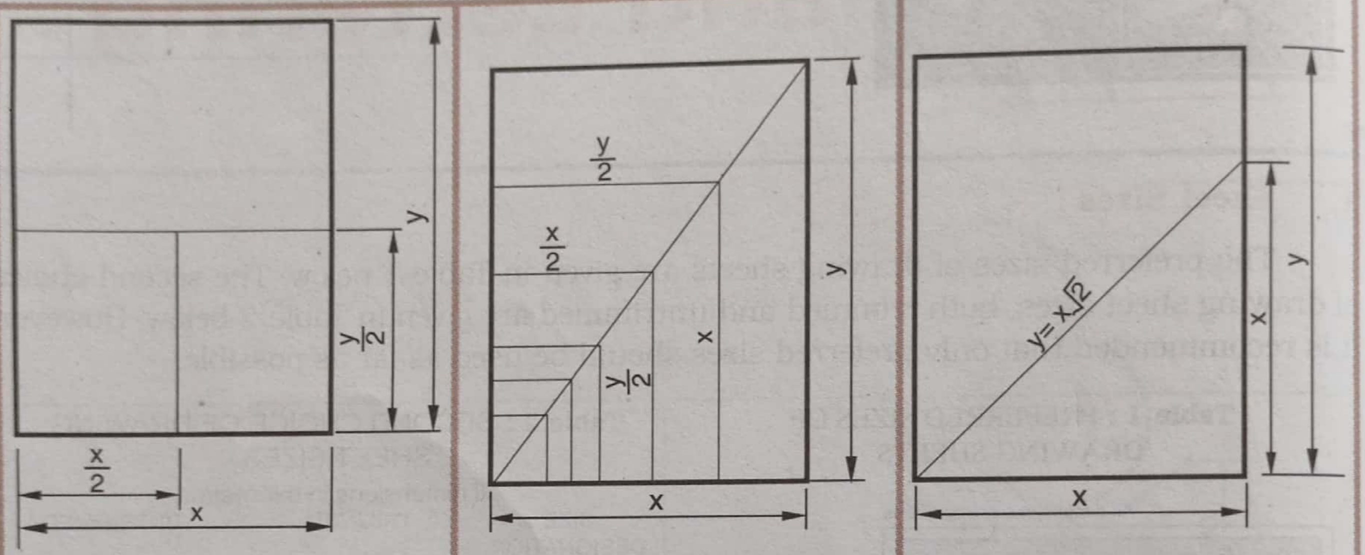
(a) Two successive preferred sizes of drawing sheets are obtained either by halving or doubling. (See Fig. 2.1A) Consequently the surface areas of two successive preferred sizes are in ratio of 1 : 2.

(b) The formats or forms of preferred sizes are geometrically similar to one another, the sides of each size being in ratio of $1 : \sqrt{2}$. (See Fig. 2.1B and Fig. 2.1C)

(c) The surface area of the basic size AO is one square metre.

2. Scales :

All drawings should be drawn to a scale. In general, the largest scale conveniently possible should be adopted. Drawings of parts or assemblies drawn larger than full size should, where practicable, include an undimensional view to actual size.



2.1 A : Halving and Doubling | 2.1 B : Similarity of Formats | 2.1 C : Relationship Between Two Sides

Fig. 2.1. Illustration of Basic Principles of Obtaining Drawing Sheets

The scale or scales of drawings should be indicated in the appropriate place in the title block. If more than one detail drawn to different scales are shown, the corresponding scale should be shown under each relevant detail.

Scales recommended for use in engineering drawings are given below :

Full scale	Reduced scale		Enlarged scale
1 : 1	1 : 2	1 : 20	10 : 1
	1 : 2.5	1 : 50	5 : 1
	1 : 5	1 : 100	2 : 1
	1 : 10	1 : 200	

3. Lines :

For general engineering drawings, the types of lines shown in Fig. 2.2 should be used. All lines should be sharp and dense to obtain good reproduction.

The thickness of lines should be chosen according to the type and size of drawing. The line group is identified by the thickest line. For a given view or section, the lines employed should be chosen from one of the lines of the group. (See Fig. 2.3)

Centre lines should project for a short distance beyond the outline to which they refer but, where necessary, to aid dimensioning or to correlate views, they may be extended.

Hidden lines to show interior or hidden surfaces should be included only where their use definitely assists in the interpretation of the drawing. An example of the use of various types of lines is shown in Fig. 2.4

Where drawings are to be reproduced to a smaller size by photographic process, the thickness of lines in the originals should be suitably accentuated to ensure sufficient legibility and clarity after reduction.

	Type	Illustration	Application
A	Continuous thick		Visible outlines
B	Continuous thin		Dimension lines, leader lines, extension lines, construction lines, outlines of adjacent parts, hatching and revolved section
C	Continuous thin-wavy		Irregular boundary lines, short break lines
D	Short dashes medium		Hidden outlines and edges
E	Long chain thin		Centre lines, locus lines, extreme positions of the moveable parts, parts situated in front of the cutting planes and pitch circles
F	Long chain thick at ends and thin elsewhere		Cutting plane lines
G	Long chain thick		To indicate surfaces which are to receive additional treatment
H	Ruled line and short zigzag thin		Long break lines

Fig. 2.2. Types of lines

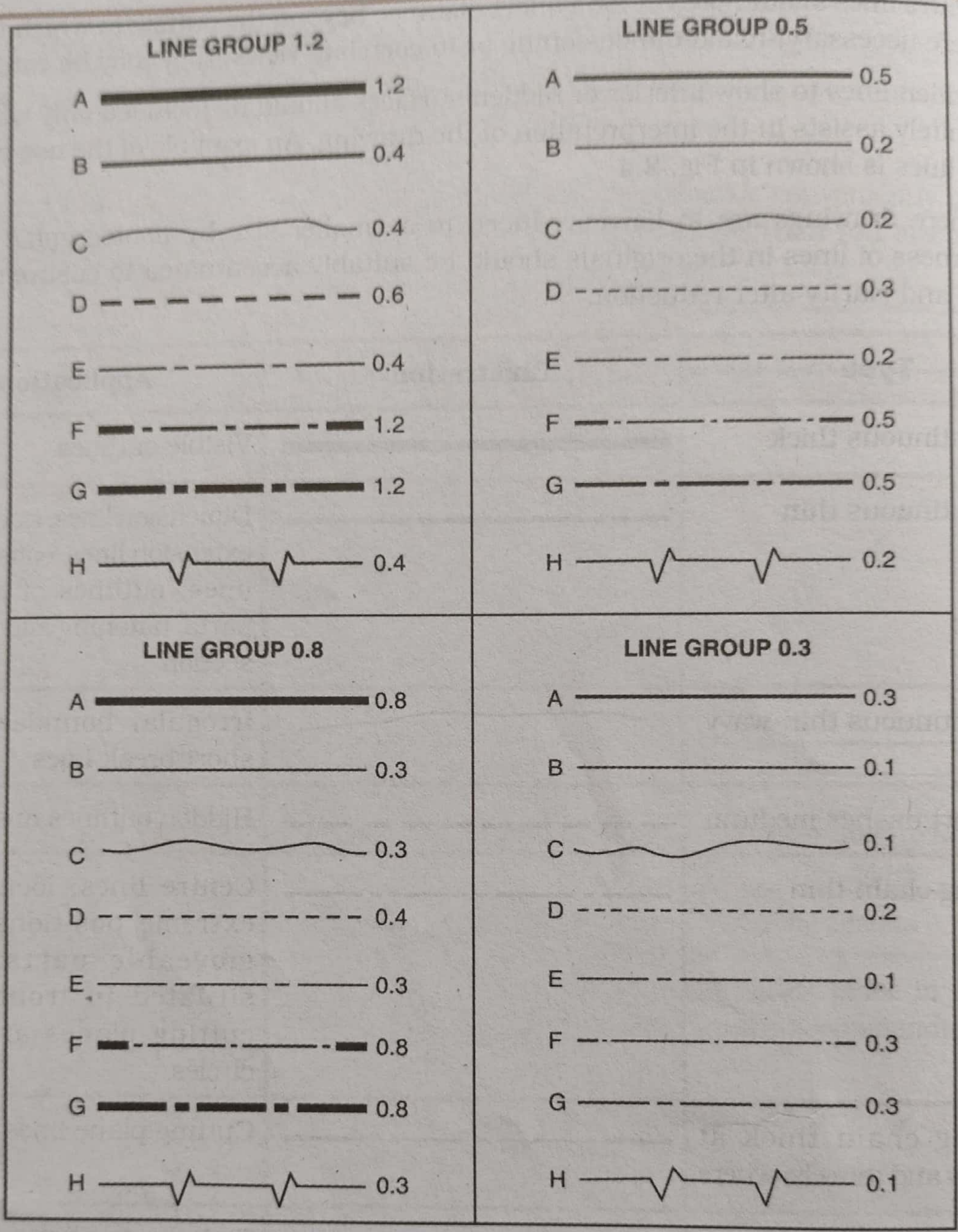


Fig. 2.3. Comparative Thickness in mm of Various Types of Line Groups

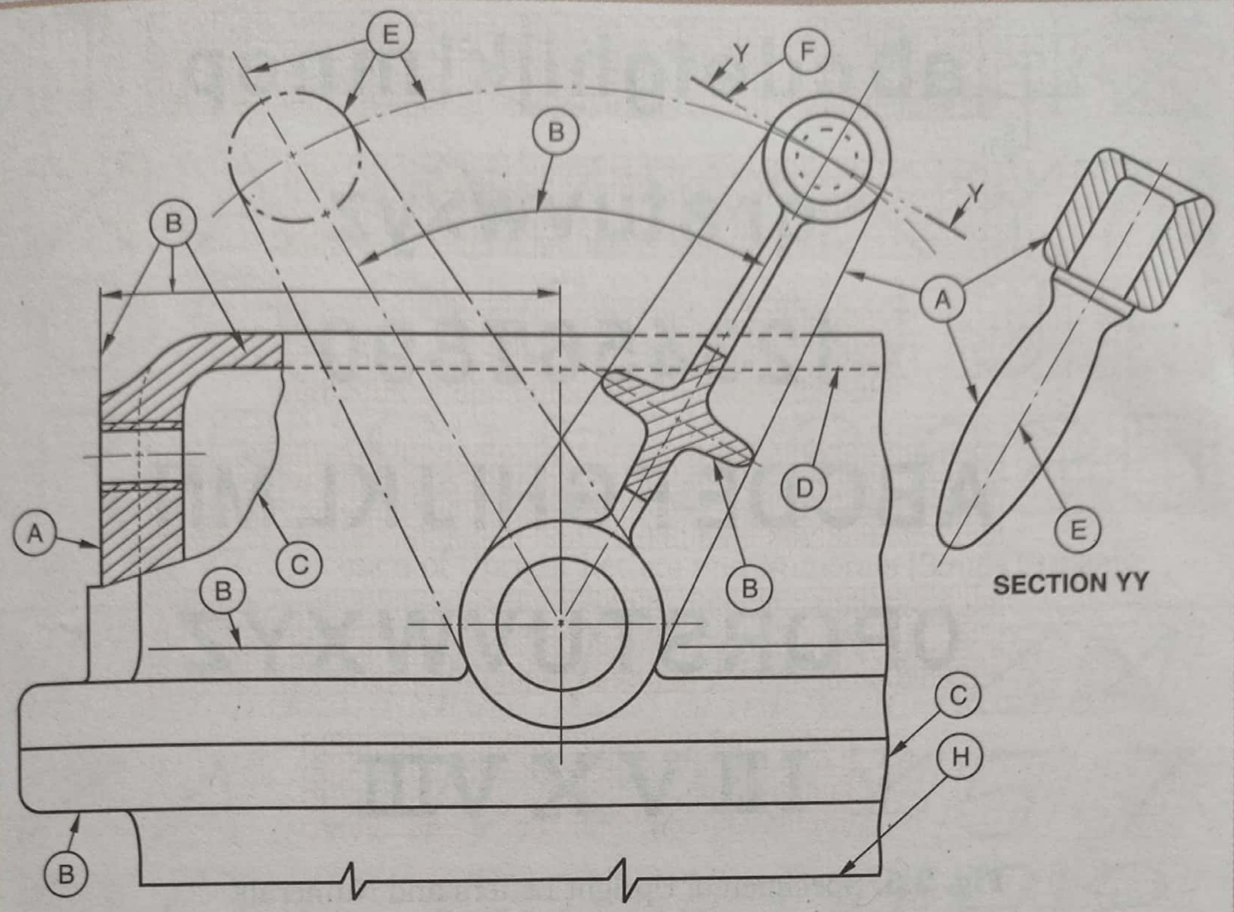


Fig. 2.4. Example for the Use of Various Types of Lines

4. Lettering :

The main requirements for 'lettering' on engineering drawings are legibility, uniformity, ease and rapidity in execution.

Both the upright and inclined letters and numerals are suitable for general use. All letters should be capitals, except where lower case letters are accepted internationally for abbreviations. If inclined letters are used, an inclination of 75° to the horizontal is recommended. Recommended specimens of letters and numerals for upright and inclined types are given in Figs.2.5 to 2.10 Fig.2.10 represents Hindi letters specified and recommended by IS : 69 6 - 19 72.

Letters and numerals are designated by their heights. Recommended sizes of letters and numerals to suit different purposes are given in Table - 3 below. However, actual sizes used depend upon the size of the drawing and purpose for which it is intended.

Lettering should be done on the drawing in such a manner that it may be read when the drawing is viewed from the bottom edge except where it is used, for dimensioning purposes.

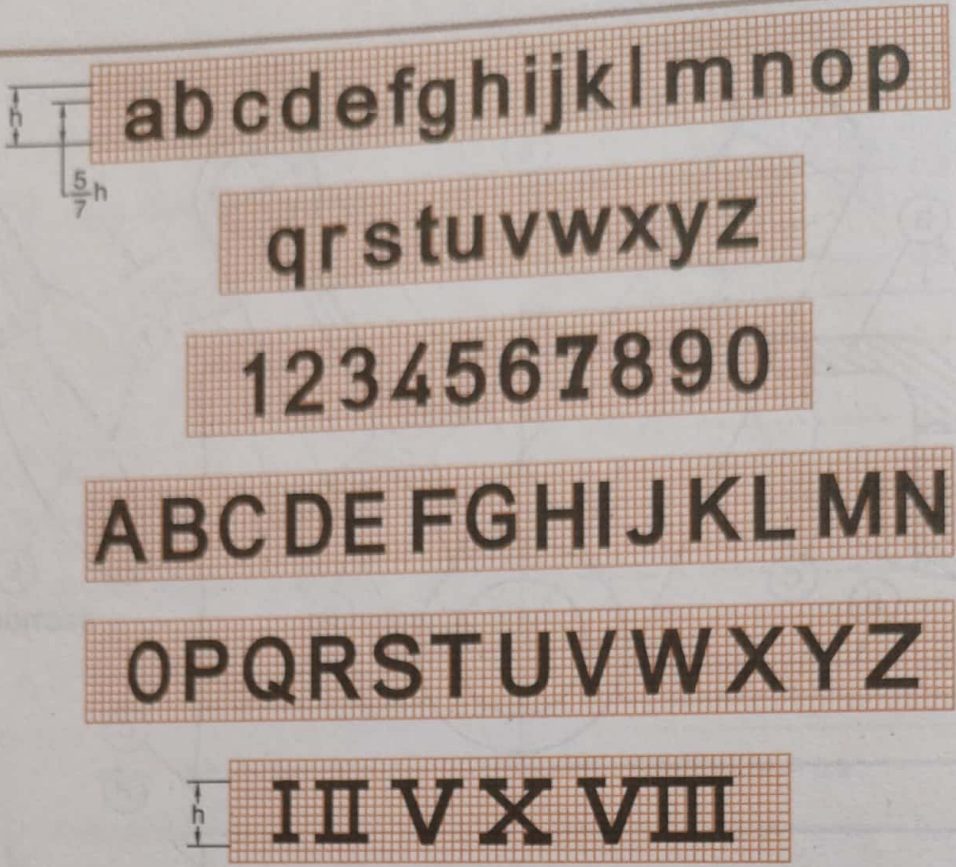


Fig. 2.5. Specimen of Upright Letters and Numerals

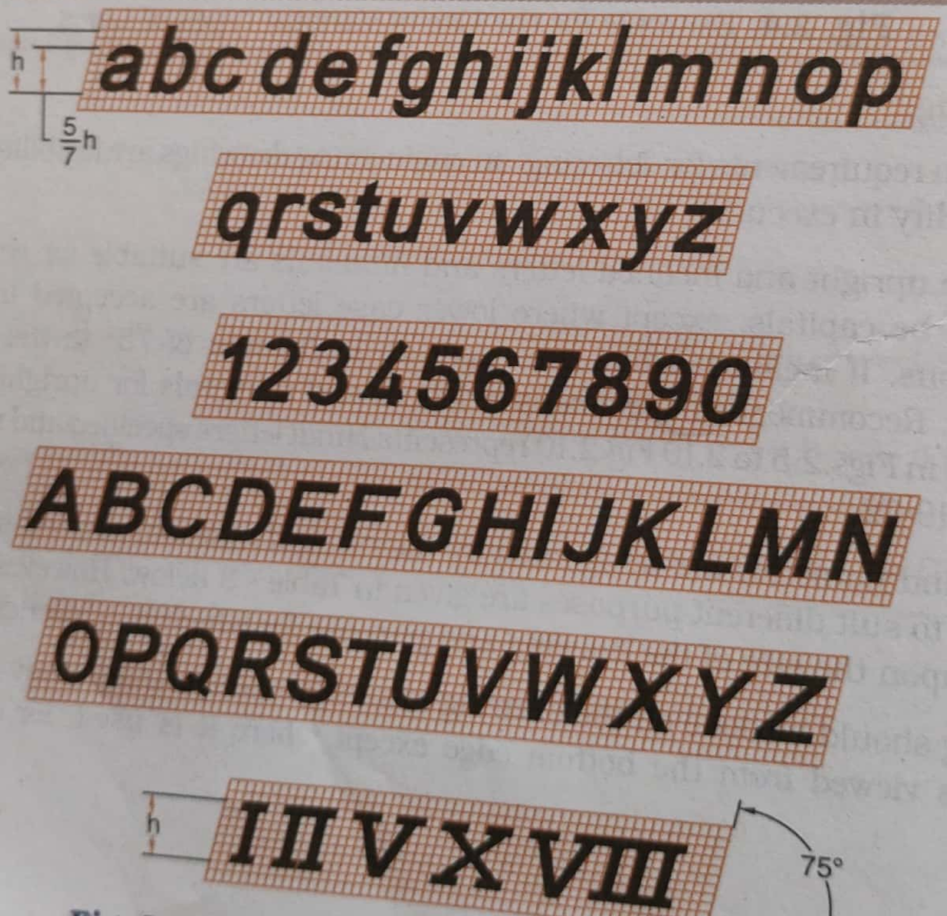


Fig. 2.6. Specimen of Inclined Letters and Numerals

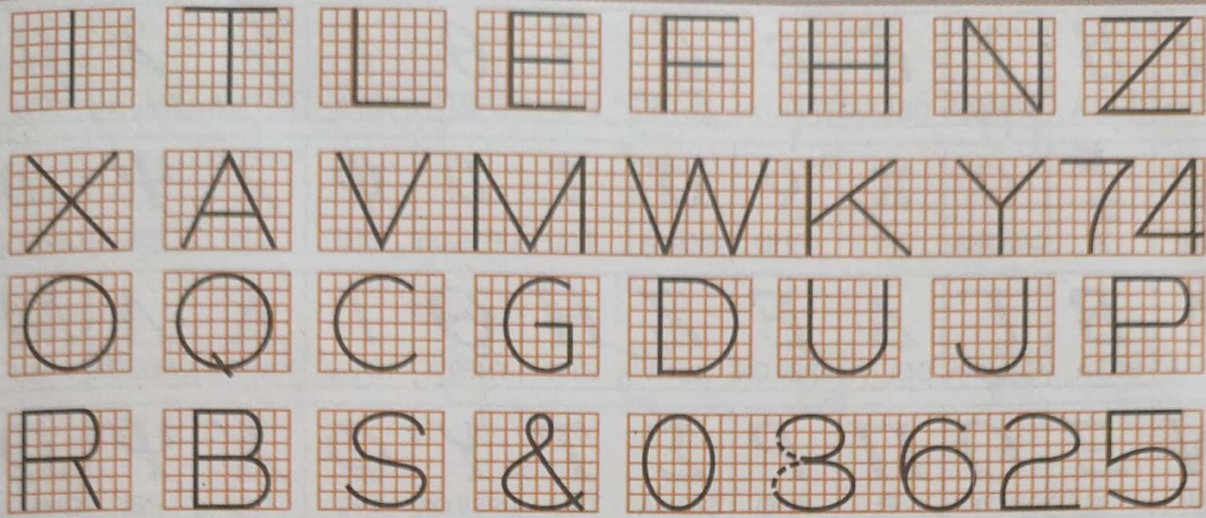


Fig. 2.7. Specimen of Upright Letters and Numerals [Single Stroke]

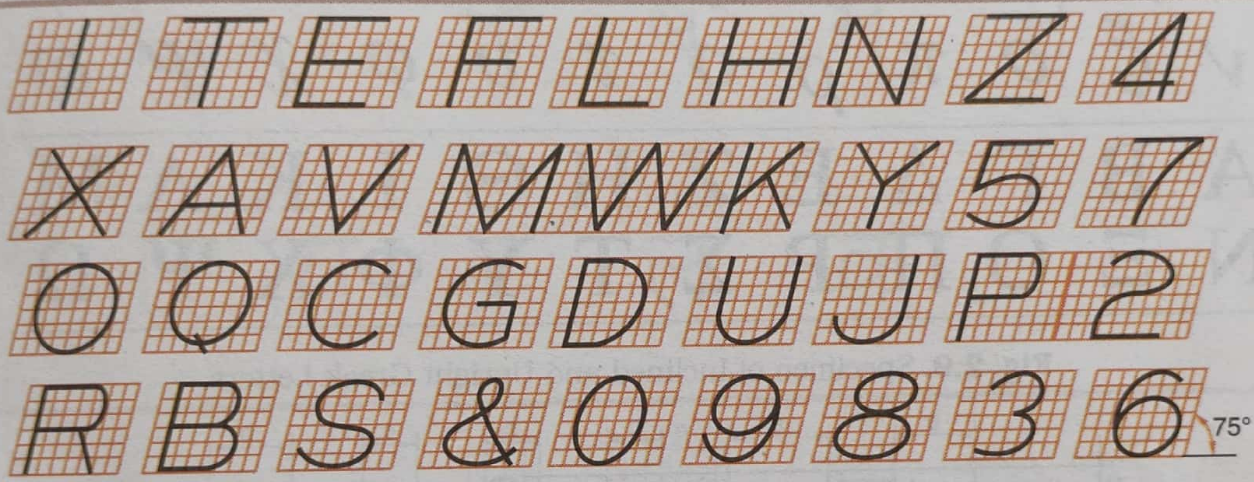


Fig. 2.8. Specimen of Inclined Letters and Numerals [Single Stroke]

Table 3 : Recommended Sizes of Letters and Numerals

Item	Size h, m.m.
Drawing number in title block and letters denoting cutting plane section	10, 12
Title of drawing	6, 8
Sub - Titles and headings	3, 4, 5, 6
Notes, such as legends, schedules, material list, dimensioning	3, 4, 5
Alteration entries and tolerances	2, 3

When drawings are to be reproduced to a smaller scale by photographic process, the size of letters in original drawing should be suitably accentuated to ensure sufficient legibility and clarity after reduction.

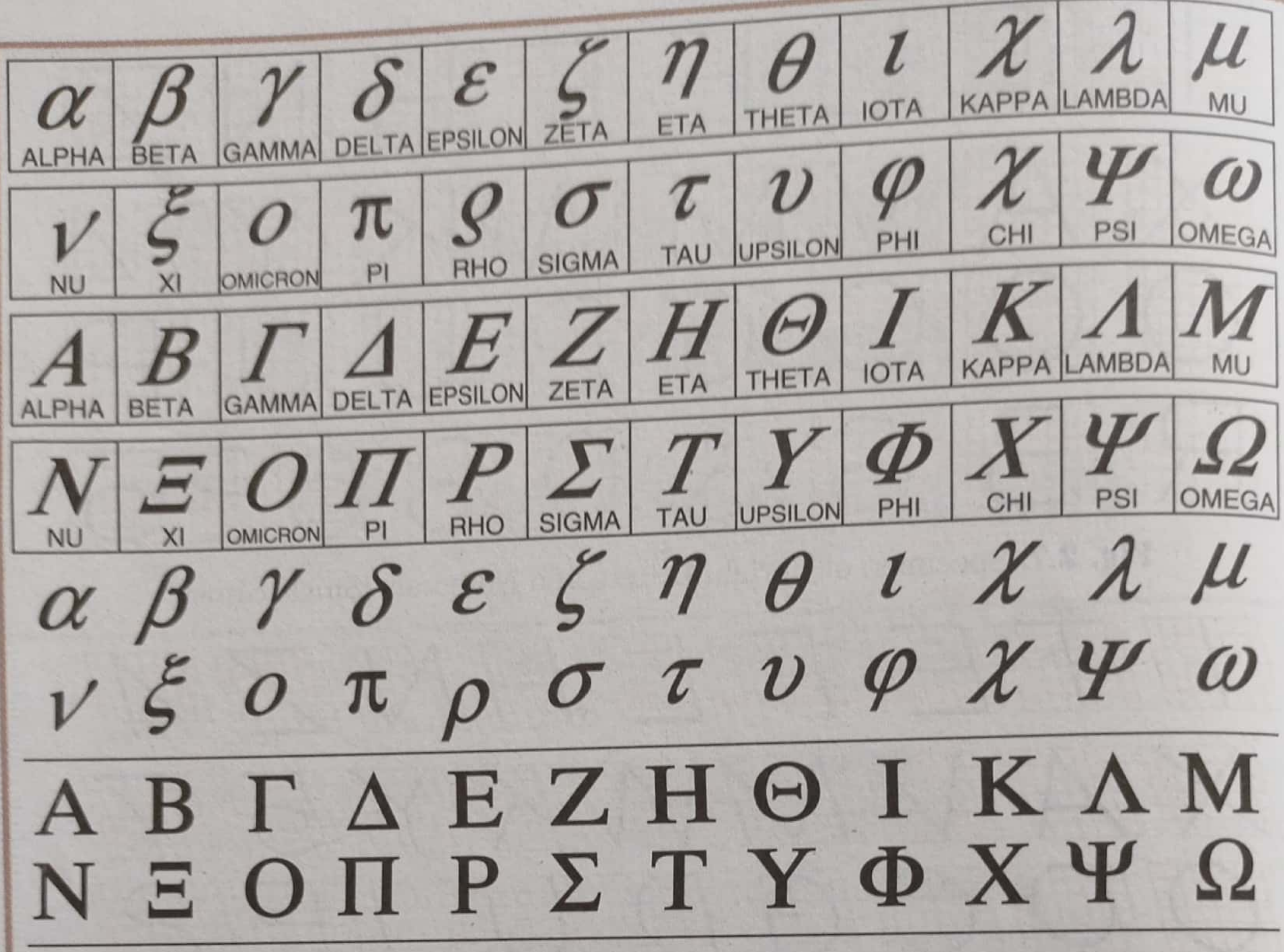
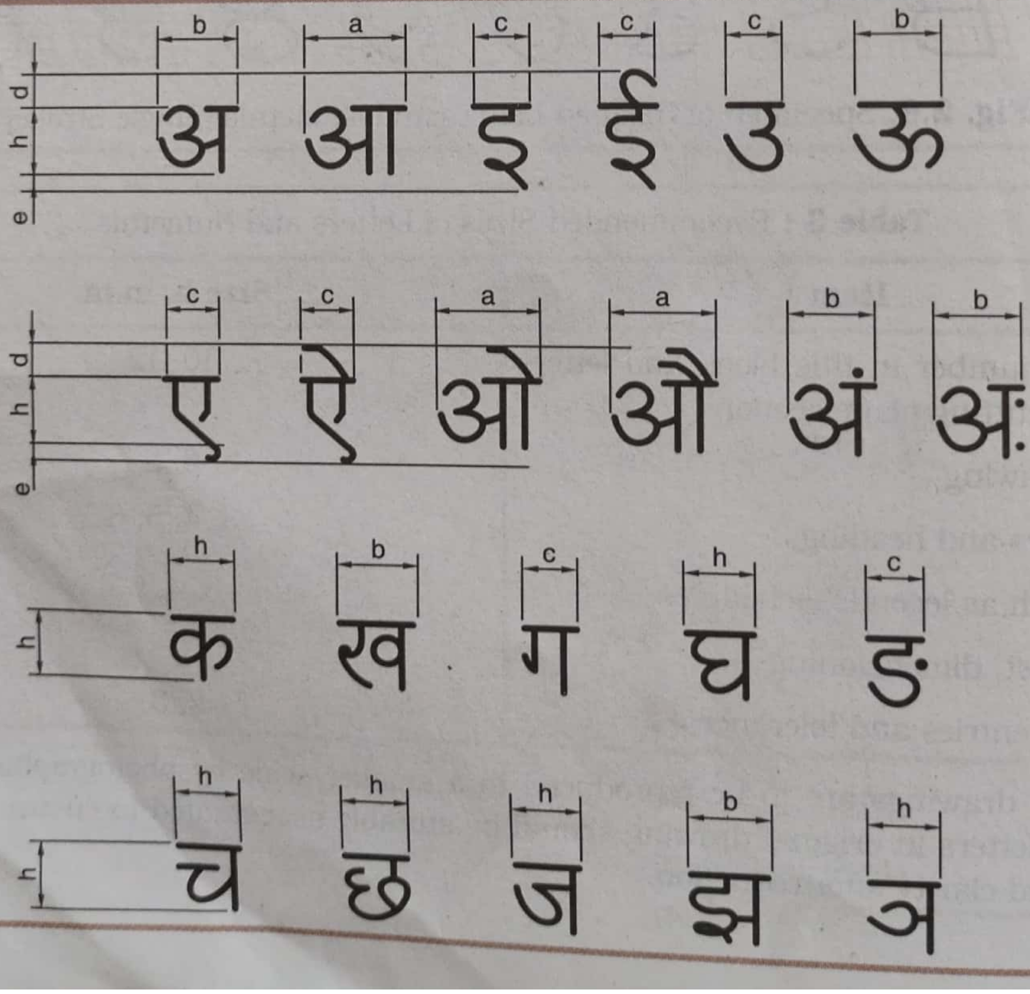


Fig. 2.9. Specimen of Inclined and Upright Greek Letters



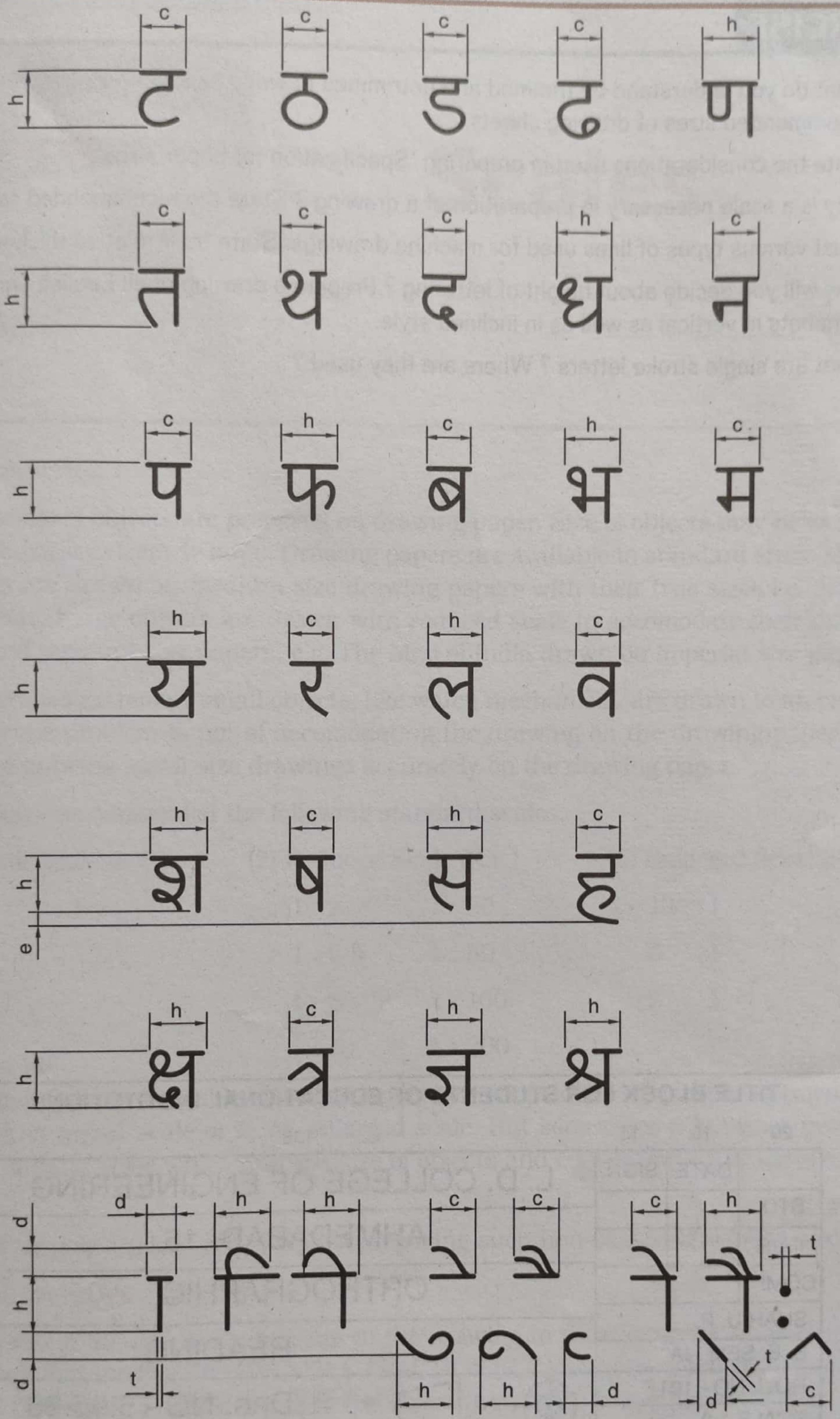
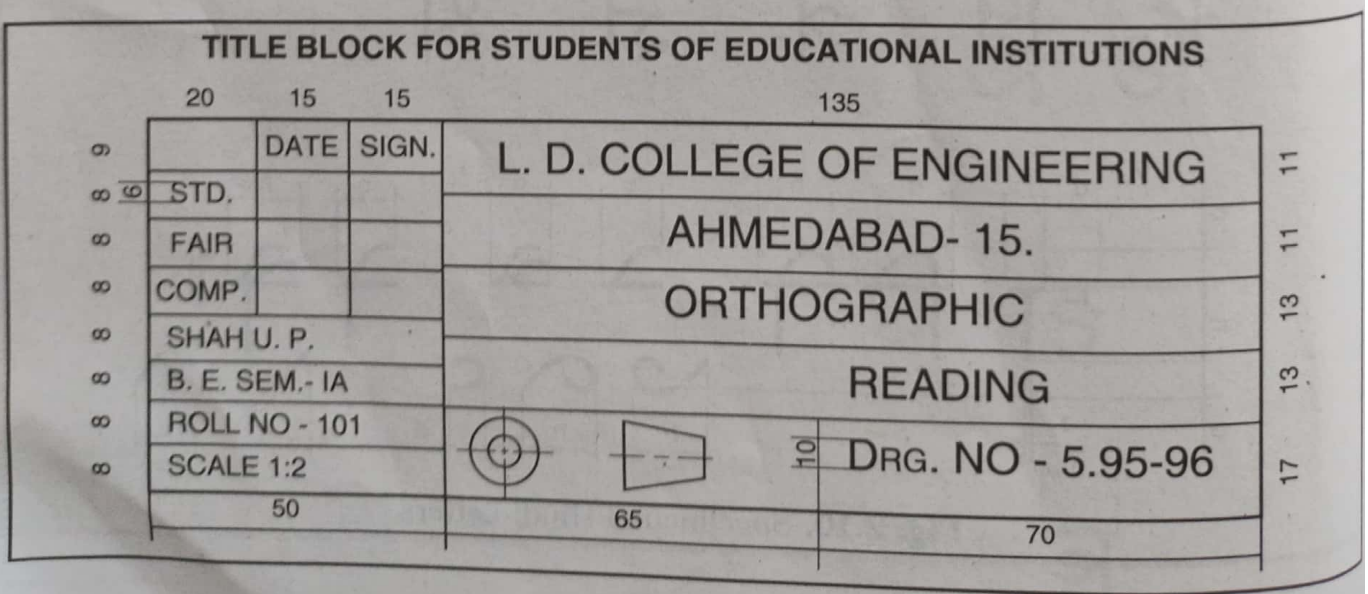


Fig. 2.10. Specimen of Hindi Letters

EXERCISE

1. What do you understand by trimmed and untrimmed drawing sheet sizes ? Prepare a list of recommended sizes of drawing sheets.
2. State the considerations used in preparing 'Specification for paper sizes'.
3. Why is a scale necessary in preparation of a drawing ? State the recommended scales.
4. Enlist various types of lines used for machine drawings. State their relative thicknesses.
5. How will you decide about height of lettering ? Prepare a drawing of all English and all Hindi alphabets in vertical as well as in inclined style.
6. What are single stroke letters ? Where are they used ?



CHAPTER - 3

Scales

1. Introduction :

Drawings of objects are prepared on drawing paper. Size of objects may be extremely small, medium or extremely large. Drawing papers are available in standard sizes. Medium size objects are drawn on medium size drawing papers with their true sizes i.e. drawn to full size scale. Large objects are drawn with reduced scale to accomodate their drawings on small and medium size papers, e.g. The Map of India drawn on imperial size paper.

Sometimes extremely small objects, like watch mechanism, are drawn to an enlarged scale. Here the problem is not of accomodating the drawing on the drawing paper but of difficulty in drawing small size drawings accurately on the drawing paper.

I.S. have recommended the following standard scales.

(1) Full Scale (R.F.)	(2) Reduced Scale (R.F.)		(3) Enlarged Scale (R.F.)
1 : 1	1 : 2	1 : 20	10 : 1
	1 : 2.5	1 : 50	5 : 1
	1 : 5	1 : 100	2 : 1
	1 : 10	1 : 200	

Normally, ready available standard scales are adopted and objects are drawn either to full scale, reduced scale or to an enlarged scale. But sometimes it becomes necessary to prepare specific scale which suit the size of objects and corresponding to the size of the paper.

We shall now study the methods of preparing such non-standard specific scales.

2. Representative Fraction :

The ratio of the size of the element in the drawing to the size of the same element in the object is called the REPRESENTATIVE FRACTION (R.F.). Suppose a line of 10 mm length in drawing represents 1 metre length of the object than the R.F. is equal to

$$\frac{10\text{ mm}}{1\text{ metre}} = \frac{10\text{ mm}}{1000\text{ mm}} = \frac{1}{100}$$

Therefore, the scale of the drawing will be 1 : 100. Here size of the drawing is smaller than the actual size of the object and hence the scale is known as reduced scale (1 : 100).

Now, suppose a line of 10 mm length in drawing represents 1 mm length of the object than the R.F. is equal to

$$\frac{10 \text{ mm}}{1 \text{ mm}} = \frac{10}{1}$$

Therefore, the scale of the drawing will be 10 : 1. Here scale 10 : 1 is an enlarged scale. Enlarged scales are used to draw extremely small objects.

3. Types of Scales :

Scales are classified in two different manner as under :

1. (a) Mechanical Engineers' Scale
   (b) Architects' Scale
   (c) Civil Engineers' Scale

2. (a) Plain Scale
   (b) Diagonal Scale
   (c) Comparative Scale
   (d) Vernier Scale
   (e) Scale of Chords
   (f) Isometric Scale.

3.1 (a) Mechanical Engineers' Scale :

These scales are 300 mm long and each unit is sub-divided. Mechanical engineers generally use following scales.

1:1, 1:2, 1 : 2.5, 1 : 5, 2 : 1 and 5 : 1 only.

(b) Architects' Scale :

Architects are required to take very small R.F., since buildings are comparatively ver big as compare to drawing paper size. Only the first main division of the architects' scal is sub-divided.

(c) Civil Engineers' Scale :

Civil engineers dealing with road maps and survey maps are required to take ver very small R.F. These scales are sub-divided on their entire lengths.

According to I.S. 962-1967 metric scales used for architectural and building drawing are as below :

(a) Topographical Maps

1 cm = 1 km $\left[\frac{1}{100000}\right]$

1 cm = 0.5 km $\left[\frac{1}{50000}\right]$

(c) Large scale surveys and layouts

1 cm = 20 m $\left[\frac{1}{2000}\right]$

1 cm = 10 m $\left[\frac{1}{1000}\right]$

1 cm = 5 m $\left[\frac{1}{500}\right]$

(b) Town Surveys

2 cm = 1 km $\left[\frac{1}{50000}\right]$ 4 cm = 1 km $\left[\frac{1}{25000}\right]$

10 cm = 1 km $\left[\frac{1}{10000}\right]$ 1 cm = 50 m $\left[\frac{1}{5000}\right]$

(d) Preliminary or sketch drawing

1 cm = 5 m $\left[\frac{1}{500}\right]$ 1 cm = 2 m $\left[\frac{1}{200}\right]$

1 cm = 1 m $\left[\frac{1}{100}\right]$

(e) Working drawings, plans, elevations and sections

1 cm = 2 m $\left[\frac{1}{200}\right]$ 1 cm = 1 m $\left[\frac{1}{100}\right]$

1 cm = 0.5 m $\left[\frac{1}{50}\right]$

(f) Large scale drawings-General details

1 cm = 20 cm $\left[\frac{1}{20}\right]$ 1 cm = 10 cm $\left[\frac{1}{10}\right]$

(g) Enlarged details

1 cm = 10 cm $\left[\frac{1}{10}\right]$ 1 cm = 5 cm $\left[\frac{1}{5}\right]$

1 cm = 2 cm $\left[\frac{1}{2}\right]$ 1 cm = 1 cm or Full size

3.2 (a) Plain Scale : See Fig. 3.1.

A plain scale is nothing but a straight line divided into suitable number of equal parts or units, the first unit of which is further subdivided. Plain scale represents a unit and its fraction or two interconnected units.

The features of the plain-scale are as under :

1. The zero is placed at the end of the first sub-divided main division.
2. From the zero mark, the units are numbered towards right side and its sub-divisions are numbered towards left side.
3. Units of the sub-divisions and of the divisions are stated either below or at the respective ends.
4. R.F. must be mentioned just below the scale.

Problem 1 : Construct a plain scale of R.F. 1 : 100 to show metres and decimetres. Maximum measurement required is 10 metres. Indicate 8 m 7 dm on the scale.

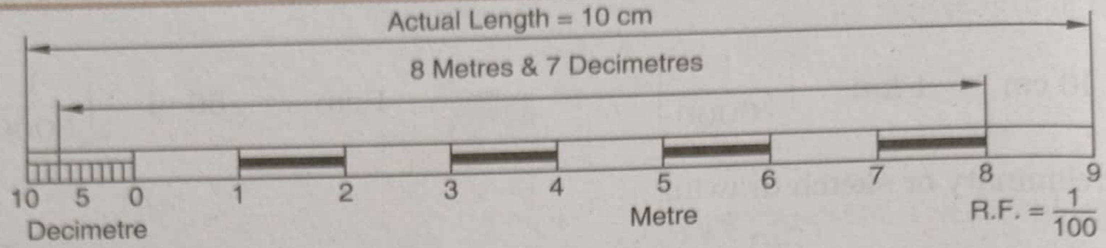


Fig. 3.1. Problem 1

For solution see Fig. 3.1 and follow the procedure as given below :

(i) First find the actual length of the scale by the equation as given below :

$$\text{Actual length of the scale} = [\text{R.F.}] \times \begin{bmatrix}\text{Units of maximum} \\ \text{length required to} \\ \text{be measured by the} \\ \text{scale.}\end{bmatrix}$$

$$= \frac{1}{100} \times 10 = 0.1 \text{ m} = 10 \text{ cm}$$

(ii) Draw a line of 10 cm length and divide it into a number of divisions (10) equal to number of units of required maximum measurement, each representing 1 m.

(iii) Sub-divide first main division into 10 sub-divisions, each representing 1 dm, since 1 metre = 10 decimetres.

(iv) Mark zero (0) at the end of the first division and towards right side mark 1, 2, ... 9 as shown.

(v) Towards left of zero (0), mark sub-divisions 5 and 10 only, since the space is limited.

(vi) Just below the sub-divided divisions write decimetre and just below 1 to 9 main divisions write metre. On the extreme right and at the bottom of the scale write R.F. as $\frac{1}{100}$

Problem 2 : Construct a plain scale of 1 cm = 1 kilometre, to show hectometres and kilometres. Scale should be long enough to measure distance between I.I.T Delhi and Rashtrapatibhavan, which is 15 kms. Indicate on the scale the distance between Caunought place and Rashtrapatibhavan, which is 3 km 7 hm or say 3.7 km.

For solution see Fig. 3.2 and follow the procedure as given below :

(i) First find actual length of the scale by the equation

$$\text{Actual length of the scale} = [\text{R.F.}] \times \begin{bmatrix}\text{Units of maximum} \\ \text{length required to} \\ \text{be measured by the} \\ \text{scale.}\end{bmatrix}$$

OCTAGON
HEPtaGON
HEXaGON

$$= \frac{1 \text{ cm}}{100000} \times 1500000 \text{ cm} = 15 \text{ cm}$$

PENTAGON

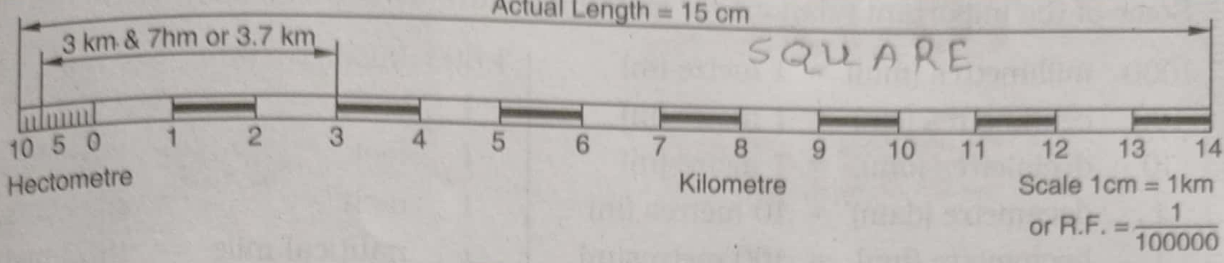


Fig. 3.2. Problem 2

(ii) Draw a line of 15 cm length and divide it into 15 equal parts, each representing 1 km.

(iii) Sub-divide first main division into 10 sub-divisions, each representing 0.1 km or 1 hectometre, since 1 km = 10 hm.

(iv) Mark zero (0) at the end of first division and towards right side mark 1, 2, ... 14 as shown. Towards left side mark sub-divisions 5 and 10 only.

(v) Just below the sub-divisions write hectometre and just below main-divisions write kilometre. On the right and at the bottom of the scale. write scale 1 cm = 1km or

$$\text{R.F.} = \frac{1}{100000}$$

(vi) Indicate 3 kms 7 hms or 3.7 kms on the scale as shown.

Problem 3 : On a map of Gujarat, 1 cm represents 5 kms. Construct a plain scale long enough to measure a distance between Ahmedabad and Baroda. Indicate on it a distance between Ahmedabad and Anand.

Distance : (1) Ahmedabad - Baroda - 100 kms.

(2) Ahmedabad - Anand - 65 kms.

UNIVERSaL METHOD.

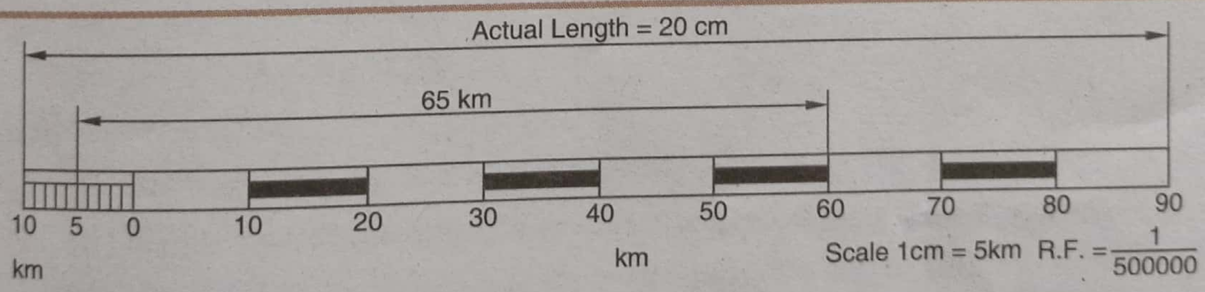


Fig. 3.3. Problem 3

(i) First find the actual length of the scale, as given below :

$$\text{Actual length of the scale} = \left[\frac{1 \text{ cm}}{5 \text{ kms}}\right] \times [100 \text{ kms}]$$

$$= 20 \text{ cms}$$

(ii) Draw a line of 20 cms length and divide it into 10 equal divisions, each representing 10 kms. Sub-divide first main division into 10 sub-divisions, each representing 1 km.

3.2 (b) Isometric Scale :

See chapter of Isometric Projection for this scale.

4. Some of the important relations for length measurement are as shown :

1000	millimetres (mm)	= 1 metre (m)	1.609	kilometre (km)	=	1 mile
100	centimetres (cm)	= 1 metre (m)	1	yard	=	3 feet
10	decimetres (dm)	= 1 metre (m)	1	foot	=	12 inches
1	decametre (dam)	= 10 metres (m)	1	inch	=	2.54 cms
1	hectometre (hm)	= 100 metres(m)	1	nautical mile	=	1852 metres
1	kilometre (km)	= 1000 metres (m)	1	metre	=	1.0936 yards

EXERCISE

1. Construct a plain scale with R.F. = $\frac{1}{50}$ to read metre and decimetre. It should be long enough to read 5 metre. Show 3.6 metre on it.
2. Construct a plain scale 1 cm = 1 dm to read decimetre and centimetre. It should be long enough to read 20 dm. Show the length 157 cm on it.
3. Construct a plain scale of R.F. = $\frac{1}{5000}$ to read hectometre and decametre. It should be able to measure 10 hectometre. Measure a distance of 7 hm 6 dm on the scale.
4. On the road map of Ahmedabad - Gandhinagar 2 kms is represented by 1 cm. Ahmedabad Gandhinagar distance is 25 kms. Gandhi-Ashram is 18 kms from Gandhinagar. Represent it after preparing the scale for the above.

CHAPTER - 4

Loci of Points

We are living in the modern age where we often come across machines and mechanisms. How the mechanism runs and what is going to be the path of various points of the mechanism will be of our interest. This information is also useful in design, force analysis and motion utilisation of the mechanism.

The path of a point keeping its distance constant from a fixed point, from a fixed straight line, from a fixed circle and various such combinations are also of our interest. In this chapter we shall study the path followed by different points of the various mechanisms along with the path followed by various points moving with given constraints or set laws.

Mechanism is the combination of various links so paired that the motion is completely constrained. Mechanisms are used to transform the motion, e.g. Rotary to reciprocating, reciprocating to rotary, rotary to oscillating, etc. In other words, by available mechanisms motion is converted into useful motion.

In this chapter we shall study following mechanisms :

(1) Slider Crank Chain.

  (a) Simple (See Fig. 4.11)

  (b) Off Set [See Fig. 4.12]

(2) Four Bar Chain.

  (a) Both the driver and driven cranks revolving (See Fig. 4.14)

  (b) Driver crank revolving and driven crank oscillating. (See Fig. 4.15)

  (c) Both driver and driven cranks oscillating. (See Fig. 4.16)

(3) Crank and connecting rod mechanism with connecting rod constrained to pass through guide named as trunnion (See Fig. 4.13)

(4) Combinations of above mechanisms (See Fig. 4.18)

Now we shall study first the path of various points moving in a plane with given conditions or set laws and then the path of various points of the mechanisms. Study will be carried out by taking concrete examples. The path followed by a point is called locus (plural loci).

Part - I

Problem 1 : Find the locus of a point P, moving in a plane, keeping its distance from the fixed point O as constant. Take constant distance equal to 20 mm (See Fig. 4.1)

Here the locus of the point P is going to be a circle of 20 mm radius and the fixed point O as the centre.

Problem 2 : Find the locus of a point P, moving in a plane, keeping its distance from the fixed straight line O_1O_2 as constant. Here the constant distance is taken as 20 mm (See Fig. 4.2)

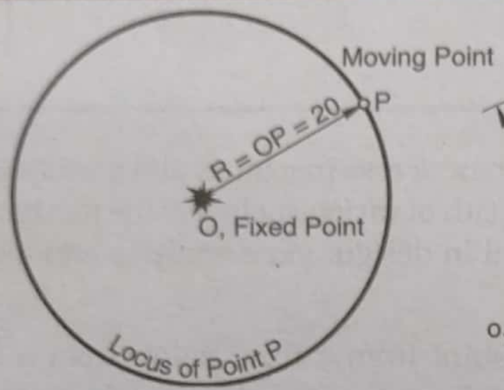


Fig. 4.1. Problem 1

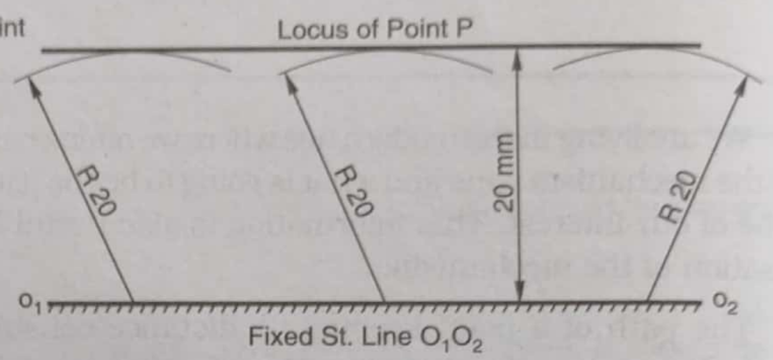


Fig. 4.2. Problem 2

Take two or three points arbitrarily on the fixed straight line O_1O_2 and with these points as centres and radius equal to 20 mm draw arcs of circles on any one side of the straight line. Now draw a line in such a way that the line becomes tangent to those arcs of circles. Obviously this locus of P is going to be a straight line parallel to the straight line O_1O_2 at 20 mm distance.;

Problem 3 : Find the locus of a point P, moving in a plane, keeping its distance from the fixed circle ⊙ (0, 20) as constant and outside it. Here constant distance is taken as 10 mm (See Fig 4.3)

Take many points arbitrarily on the periphery of the circle ⊙(0, 20) as centres and with radius equal to 10 mm draw arcs in the plane of a circle, and outside it.

Now draw a curve in such way that this curve becomes tangent to all the arcs that you have drawn. Obviously this locus of P is going to be a circle of 30 mm radius with O as the centre.

If it is required to draw the locus inside then it will be a circle of 10 mm radius with the same point O as the centre.

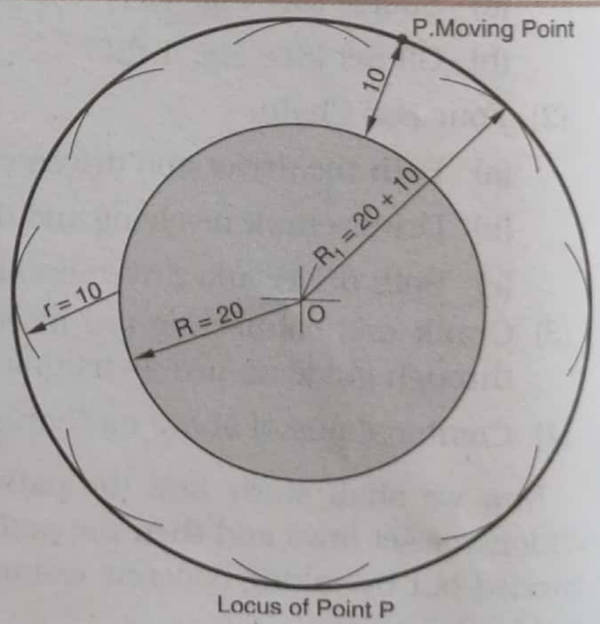


Fig. 4.3. Problem 3

Problem 4 : Find the locus of point P, moving in a plane, keeping its distances equal from the two fixed points O_1 and O_2. (See Fig. 4.4)

The locus of a point equidistant from the two fixed points is a perpendicular bisector of the line joining the two fixed points.

So to get the locus of point P equidistant from the fixed points O_1 and O_2, first join O_1 and O_2. Now with O_1 and O_2 as centres and R as radius draw two intersecting arcs on each side of O_1O_2 at P and Q as shown in figure. Take $R > \frac{O_1 O_2}{2}$

Join the points of intersection P and Q to get perpendicular bisector of O_1O_2. This perpendicular bisector line PQ of O_1O_2 is the locus of point P which is equidistant from O_1 and O_2.

Problem 5 : Find the locus of a point P, moving in a plane, keeping its distances equal from two Non-parallel fixed straight lines O_1O_2 and O_1O_3. (See Fig. 4.5)

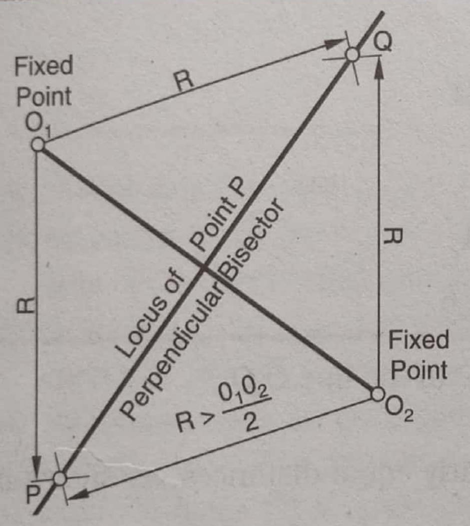


Fig. 4.4. Problem 4

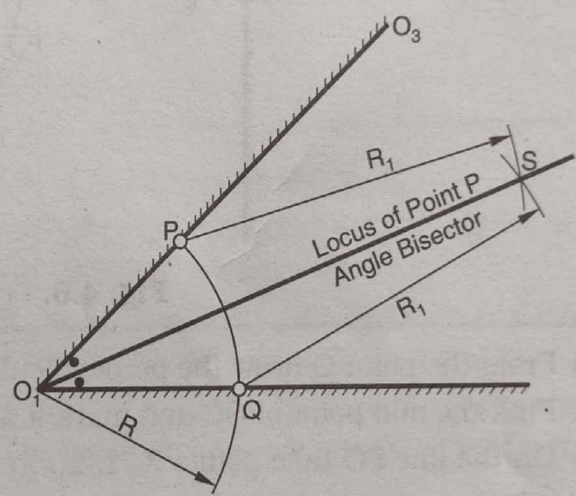


Fig. 4.5. Problem 5

If the two given lines do not intersect in a given space then draw two intersecting lines parallel to and equidistant from the given two straight lines and then proceed for solution.

Locus of a point, equidistant from two fixed straight lines, is an angle bisector of the angle between the two fixed straight lines.

To get locus of a point P we must draw angle bisector of angle $O_3O_1O_2$. To get angle bisector follow the procedure as given below :

(1) With O_1 as the centre and the radius equal to R draw an arc cutting O_1O_3 at P and O_1O_2 at Q.

(2) Now with points P and Q as centres and radius equal to R_1 draw two arcs intersecting at S as shown in figure.

(3) Join O_1S which is the angle bisector of $O_3O_1O_2$ and so it is the locus of the point P equidistant from the two fixed straight lines O_1O_3 and O_1O_2.

Problem 6 : Find the locus of the point P, moving in a plane, keeping its distances equal from a fixed straight line O_1O_2 and a fixed point O. (See Fig. 4.6)

To get locus of the point P equidistant from the line O_1O_2 and the point O, follow the procedure as given below :

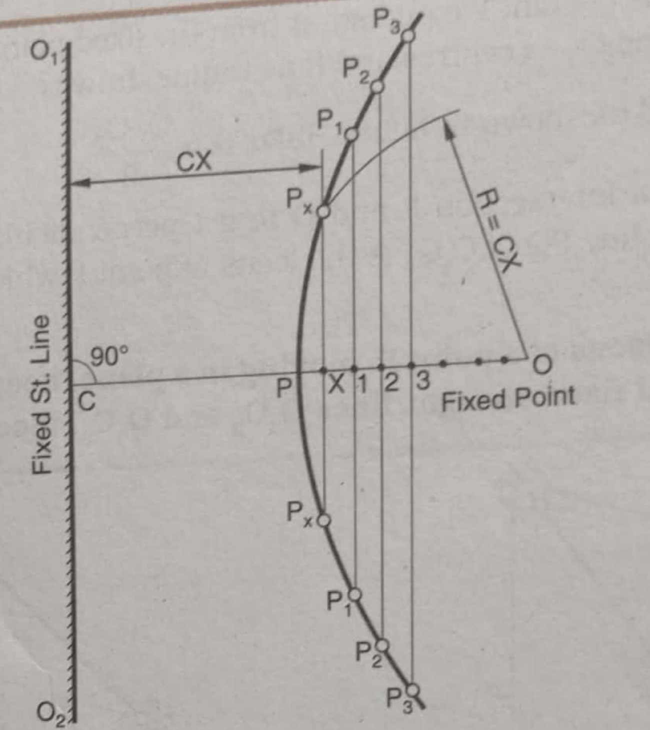


Fig. 4.6. Problem 6

(1) From the point O draw the perpendicular O C to the line O_1O_2.
(2) Find the mid point of OC and mark it as P.
(3) On the line PO take points X, 1, 2,...n at nearly equal distances, as shown in the figure.
(4) Through these points, X, 1, 2, 3,.....n draw lines parallel to O_1O_2.
(5) Now with O as the centre and the radius equal to CX, C1, C2.....Cn, draw two arcs one on each side of CO to intersect the parallel lines through X, 1, 2,......n respectively, as shown in the figure.
(6) On the parallel lines through X, 1, 2,....n, we get the points of intersections as (P_x, P_x), $(P_1, P_1)(P_2, P_2)$.....(P_n, P_n), as shown in figure.
(7) Join points P_n,.......P_2, P_1, P_x, P, P_x, P_1, P_2.......P_n by a smooth curve. This smooth curve is the locus of the point P which is equidistant from the fixed line $O_1 O_2$ and the fixed point O. This curve is a Parabola.

Problem 7 : Find the locus of a point P, moving in a plane, keeping its distances equal from a fixed point O_2 and a fixed circle ⊙ $(O_1, 25)$. O_2 is 70 mm away from the centre O_1. (See Fig. 4.7)

To get the locus of the point P, equidistant from the fixed point O_2 and fixed circle $(O_1, 25)$, follow the procedure as given below:

(1) Join O_1 and O_2 and mark the point A where O_1O_2 intersects circle $(O_1, 25)$.
(2) Find mid point P of the line AO_2.

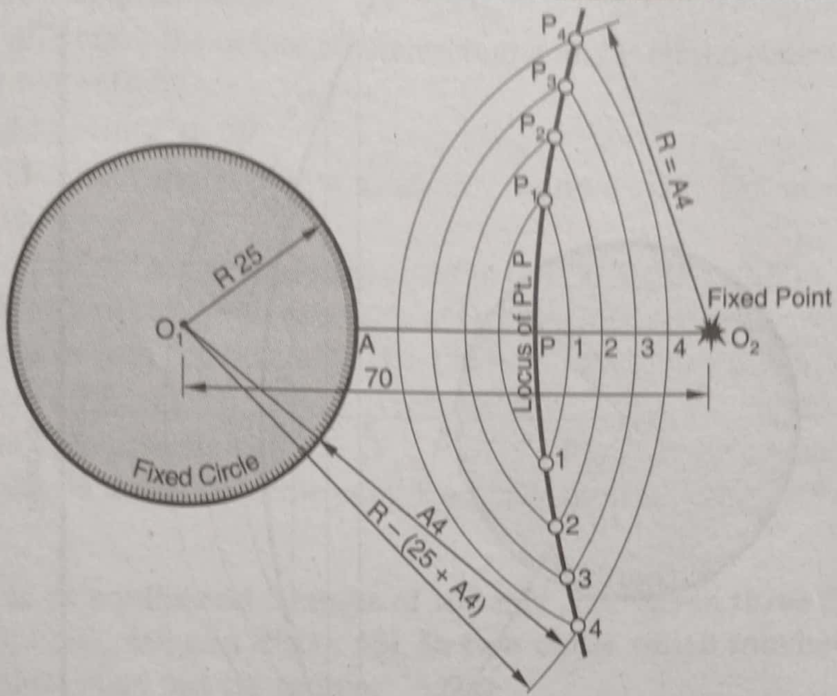


Fig. 4.7. Problem 7

(3) On the line PO_2 take points 1, 2,.......n at nearly equal distances.

(4) Now with O_1 as the centre and radii equal to (25 + A1), (25 + A2),......(25 + An) draw arcs of circles on each side of O_1O_2 as shown in figure.

(5) Now with O_2 as the centre and radii equal to A1, A2,.....An draw arcs of circles on each side of O_1O_2 to intersect previously drawn corresponding arcs of the circles.

(6) By intersections we get points (P_1, P_1'), (P_2, P'_2)....(P_n, P_n'). Join all these points by a smooth curve. The smooth curve is the locus of the point P equidistant from a fixed point O_2 and a fixed circle (O_1, 25).

Problem 8 : Find the locus of a point P, moving in a plane, keeping its distances equal from a fixed straight line O_2O_3 and a fixed circle (O_1, 25). (See Fig. 4.8).

To get locus of a point P equidistant from a fixed straight line O_2O_3 and fixed circle (O_1, 25), follow the procedure as given below :

(1) From the point O_1 draw perpendicular O_1C to the fixed straight line O_2O_3.

(2) Mark the point A where O_1C intersects the fixed circle. Find the midpoint of AC and mark it as the point P.

(3) On the line PA take points 1, 2, 3,.....n at nearly equal distances, as shown in figure.

(4) From points 1, 2,.....n draw straight lines parallel to the fixed line O_2O_3, as shown in figure.

(5) Now with O_1 as the centre and radii equal to (25 + C1), (25 + C2), ...(25 + Cn) draw arcs of circles on each side of O_1C to intersect parallel lines from 1, 2,.....n at the points (P_1, P_1') (P_2, P_2').......(P_n, P_n') respectively.

(6) Join points P_n',....P_2',P_1', P, P_1, P_2,.....P_n by smooth curve. This curve is the locus of the point 'P' equidistant from the fixed straight line O_2O_3 and the fixed circle (O_1, 25).

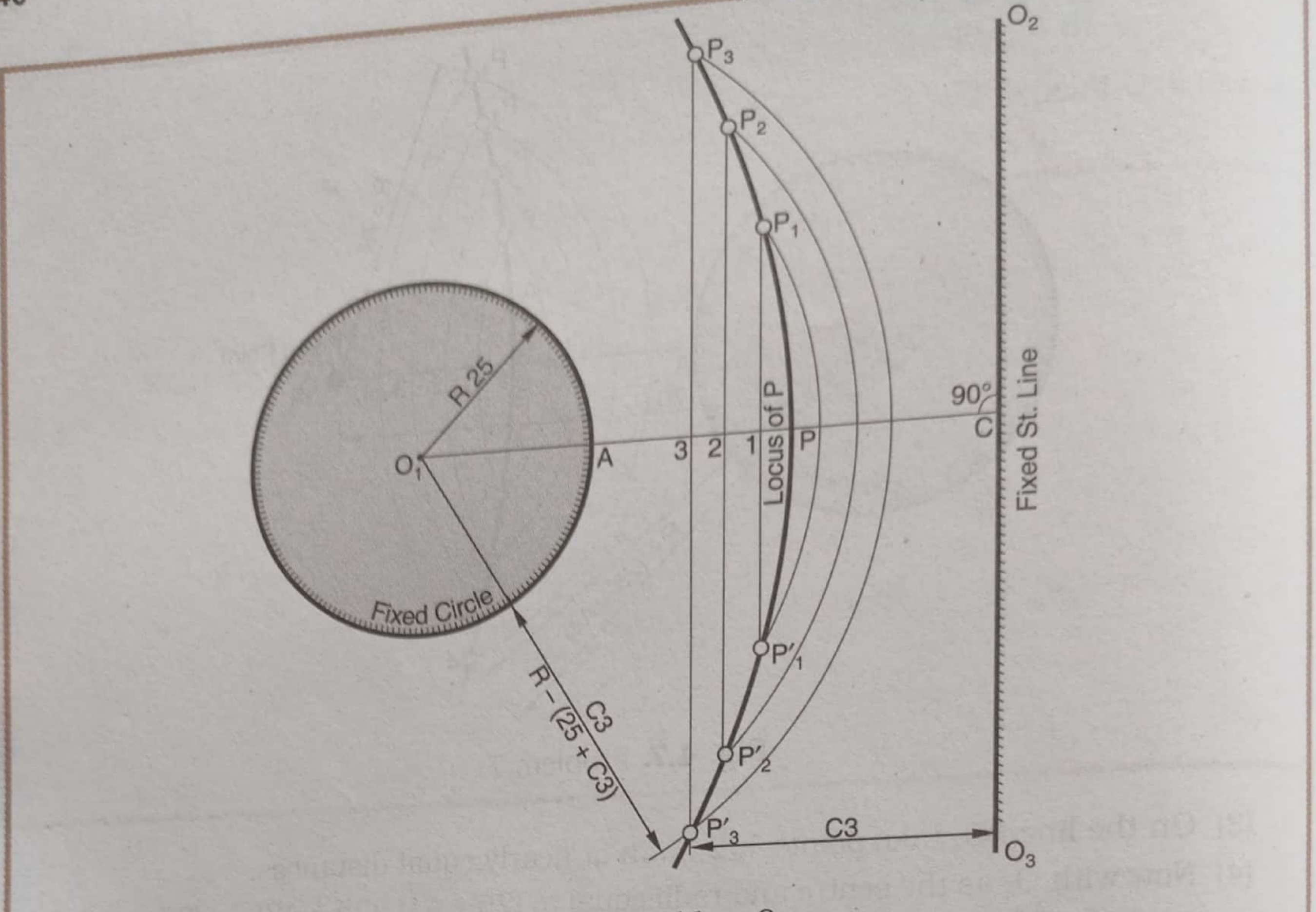


Fig. 4.8. Problem 8

Problem 9 : Find the locus of a point P, moving in a plane, keeping its distances equal from two fixed circles. Here the two fixed circles are (O_1, 50) and (O_2, 30). Take distance between O_1 and O_2 as 110 mm (See Fig. 4.9)

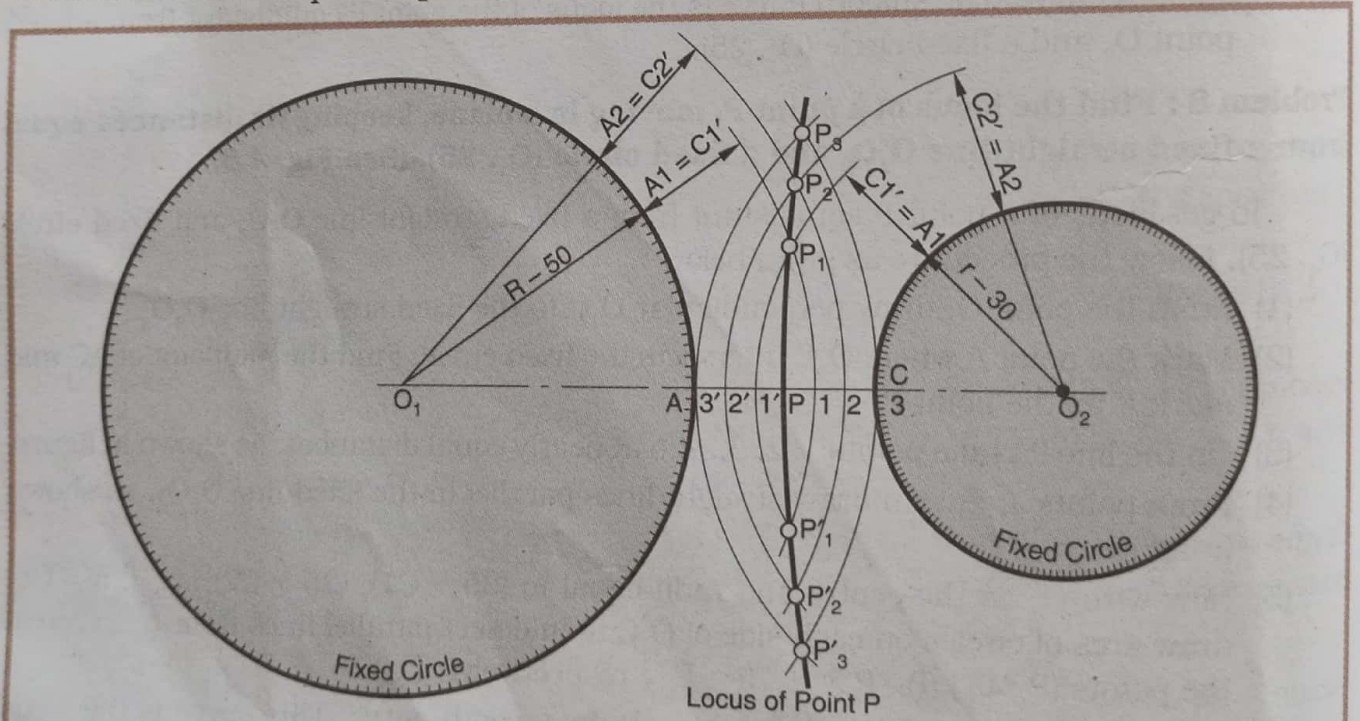


Fig. 4.9. Problem 9

Follow the procedure as given below :

(1) Join O_1O_2 and mark the points of intersection A and C on the circles ⊙ (O_1, 50) and ⊙ (O_2, 30) respectively.

(2) Mark the mid point P of AC.

(3) On PC and on PA mark points 1, 2, ...n and 1', 2'......n' at equal intervals respectively.

(4) Now with O_1 as the centre and radii equal to $O_1$1(50 + A1); $O_1$2(50 + A2);......O_1n(50 + An) draw arcs of circles on two sides of O_1 O_2 to intersect with arcs of circles with O_2 as the centre and radii equal to $O_2$1' (30 + C1'); $O_2$2'(30 + C2'),......O_2n'(30 + Cn') to get points P_1, P_1'; P_2, P_2';....P_n, P_n□ respectively.

(5) Join points of intersections P'_n,.......P_2', P'_1,P, P_1, P_2,.......P_n by a smooth curve. This smooth curve is the locus of the point P equidistant from ⊙ (O_1, 50) and ⊙ (O_2, 30).

Problem 10 :

(a) $\Delta O_1O_2O_3$, is an equilateral triangle of 100 mm size. Given three fixed circles an ⊙ (O_1, 30); ⊙ (O_2,40) and ⊙ (O_3, 45). Draw a circle which touches all the three given circles. Find out its radius.

(b) Given the fixed line O_4O_5 parallel to O_1O_2 and at a distance of 70 mm. Draw a circle which touches the two circles of O_1 and O_2 as centres and line O_4O_5. See Fig 4.10 for solution of (a) as well as of (b).

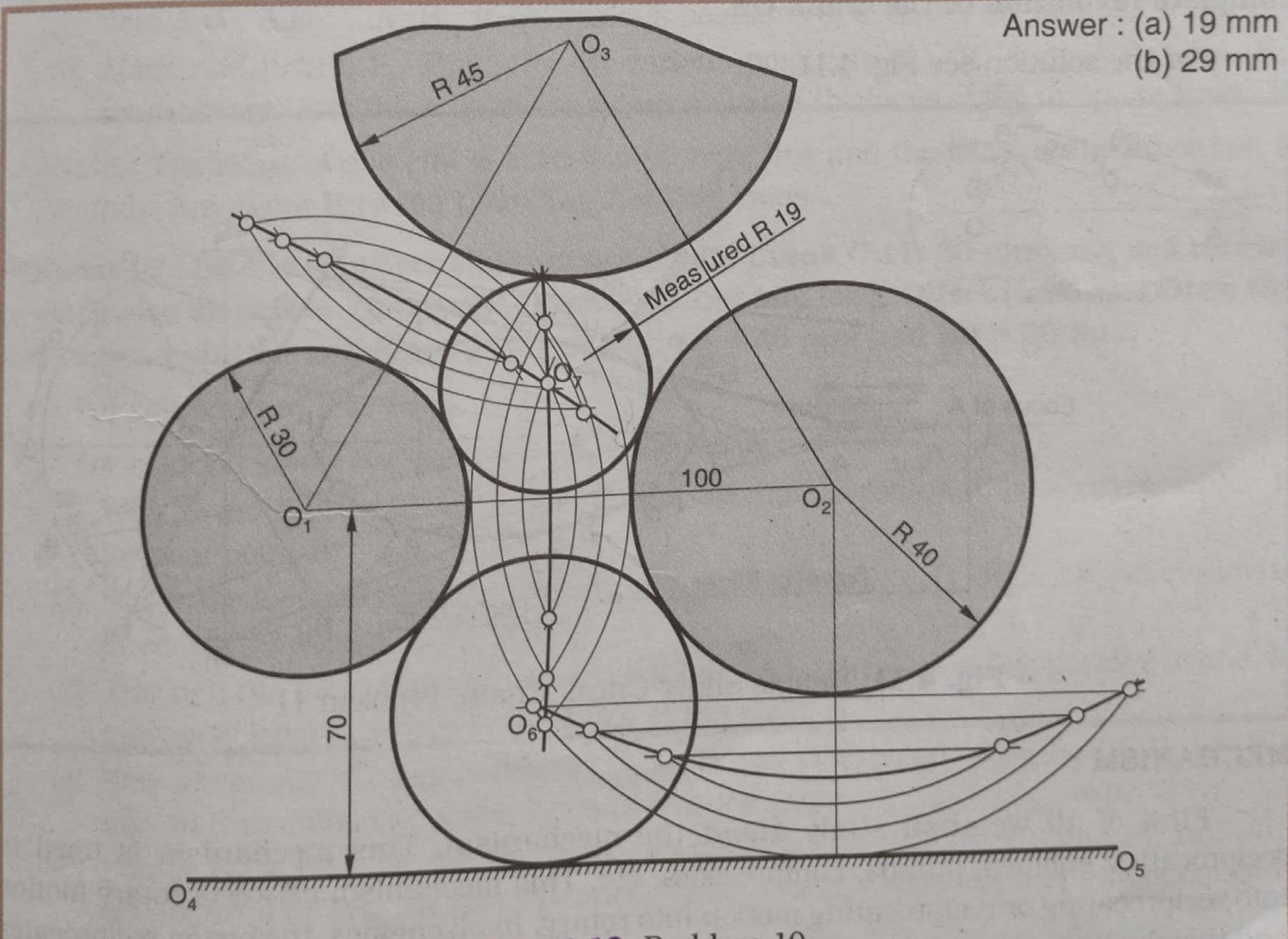


Fig. 4.10. Problem 10

To get the solution, follow the procedure as given below :

(a) Part (i) following the procedure of problem 9, find a curve equidistant from circles ⊙ (O_1, 30) and ⊙ (O_2, 40) and similarly, a curve equidistant from circles ⊙ (O_1, 30) and ⊙ (O_3, 45). These two curves intersect at the point O_7. (ii) Draw a circle, with O_7 as the centre and of suitable radius, which touches the given three fixed circles. Measure the radius. Ans. 19 m.m.

(b) (i) Following the procedure of problem 9, find a curve equidistant from the two circles ⊙ (O_1, 30) and ⊙ (O_2, 40).

(ii) Following the procedure of problem 8, find a curve equidistant from one fixed circle, ⊙ (O_2, 40) and one fixed straight line O_4O_5.

(iii) Intersection of above two curves gives the point O_6 as shown in the figure. Therefore, the point O_6 is equidistant from ⊙ (O_1, 30); ⊙ (O_2, 40) and the line O_4O_5.

(iv) Now draw a circle with O_6 as the centre and with a suitable radius so that the circle touches the two given circles and the given straight line. Measure the radius. Ans:-29 m.m.

Part - II

Problem 11 : OBA is a simple slider crank chain. OB is a crank of 30 mm length. BA is a connecting rod of 90 mm length. Slider A is sliding on a straight path passing through point O. Draw the locus of the mid-point of the connecting rod AB for one complete revolution of the crank OB.

For the solution see Fig. 4.11.

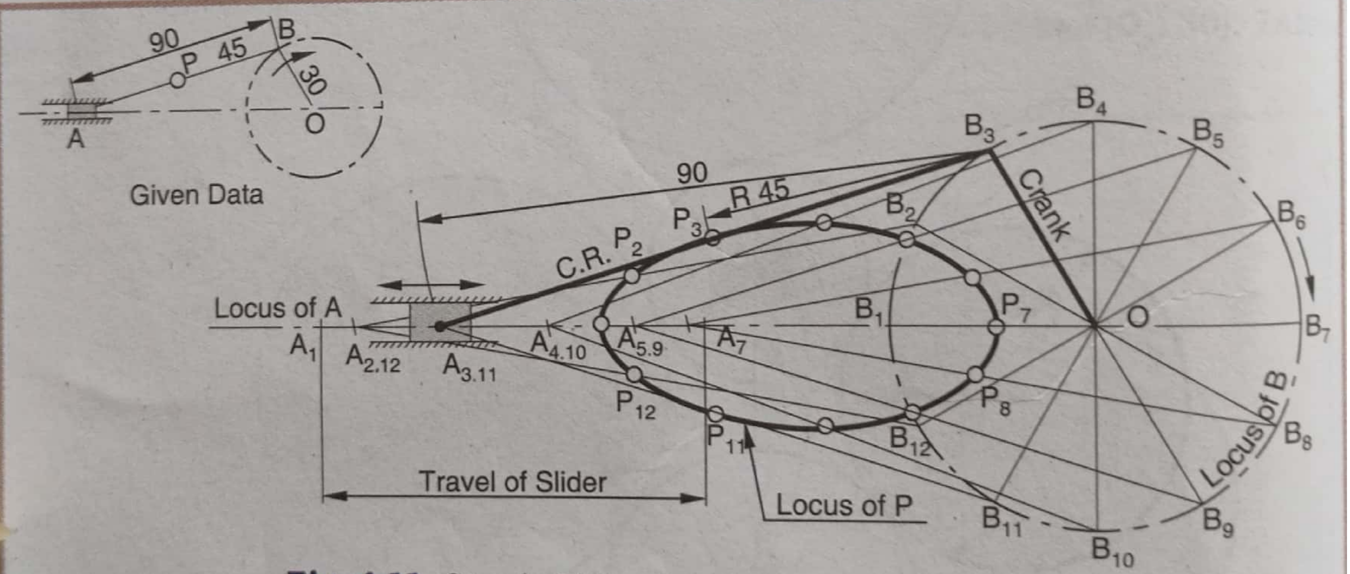


Fig. 4.11. Simple Slider Crank Chain. Problem 11

MECHANISM :

First of all we shall study about the mechanism. This mechanism is used in reciprocating engines, pumps, compressors, etc. This mechanism converts rotary motion into reciprocating or reciprocating motion into rotary. In oil engines, the piston reciprocates and thereby the crank revolves. In compressors the crank is given rotation by electric motor and thereby the piston reciprocates and compresses air.

In this mechanism there are four links : (i) crank (ii) connecting rod (iii) slider and (iv) fixed body having bearings for the crank and guide for the slider.

The crank is able to revolve about bearings provided at O. The big end of the connecting rod is connected to the crank by the turning pair (B) so that they can have relative angular velocity between them i.e. angle between them can change. The small end of C.R. is connected to the slider, also by the turning pair (A). Slider is able to reciprocate inside the guide provided in the main body which is fixed.

If the line of reciprocation of slider passes through the bearing of the crank i.e. here the point O, then it is called a simple slider crank chain. If it is offset then it is said to be an offset slider crank chain. (See Fig. 4.12)

For solution of the problem follow the procedure as given below. (See Fig. 4.11)

(1) Draw a circle ⊙ (0, 30). This circle is the locus of the point B.
(2) Divide this circle into 12 equal parts and mark them as B_1, B_2,...B_{12} as shown in the figure in the direction of rotation. OB_1, OB_2......OB_{12} represent the positions of the crank during the rotation.
(3) Draw a line through O preferably horizontal on which the slider A is to travel.
(4) Draw arcs of circles with B_1, B_2,......B_{12} as centres and length of C.R. as the radius to cut the straight line, locus of slider A, at various points. Mark those points as A_1, A_2......A_{12} respectively.
(5) Join A_1B_1, A_2B_2.....$A_{12}B_{12}$ to get various positions of C.R. during motion.
(6) Mark mid points P_1, P_2,......P_{12} on various positions of C.R. A_1B_1, A_2B_2......$A_{12}B_{12}$ respectively. Join these points in sequence to get the locus of the midpoint P of C.R.

Note : The locus of one end of C.R. is a straight line and the locus of the other end is a circle. Any point between them has elliptical locus.

Problem 12 : OBA is an offset slider crank chain. Crank OB is 30 mm long and rotates in clockwise direction. Connecting rod AB is 128 mm long. Offset is 40 mm. Draw the loci of two points P and R as shown in Fig. PB = 45 mm and BR = 30 mm.

For solution see Fig. 4.12.

For solution follow the procedure as given below :

(1) With O as the centre and crank length 30 mm as radius draw a circle, which is locus of point B.
(2) Divide the locus of B in 12 divisions and mark the points, B_1, B_2,.....B_{12} in clockwise direction, as shown in the figure.
(3) Through the point O draw a horizontal line and then draw a line parallel to and 40 mm away from it, which is the locus of the slider A.
(4) Now draw arcs of circles with B_1, B_2,.....B_{12} as centres and length of C.R. i.e. 128 mm as the radius to cut the locus of A at the points A_1, A_2,.....A_{12} respectively.
(5) Join A_1B_1, A_2B_2,.....$A_{12}B_{12}$ and mark on them points P_1,P_2,.....P_{12} and R_1, R_2,....R_{12} taking 45 mm and 30 mm distances from B and in the opposite direction. of B respectively, as shown in the figure.
(6) Join P_1, P_2,.......P_{12} to get the locus of P and join R_1, R_2,.....R_{12} to get the locus of R.

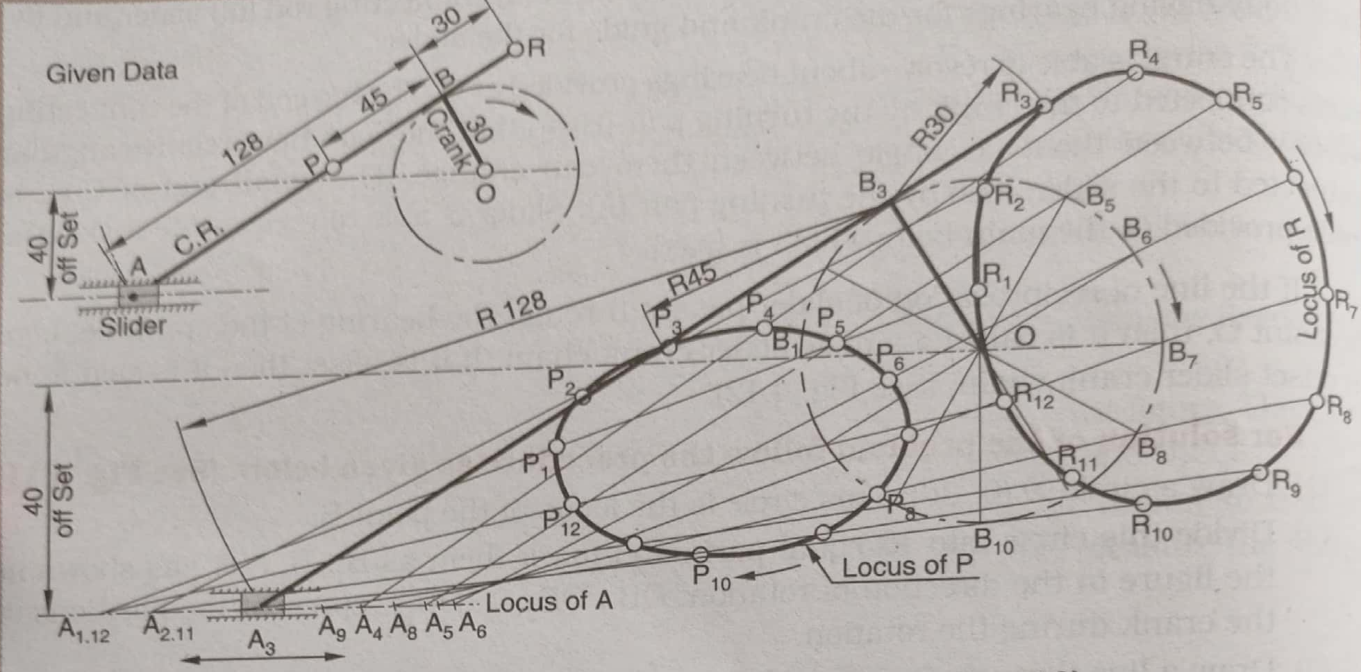


Fig. 4.12. Off-set Slider Crank Chain (Problem 12)

Problem 13 : Fig. 4.13 shows a mechanism in which OB is a crank of 30 mm lengtl revolving in clockwise direction. BC is a rod connected to the crank at the poin B by turning pair and rod BC is constrained to pass through the guide at O_1 calle trunnion. Draw the loci of points P and C for one revolution of the crank. The poin P is 30 mm from B on the rod BC. Length of BC is 150 mm (See Fig. 4.13). Point 0 is 80 mm on the right and 15 mm below the point O.

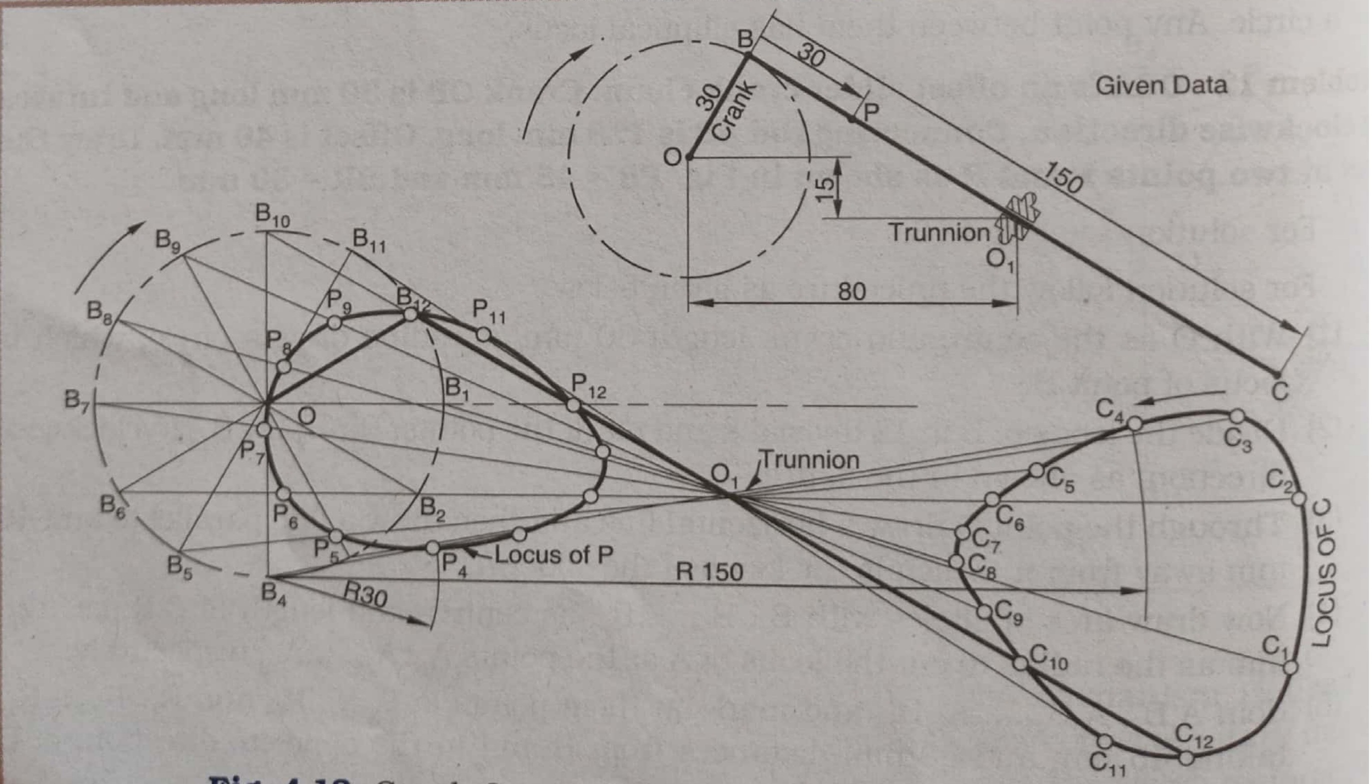


Fig. 4.13. Crank-Con-Rod-Trunnion Mechanism (Problem 13)

Follow the procedure as given below for the solution :

(i) With any point O as the centre and crank length 30 mm as the radius draw a circle which is the locus of point B.

(ii) Divide locus of B (circle) into 12 equal parts and mark them as B_1, B_2,.......B_{12} in the direction of rotation. OB_1, OB_2,.....OB_{12} represent positions of crank during rotation.

(iii) From B_1, B_2,......B_{12} draw lines passing through trunnionO_1 and of 150 mm length to get points C_1, C_2,.....C_{12} respectively.

(iv) On lines B_1 C_1, B_2 C_2,.....$B_{12}C_{12}$ mark points P_1, P_2,....P_{12} respectively at a distance of 30 mm from B towards C on rod.

(v) Join the points P_1, P_2,.....P_{12} and P_1 and C_1, C_2,.....C_{12} and C_1 by means of a smooth curve to get the loci of the points P and C respectively.

Problem 14 : O_1ABO_2 is a four bar chain with the link O_1O_2 as the fixed link. Driving crank O_1A is 30 mm long. Driven crank O_2B is also 30mm long. Connecting link AB is 90 mm long. Distance between O_1 and O_2 is 90 mm. Two cranks are in opposite directions as shown in the figure. Draw the loci of points P and R for one complete revolution of the driving crank. The point P is the mid point of the connecting link AB and the point R is 35 mm from A on BA extended.

See the Fig. 4.14 for solution and follow the procedure as given below :

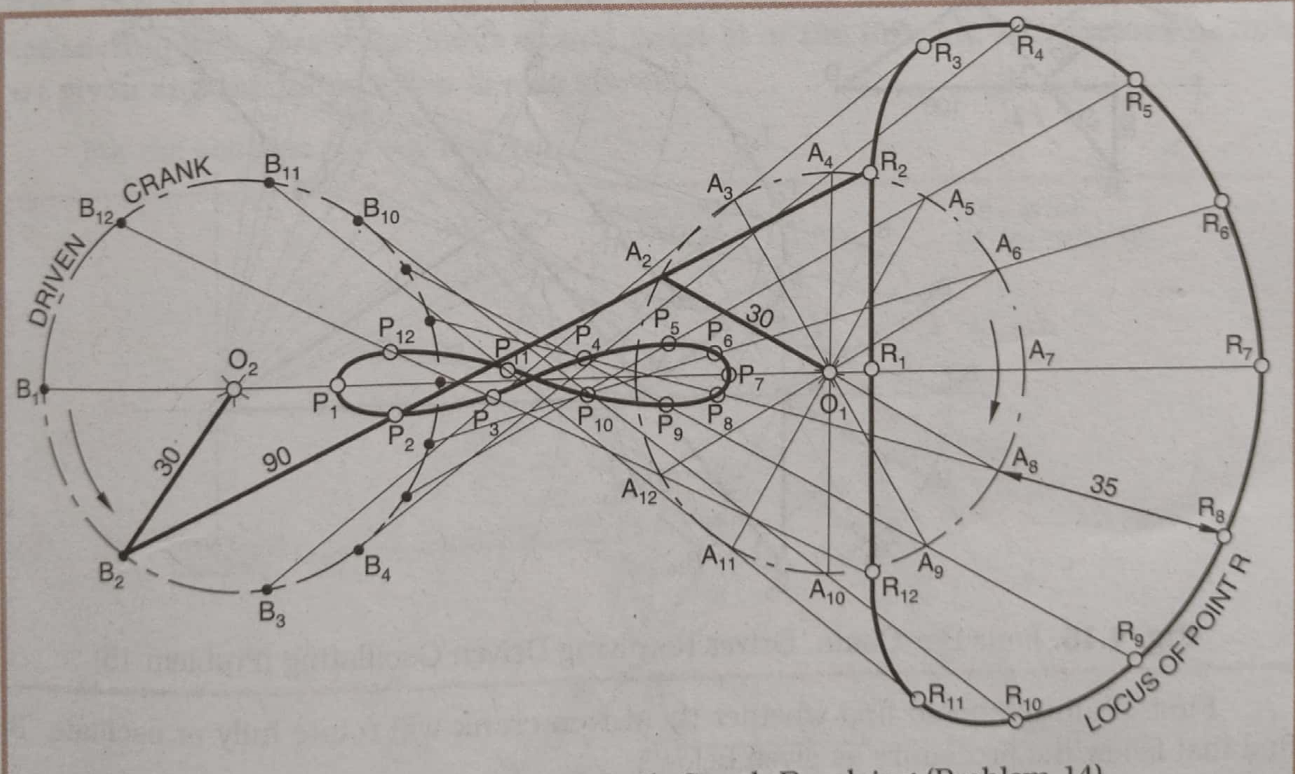


Fig. 4.14. Four Bar Chain. Both Crank Revolving (Problem 14)

(i) With O_1 as the centre and driving crank length 30 mm as the radius draw a circle which is the locus of the point A. Divide that circle into 12 equal parts and mark on them A_1, A_2,.....A_{12} in the direction of rotation.

(ii) Draw a horizontal line from O_1 and on it mark O_2 at 90 mm distance from O_1. With O_2 as centre and driven crank length 30 mm as the radius, draw a circle which is the locus of the point B.

(iii) With A_1, A_2,.......A_{12} as the centres and the radius equal to the connecting link length 90 mm draw arcs to cut the locus of the point B at points B_1, B_2,.......B_{12} as shown in the figure respectively. This cutting should be done on opposite side of O_1O_2 on the circle.

(iv) Join A_1B_1, A_2B_2,....$A_{12}B_{12}$ and mark on them midpoints P_1, P_2,.....P_{12} respectively. Join P_1, P_2......P_{12} & P_1 to get the locus of the point P.

(v) Extend B_1A_1, B_2A_2,.....$B_{12}A_{12}$ by 35 mm to get points R_1, R_2,.....R_{12} respectively. Join points R_1, R_2,.......R_{12} & R_1 to get the locus of the point R.

Problem 15 : ABCD is a four bar chain with the link AD fixed and of 100 mm length. AB is a driving crank of length 30 mm. CD is a driven crank of 60 mm length. BC is 80 mm long connecting link. TM is a rod of 20 mm length which is attached to the connecting link BC at right angles to it and at the point M 40 mm from B on BC. S is a point on extension of BC 40 mm from C. Draw the loci of points T and S for one complete revolution of driving crank.

For solution see Fig. 4.15 and follow the procedure as given below :

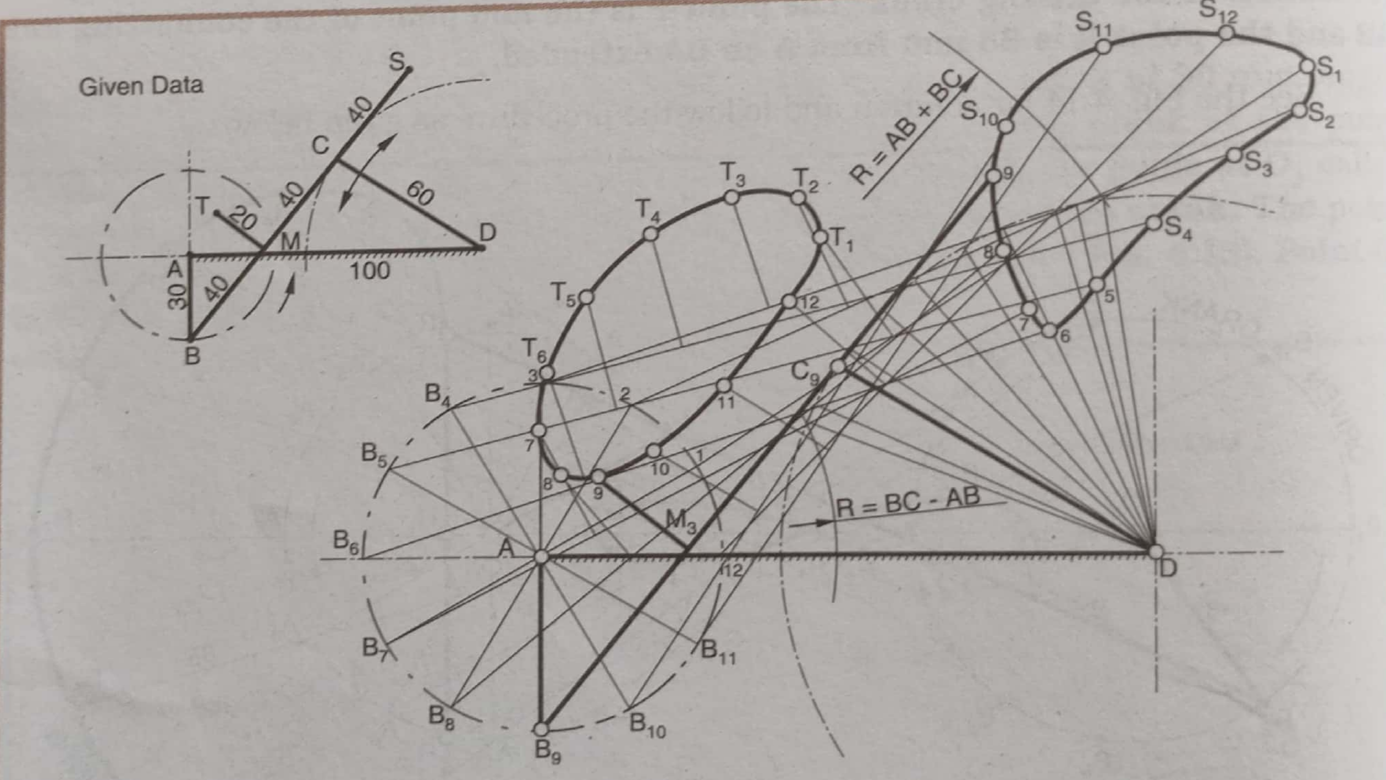


Fig. 4.15. Four Bar Chain. Driver Revolving Driven Oscillating (Problem 15)

First of all we should find whether the driven crank will rotate fully or oscillate. To find that follow the procedure as given below :

(i) Draw a circle ⊙ (D, 60) which is a locus of the point C.

(ii) With A as the centre and (AB + BC) i.e. 30 + 80 = 110 mm as the radius draw an arc of the circle and observe, whether the arc is going beyond the circle ⊙ (D, 60) or cutting it at two points. This is done because the maximum distance that the point C can go from A is [30 + 80 = 110] that is when both the links AB and BC are in one line. If that arc is beyond the circle ⊙ (D, 60), point C can travel anywhere on the circle ⊙ (D, 60) i.e. driven crank can rotate fully. If not, then the two cutting points of the arc with the circle ⊙ (D, 60) are the extreme positions of the point C. Here in this problem other extreme position of the point C is achieved by drawing an arc with A as centre and (BC - AB) i.e. (80 - 30 = 50) as the radius. In this condition BC and AB will be along one line but overlapping.

Now after finding the two extreme positions of the point C on the circle ⊙ (D, 60) follow the procedure as given below :

(i) With A as centre and the length of the driving crank 30 mm as the radius draw a circle. Divide it in 12 equal parts B_1, B_2, ...B_{12}.

(ii) With B_1, B_2,.....B_{12} as the centres and the radius equal to the length of the connecting link BC = 80 mm draw arcs cutting the circle ⊙ (D, 60), between two extreme positions of point C, at points C_1, C_2,......C_{12}. Join B_1C_1, B_2C_2,....$B_{12}C_{12}$ and mark on them points M_1, M_2,....M_{12} at distance 40 mm from points B_1, B_2,......B_{12} respectively.

(iii) At the points M_1, M_2,......M_{12} draw right angles of 20 mm length to the lines B_1C_1, B_2C_2,......$B_{12}C_{12}$ and mark the points T_1, T_2,......T_{12}. Join T_1, T_2,.....T_{12} and T_1 to get the locus of the point T.

(iv) Now extend lines B_1C_1, B_2C_2,......$B_{12}C_{12}$ by 40 mm and get points S_1, S_2,.....S_{12} respectively. Join the points S_1, S_2,......S_{12} and S_1 to get the locus of the point S.

Problem 16 : Fig. 4.16 shows four bar chain mechanism O_1ABO_2 with O_1O_2 as the fixed link. O_1A and O_2B are driver and driven cranks respectively and link AB is a connecting link. Draw the locus of mid point M of the link AB. Dimensions of links are given and initial position is also shown.

For the solution see the Fig. 4.16.

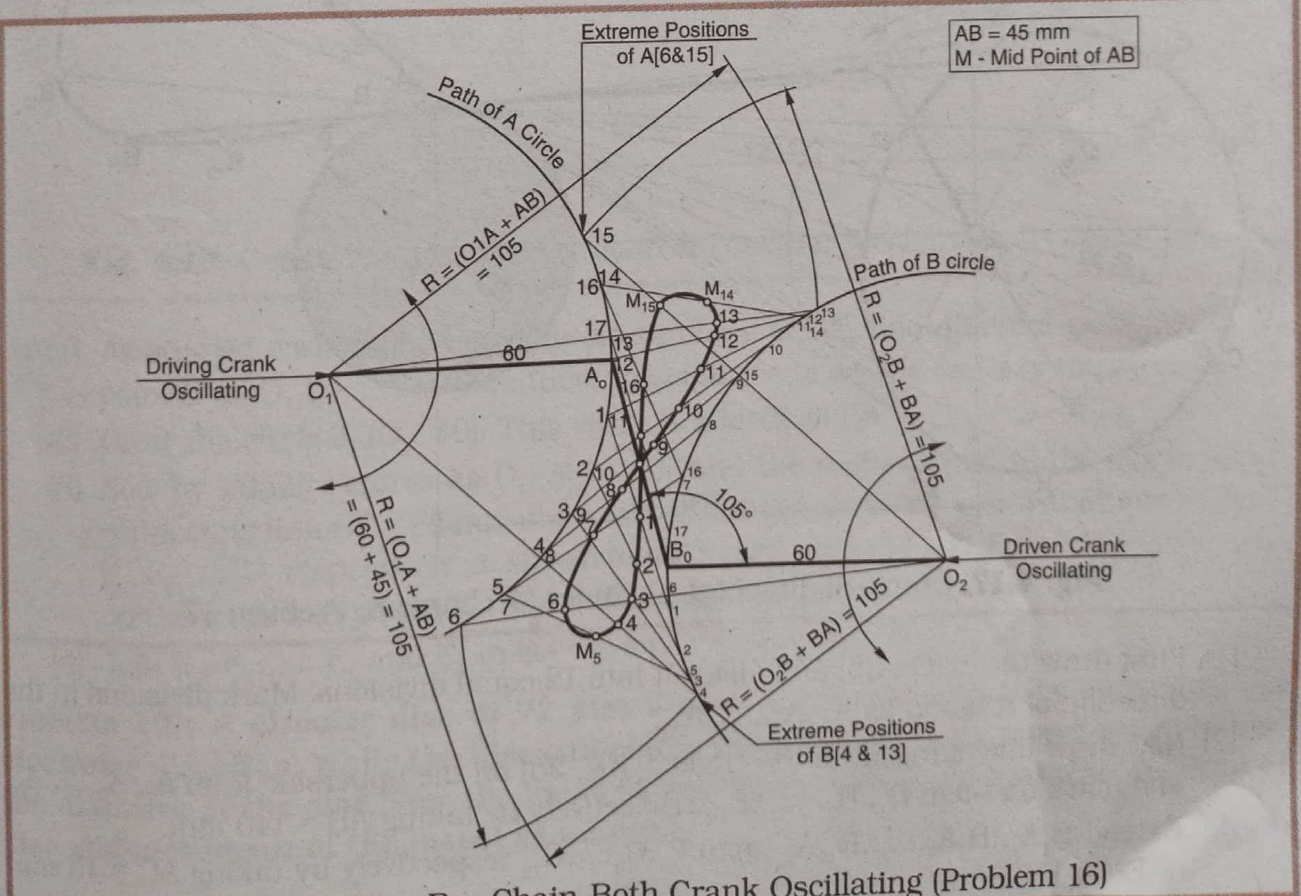


Fig. 4.16. Four Bar Chain-Both Crank Oscillating (Problem 16)

First of all find the extreme positions of the point A on the circle ⊙ $(O_1, 60)$ with the help of intersection of arc of the circle ⊙ $[O_2, (60 + 45)]$. Similarly find extreme positions of the point B on circle ⊙ $[O_2, 60]$ with the help of intersection of the arc of the circle ⊙ $[O_1, (60 + 45]$.

As shown in the figure, between the two extreme positions take different positions of the driver crank at nearly equal intervals, from the given configuration, in the direction of rotation and then back. Mark points A_1, A_2,....A_6.....A_{15}......A_{17}.

Now with these points A_1, A_2,....A_6......A_{15}......A_{17} as centres and the radius equal to the length of connecting link AB, draw arcs of circles to intersect the circle ⊙ $(O_2, 60)$ at points B_1, B_2,.......B_4.......B_{13}.......B_{17}. Join A_1B_1, A_2B_2,.......$A_{17}B_{17}$ and find their mid points M_1, M_2,.......M_{17}. Now join M_1, M_2,....M_{17} and M_1 to get the locus of the point M.

[Note : Points are marked only by suffix due to less available space]

Problem 17 : The crank O_1A is 35 mm long and rotates about the point O_1 in the clock wise direction. The link AB is connected to the crank by turning pair at the point A. The link AB glides/slides over a fixed cylinder for which the circle ⊙ $(O_2, 25)$ is shown in the figure. O_1O_2 = 100 mm, AB = 140 mm, AC = 15 mm ; BC = 155 mm. Draw the loci of the points B and C for one revolution of the crank.

For the solution see Fig. 4.17 and follow the procedure as given below :

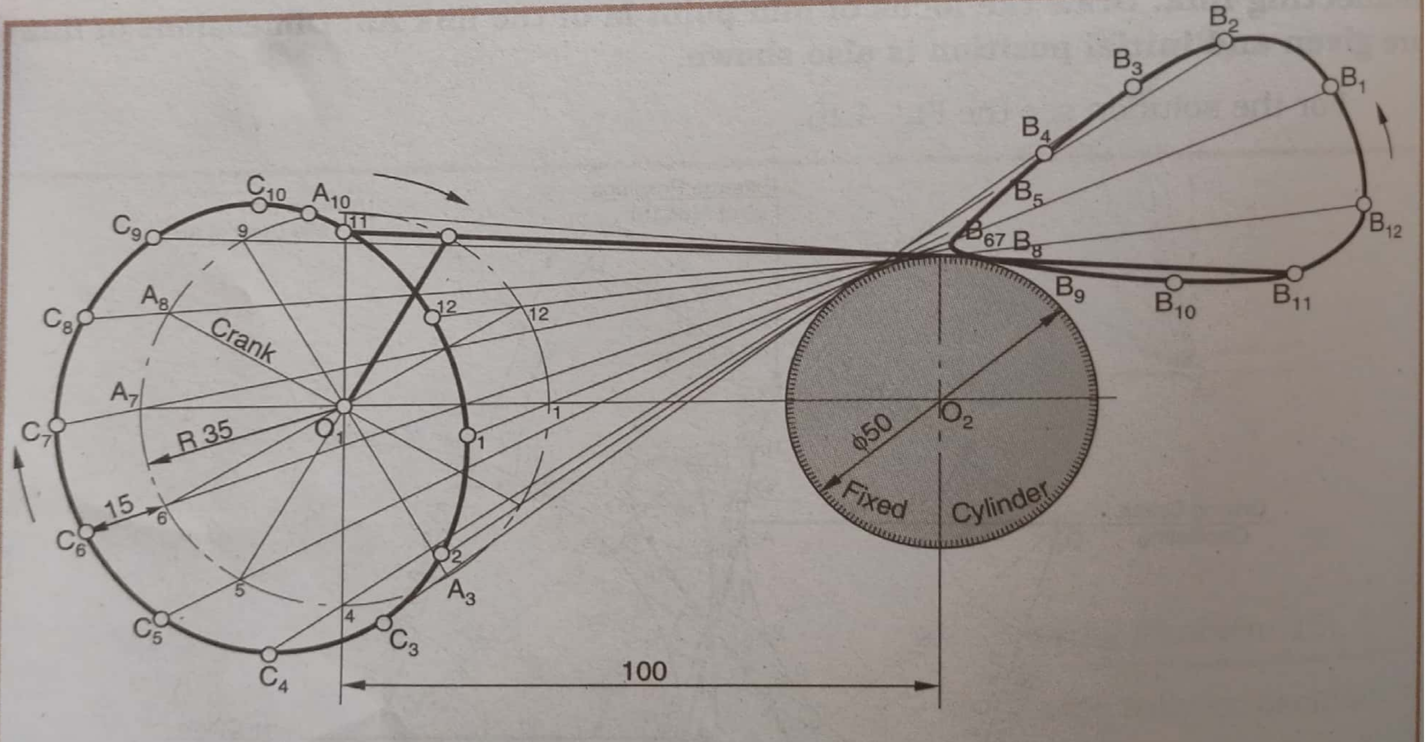


Fig. 4.17. Crank-Sliding Link-Cylinder Mechanism. Problem 17

(1) First draw the ⊙ $(O_1, 35)$ and divide it into 12 equal divisions. Mark divisions in the direction of rotation as A_1, A_2,.......A_{12}.

(2) Now draw lines tangent to the circle ⊙ $(O_2, 25)$ on the upperside from A_1, A_2,.....A_{12} and mark on them B_1, B_2, ...B_{12} respectively by taking AB = 140 mm.

(3) Extend B_1A_1, B_2A_2,.......$B_{12}A_{12}$ upto C_1,C_2,....C_{12} respectively by taking AC = 15 mm or BC = 155 mm.

(4) Join the points B_1, B_2, ...B_{12} and B_1 and C_1, C_2,.......C_{12} and C_1 to get the loci of the points B and C respectively.

Problem 18 : In the mechanism shown in Fig. 4.18, one end of the connecting link DF is connected to the point D of the connecting rod BA by a pin joint/turning pair and other end is connected to the oscillating crank O_2F also by a pin joint or a turning pair. Draw the loci of the point D and the mid point E of the connecting link DF for one revolution of the driving crank O_1B.

For the solution see the Fig. 4.18 and follow the procedure as given below :

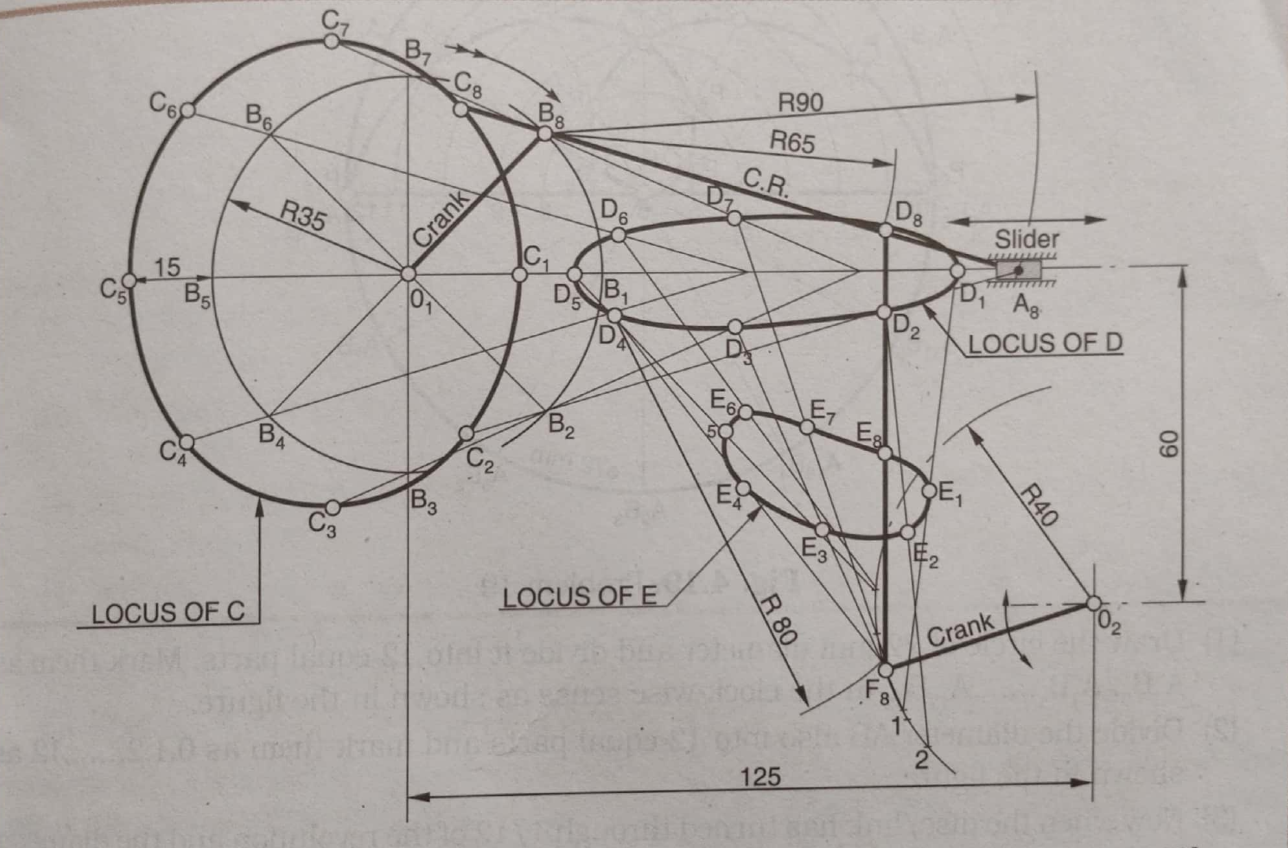


Fig. 4.18. Combine Slider Crank Chain & Four Bar Mechanism [Problem 18]

(1) As studied earlier, for the slider crank chain O_1BA, find different positions of the point D as D_1,D_2,.....D_8. Join them in sequence to get the locus of the point D.

(2) Draw the circle ⊙ (O_2, 40). This circle is a locus of F.

(3) Now by taking centres as D_1, D_2,.....D_8 and the radius equal to the length of the connecting link DF = 80 mm draw arcs to cut the circle ⊙ (O_2, 40) at different points F_1, F_2,......F_8 respectively as shown in the figure. Join D_1F_1, D_2F_2,......D_8F_8 and mark on them the mid points E_1, E_2,......E_8 respectively.

(4) Join E_1, E_2,......E_8 and E_1 to get the locus of the point E.

Problem 19 : A circular disc of 72 mm diameter rotates about its centre in the clockwise direction. While the disc completes one revolution, an insect walks across the diameter of the disc. Plot the locus of the insect, assuming both the rotation of disc and movement of the insect as uniform.

[OR]

A link AB of 72 mm length rotates about its centre in the clockwise direction. While the link completes one revolution, the insect walks across the length from one end to the other. Plot the locus of the insect assuming the rotation of the link and the motion of the insect as uniform.

For the solution see Fig. 4.19 and follow the procedure as given below :

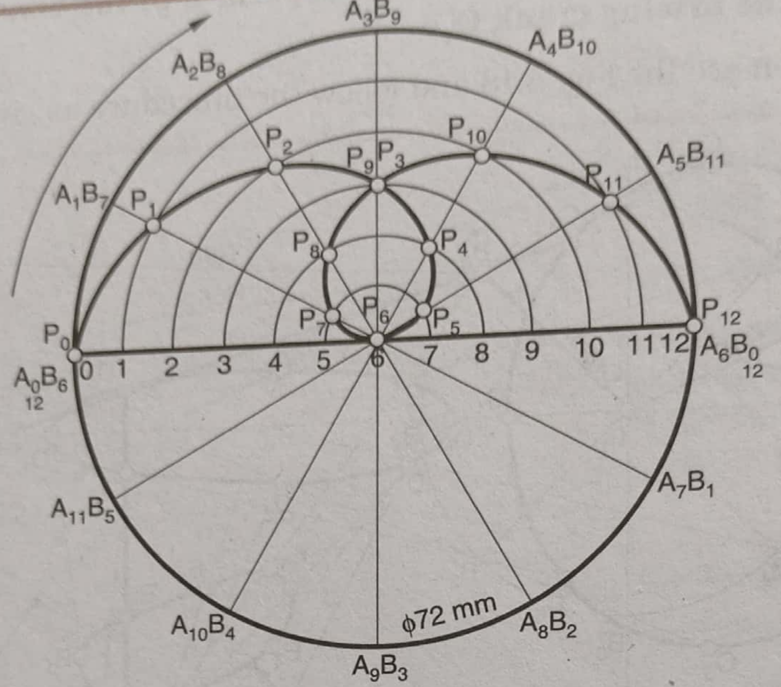


Fig. 4.19. Problem 19

(1) Draw the circle of 72 mm diameter and divide it into 12 equal parts. Mark them as A_0B_0, A_1B_1,.......$A_{12}B_{12}$ in the clock-wise sense as shown in the figure.

(2) Divide the diameter AB also into 12 equal parts and mark them as 0,1,2,.......12 as shown in the figure.

(3) Now when the disc/link has turned through 1/12 of the revolution and the diameter A_0B_0 must have achieved the position A_1B_1 and by that time the insect initially at A_0 must have walked 1/12 of the length of AB from A towards B.
Therefore, draw an arc of the circle passing through 1 to cut A_1B_1 at the point P_1, as shown in figure. Here the link/disc has turned from A_0B_0 to A_1B_1 and by that time the insect has moved by the distance A_1P_1(1/12 of diameter of disc or length of link).

(4) In the similar manner draw arcs of circles through the points 2,3,......12 to cut the positions A_2B_2, A_3B_3,.......$A_{12}B_{12}$ at points P_2, P_3,.......P_{12} respectively. Join P_0, P_1,....... P_{12} in a sequence to get the path of the insect as shown in the figure.

Problem 20 : As seen in the plan, AD and DB are two equal size portions of a folding door hinged joint or pinned joint at D. Span CB of the door is 150 mm. The end B is fixed and the end A is constrained to move along the line BC. Draw the locus of the mid point P of AD for a complete movement of the folding door.

For the solution see Fig. 4.20 and follow the procedure as given below :

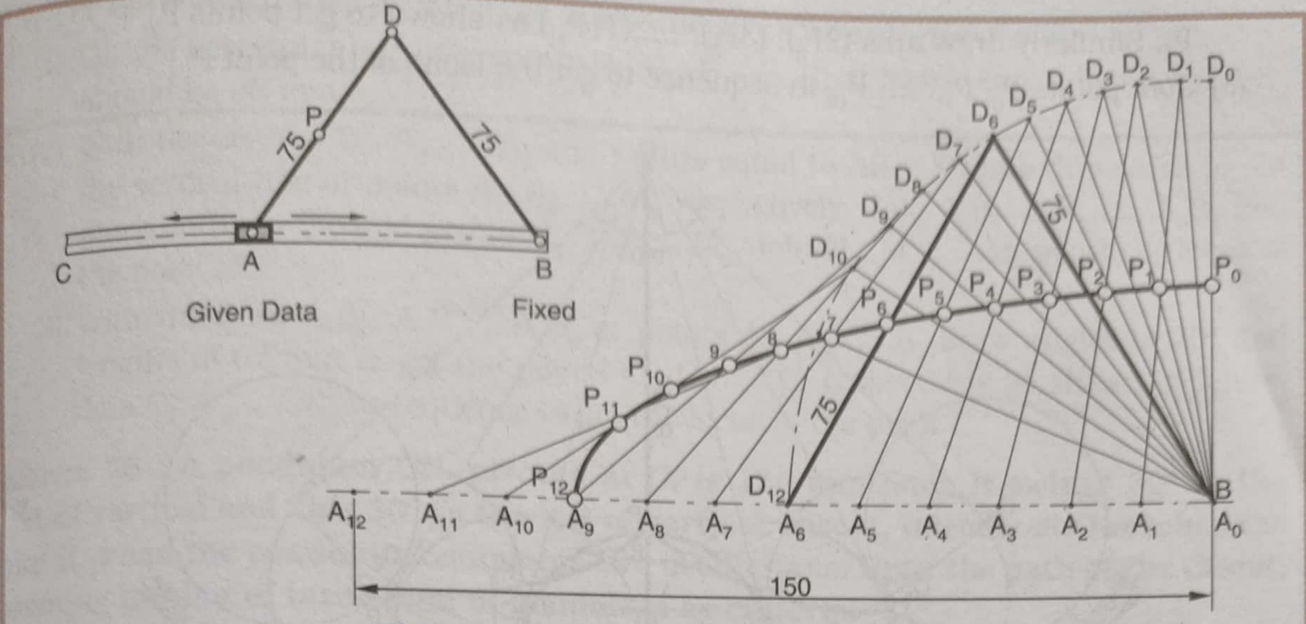


Fig. 4.20. Folding Door [Problem-20]

(1) Take the complete open position of the door as the initial position i.e. BD_0A_0 as shown in the figure.

(2) The point D will move along 1/4th of the circle ⊙ (B,75) and therefore, draw that 1/4 circle.

(3) Divide the total movement of A i.e. BC = 150 mm into 12 equal parts and mark them as $A_0, A_1, \ldots\ldots A_{12}$.

(4) With $A_0, A_1, \ldots\ldots A_{12}$ as the centres and the radius equal to AD = 75 mm draw arcs to cut the previously drawn 1/4th of the circle at points $D_0, D_1, \ldots\ldots D_{12}$ respectively. Join $A_0D_0, A_1D_1, \ldots\ldots A_{12}D_{12}$ and mark on them their mid points $P_0, P_1, \ldots\ldots P_{12}$ respectively.

(5) Join $P_0, P_1, \ldots\ldots P_{12}$ in sequence to get the locus of the mid point P of AD.

Problem 21 : Link OC, hinged at O, is 100 mm long. It carries a circular disc at C of radius 25 mm capable of rotating about the centre point C. Link, OC initially vertical, turns uniformly towards the right side by an angle of 45° and then towards the left side by the total angle 90° and then to the initial vertical position and during the same time the disc revolves uniformly in the clockwise sense through one complete revolution. Draw the locus of the point P on the disc, initially at the lowest position.

For the solution see Fig. 4.21 and follow the procedure as given below :

(1) Draw vertical line OC_0 of 100 mm length and draw the circle with C_0 as the centre and 25 mm as the radius to represent the initial position. Mark P_0 on the disc at the lowest position.

(2) At O, draw 45° angle to the right as well as to the left of the vertical OC_0 and divide each of them into three equal parts of 15°. In this way the total movement of the link OC is divided into 12 equal parts. Mark the positions of the centre C as $C_0, C_1, C_2, \ldots\ldots C_{12}$ as shown in the figure.

(3) Divide the circle of the disc into 12 equal parts 1,2,.....12 as shown.

(4) When the link has turned through 1/12th of its total angular movement(OC_0 □ OC_1) the line CP of the disc must have turned by 30° which is 1/12th of one revolution. So draw arc of a circle $1P_1$ with o as centre as shown in the figure and mark point

P_1. Similarly draw arcs ($2P_2$), ($3P_3$), ...($11P_{11}$) as shown to get points P_2, P_3, ...P_{11}.

(5) Join points P_0, P_1........P_{12} in sequence to get the locus of the point P.

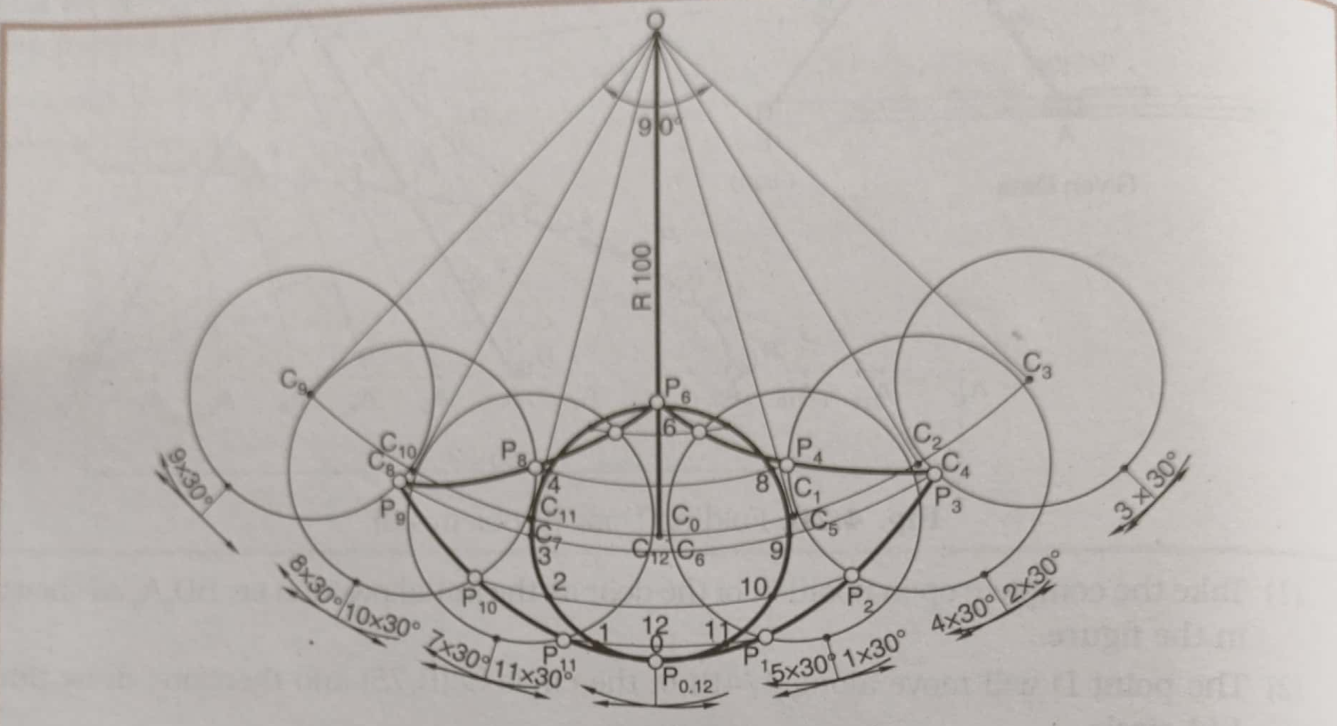


Fig. 4.21. Problem 21

Problem 22 : AB and AC are two links welded together at the point A at an angle of 70° to each other. The ends A and B of the link AB are constrained to slide in the vertical and the horizontal guides respectively. Draw the loci of the point C and the mid point M of the link AB as the link AB moves from the vertical to the horizontal position. AB = 80 mm and AC = 67 mm.

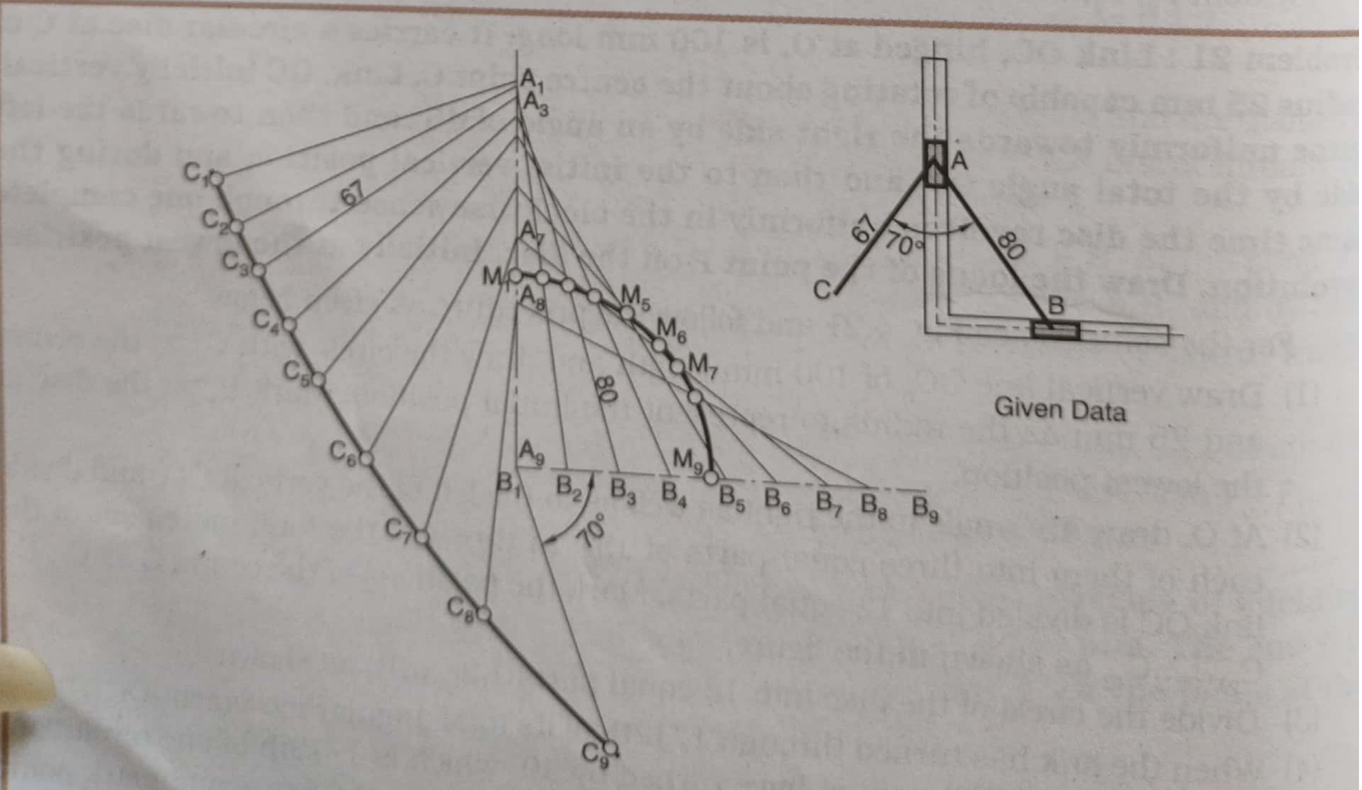


Fig. 4.22. Problem 22

For the solution see Fig. 4.22 and follow the procedure as given below :

(1) On the horizontal line take points B_1, B_2, B_3,.......B_9 at nearly equal distances. B_1B_9 should be 80 mm.

(2) With the centres B_1, B_2,.......B_9 and radius equal to AB = 80 mm draw arcs to cut the vertical line at points A_1, A_2, ...A_9 respectively. Join A_1B_1, A_2B_2,.......A_9B_9 and mark on them the mid points M_1, M_2,.......M_9. Join M_1, M_2,.......M_9 to get the locus of the point M.

(3) With the lines A_1B_1, A_2B_2,....A_9B_9 at points A_1, A_2,.......A_9 draw angles of 70° and lengths of 67 mm to get the points C_1, C_2,.......C_9 respectively as shown in figure. Join C_1, C_2,.......C_9 in sequence to get the locus of the point C.

Problem 23 : A pendulum OC, pivoted at O, is 120 mm long. It swings 30° to the right of vertical and also 30° to the left of vertical. Insect, initially at O reaches the point C, when the pendulum completes two oscillations. Draw the path of the insect, assuming motion of insect and of pendulum as uniform.

For the solution see Fig.No. 4.23 and follow the procedure as given below.

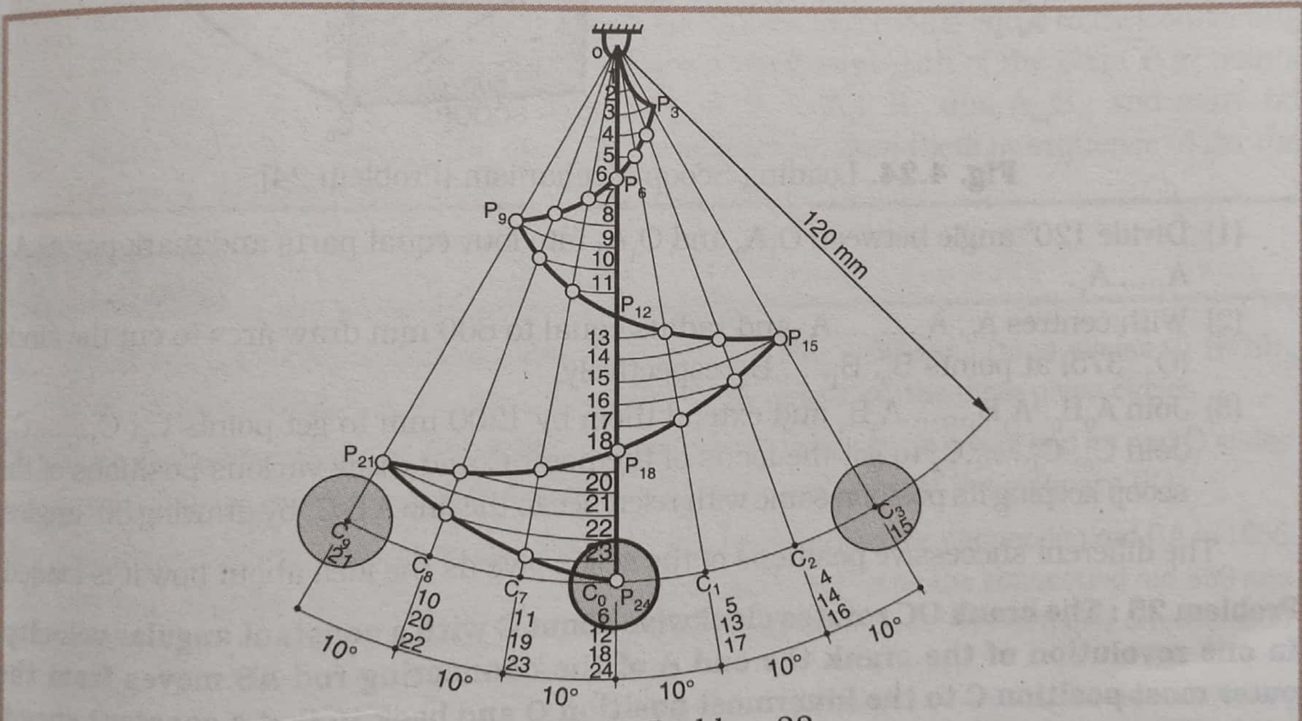


Fig. 4.23. Problem 23

(1) As the total angle of oscillation is (30 + 30 + 30 + 30) = 120°, divide it into 12 divisions by taking each division of 10° as shown.

(2) Divide the total motion of insect of 120 mm into (12x2) = 24 divisions by taking 5 mm each as shown.

(3) Rest is clear from the figure.

Problem 24 : Fig. No. 4.24 shows a mechanism for a loading scoop. O_1A is a crank of length 450 mm pivoted at O_1. AB is a connecting link of 600 mm length and its extension BC is 1200 mm long. O_2B is a crank of 375 mm length pivoted at O_2. At the point C the scoop is welded. Draw different positions of the scoop and the locus of the point C when the crank O_1A moves downward from O_1A_0 to O_1A_4 through and angle of 120°.

For the solution see Fig. 4.24 and follow the procedure as given below :

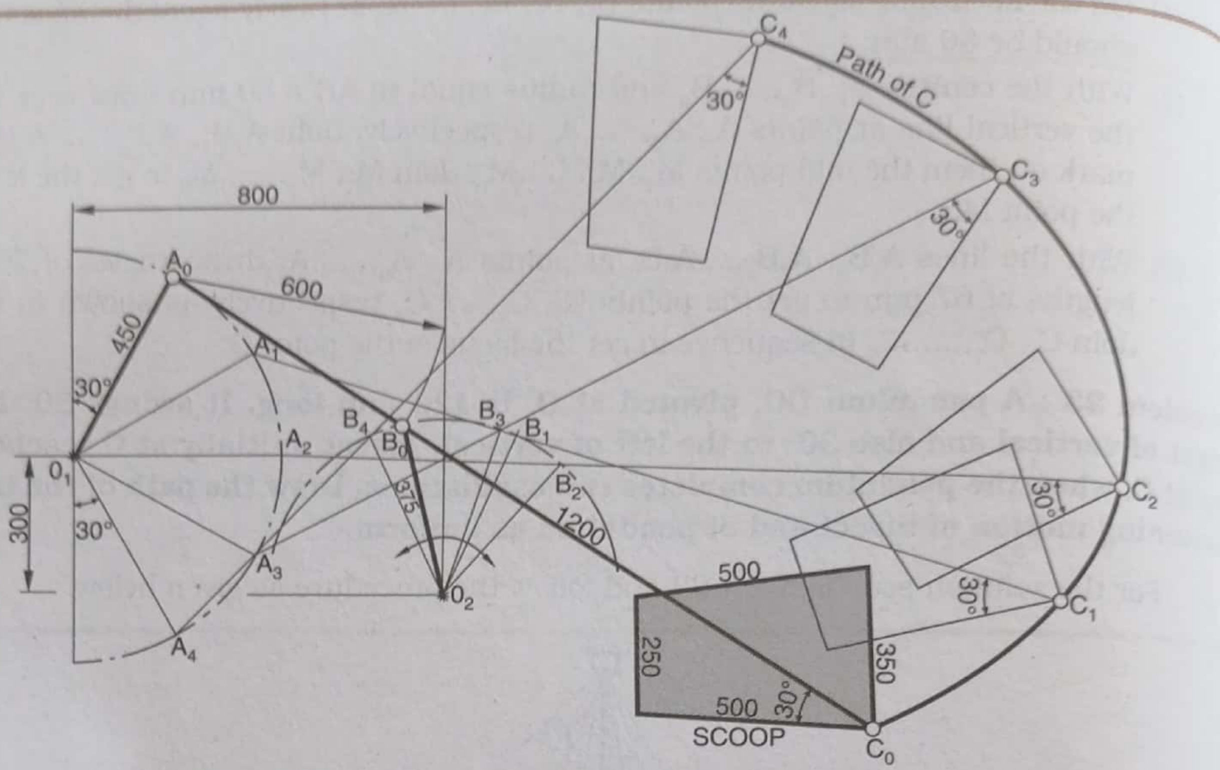


Fig. 4.24. Loading Scoop Mechanism [Problem 24]

(1) Divide 120° angle between O_1A_0 and O_1A_4 into four equal parts and mark points A_0, A_1,....A_4.

(2) With centres A_0, A_1,........A_4 and radius equal to 600 mm draw arcs to cut the circle (O_2, 375) at points B_0, B_1, ...B_4 respectively.

(3) Join A_0B_0, A_1B_1,.......A_4B_4 and extend them by 1200 mm to get points C_0, C_1,.......C_4. Join C_0, C_1,.......C_4 to get the locus of the point C and draw various positions of the scoop keeping its position same with reference to the line A B C, by drawing 30° angles.

The different successive positions of the scoop give us the idea about how it is loaded.

Problem 25 : The crank OC rotates clockwise about O with a constant angular velocity. In one revolution of the crank the end A of the connecting rod AB moves from the outer most position C to the innermost position O and back to C at a constant speed while the end B moves along a horizontal line through O. Draw the loci of the point A and the mid point M of AB for one revolution of the crank OC.

For the solution see Fig. 4.25 and follow the procedure as given below :

(1) With O as the centre and radius equal to the crank length OC = 48 mm, draw a circle and divide the circle into 12 equal parts.

(2) As the end A moves on the crank from the outermost to the innermost position and back, divide the crank length into 12/2 = 6 equal parts and mark them as a, b,.....e, f as shown in the figure.

(3) When the crank has turned through 1/12 of a revolution the point A must have moved inside by 1/6. OA and so mark A_{1a} on O_1 by drawing an arc through a. Similarly mark points A_{2b}, A_{3c},....A_{11a} and A_{12}. Join A_0, A_{1a}, A_{2b},....A_{11a} and A_{12} in sequence to get the locus of the point A.

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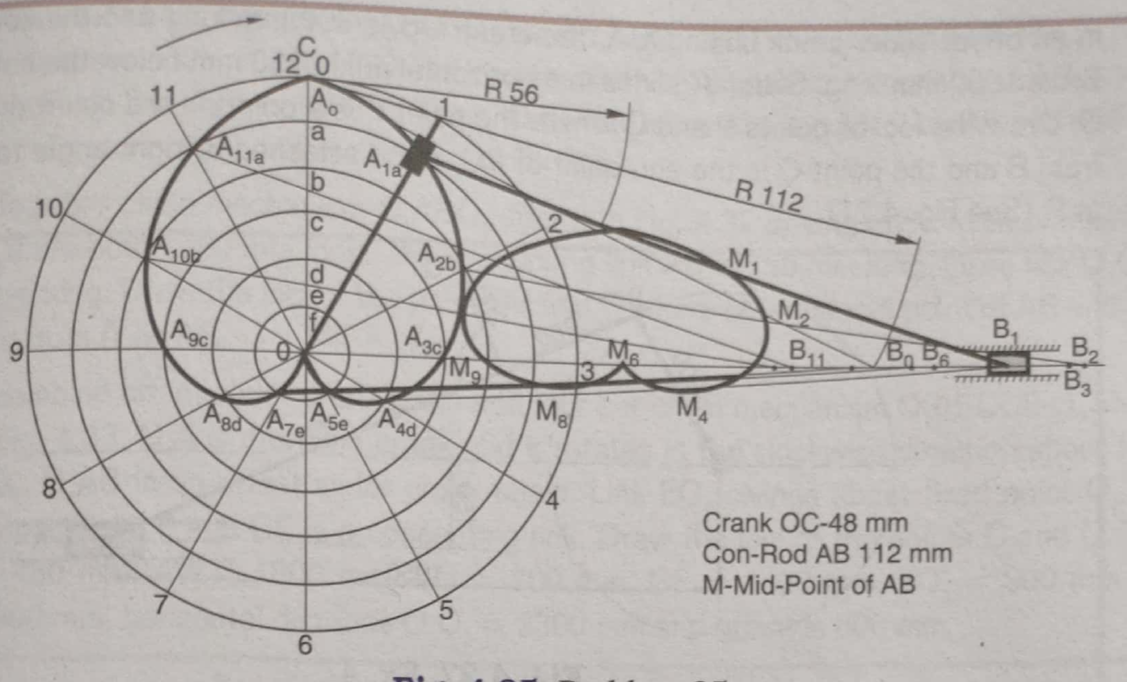


Fig. 4.25. Problem 25

(4) Now with A_0, A_{1a}, A_{2b},....A_{11a} and A_{12} as the centres and radius equal to the connecting rod length AB = 112 mm, draw arcs to cut horizontal path of the point B at points B_0, B_1,B_2,....B_{12}, respectively. Join A_0B_0, $A_{1a}B_1$,....A_{11a} B_{11} and $A_{12}B_{12}$ and mark on them the mid points M_0, M_1,M_2,....M_{12} respectively. Join them in sequence to get the locus of the point M.

EXERCISE

1. ΔPQR has sides PQ = 100 mm, QR = 75 mm and RP = 50 mm. Three circles ⊙ (P,30), ⊙ (Q, 40) and ⊙ (R, 15) are given. Draw two circles touching the three given circles.
2. The end P of the staircase PQ, 3000 mm long, slides vertically on a wall and its end Q slides horizontally away from the wall. Find the locus of the mid point of the staircase PQ.
3. In a slider crank chain OBA, the crank OB is 350 mm long and the connecting rod BA is 1050 mm long. Plot the loci of points P, Q and R where (i) Point P is on the connecting rod 350 mm from B, (ii) Point R is on extension of C.R. BA and 250mm from A (iii) Point Q is on extension of C.R. AB and 500 mm from B, see Fig. 4.26.

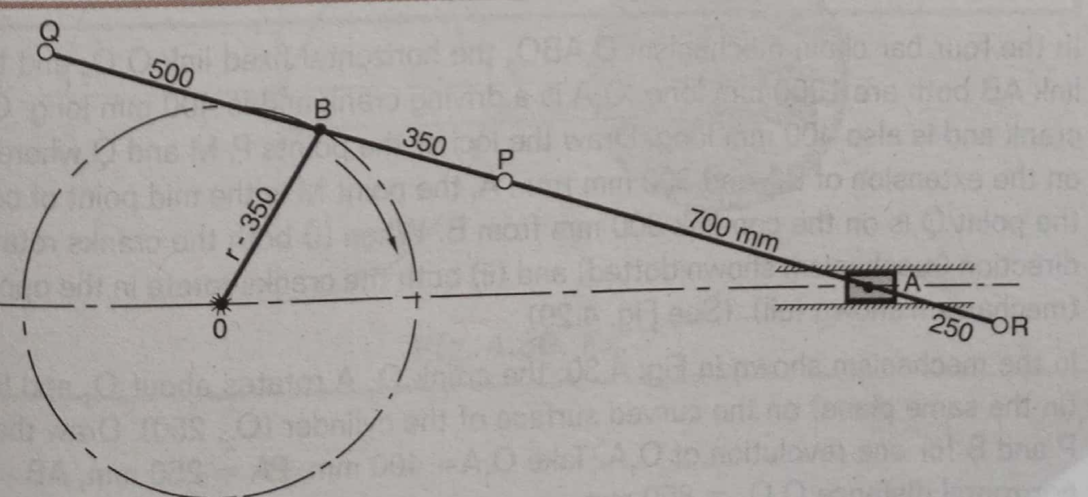


Fig. 4.26. Ex. 3

4. In an offset slider crank chain OBA, the crank OB is 300 mm long and the connecting rod BA is 1000 mm long. Slider 'A' slides in a horizontal guide 150 mm below the horizontal from O. Draw the loci of points P and Q where the point P is a point on the con-rod BA, 250 mm from B and the point Q is the end point of PQ, a rod attached at right angle to con-rod AB at P. (See Fig. 4.27)

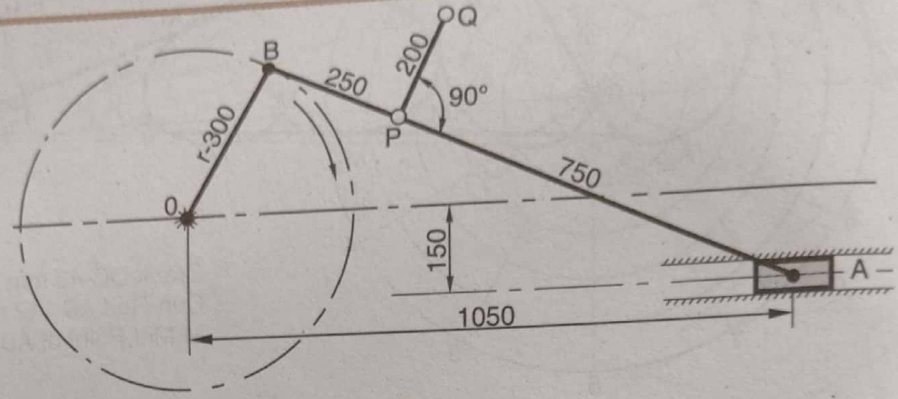


Fig. 4.27. Ex. 4

5. In the crank-connecting rod-trunnion mechanism shown in Fig. 4.28; crank OB is 400 mm long. Connecting rod BA is 1800 mm long and trunnion C is located 1250 mm on the right of O and 150 mm below O. Draw the loci of the points A, P and M where the point P is on the extension of AB and 300 mm from B and the point M is the mid point of AB.

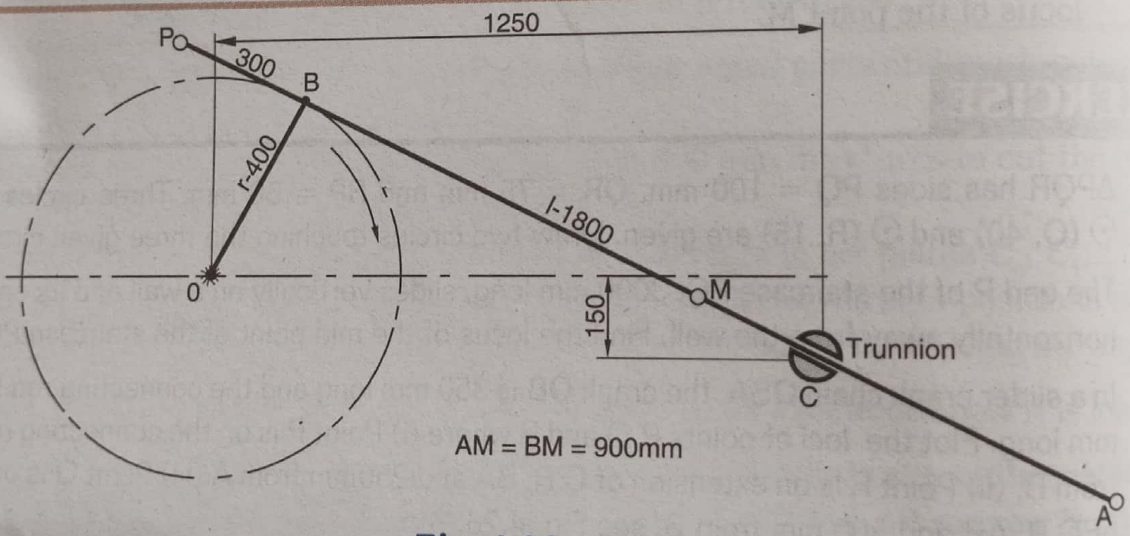


Fig. 4.28. Ex. 5

6. In the four bar chain mechanism O_1ABO_2 the horizontal fixed link O_1O_2 and the connecting link AB both are 1200 mm long. O_1A is a driving crank and is 400 mm long. O_2B is a driven crank and is also 400 mm long. Draw the loci of the points P, M and Q where the point P is on the extension of BA and 300 mm from A, the point M is the mid point of con-link AB and the point Q is on the con-link 300 mm from B. When (i) both the cranks rotate in the same direction (mechanism shown dotted) and (ii) both the cranks rotate in the opposite direction (mechanism shown full). (See Fig. 4.29)

7. In the mechanism shown in Fig. 4.30, the crank O_1 A rotates about O_1 and link PAB slides (in the same plane) on the curved surface of the cylinder (O_2, 250). Draw the loci of points P and B for one revolution of O_1A. Take O_1A= 400 mm, PA = 250 mm, AB - 1200 mm and horizontal distance O_1O_2 = 850 mm.

8. In the four bar chain mechanism O_1ABO_2 shown in Fig. 4.31, two cranks O_1A (driver) and O_2B (driven) are 400 mm and 1000 mm long respectively and the connecting link AB is 1300 mm long. Fixed link O_1O_2 is 1600 mm long. Draw the locus of the point P, on AB and 400 mm from A, for one revolution of the driving crank O_1A.

9. In the four bar chain mechanism O_1ABO_2 shown in Fig. 4.32 driving and driven cranks O_1A and O_2B are both 1000 mm long. The connecting link AB is 750 mm long. Fixed link O_1O_2 is 2500 mm long. Draw the loci of the points M and Q where M is the mid point of AB and Q is 200 mm from A on AB.

10. In the combine offset slider crank chain and four bar chain mechanism $O_1ABCDE\ O_2$ shown in the Fig. 4.33, O_1A is a driving crank and it rotates in the clockwise direction about fixed point O_1. O_1AB is an offset slider crank chain. Link EO_2 swings about fixed point O_2. AB is a connecting rod and CE is a connecting link. Draw the loci of the points C and D. Take O_1A = 450 mm, AB = 1800 mm, BC = 700 mm, CE = 1400 mm, EO_2 = 900 mm and DE = 900 mm, horizontal distance O_1O_2 = 2300 mm and offset is 600 mm.

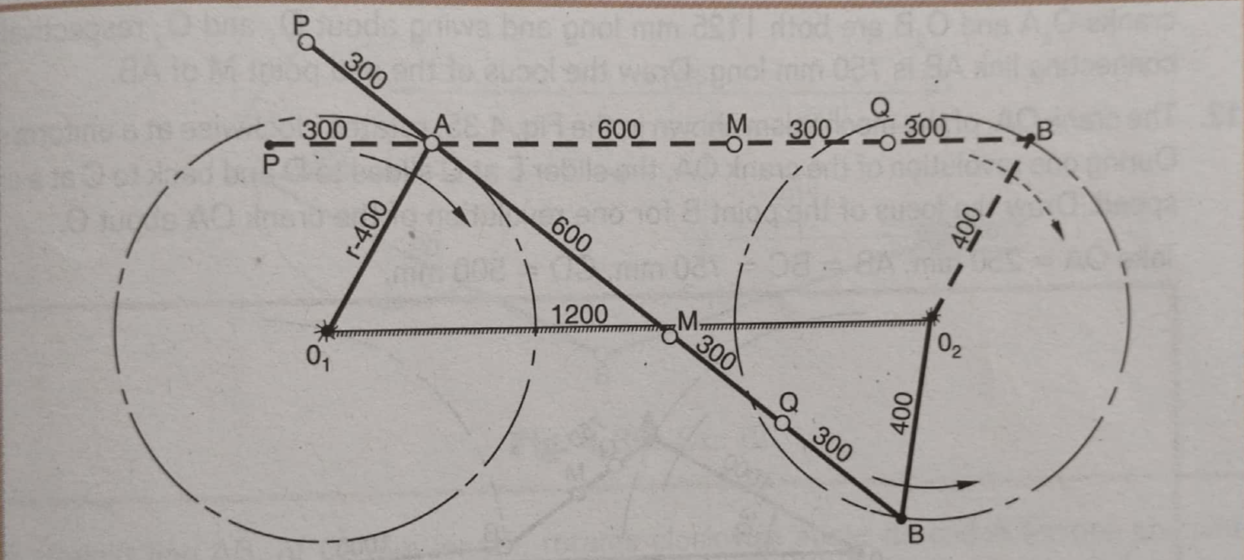


Fig. 4.29. Ex. 6

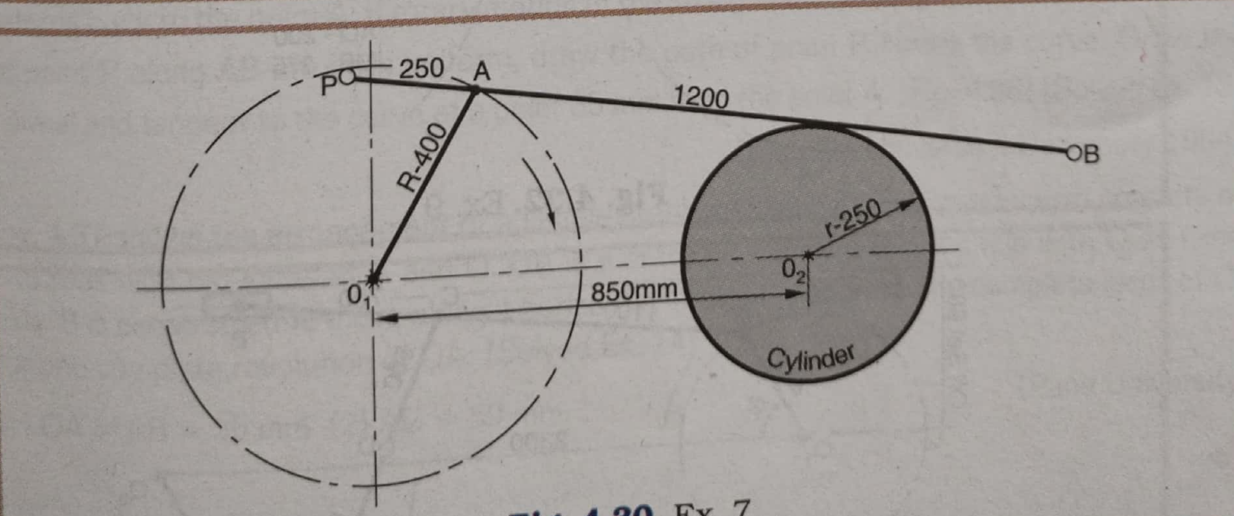


Fig. 4.30. Ex. 7

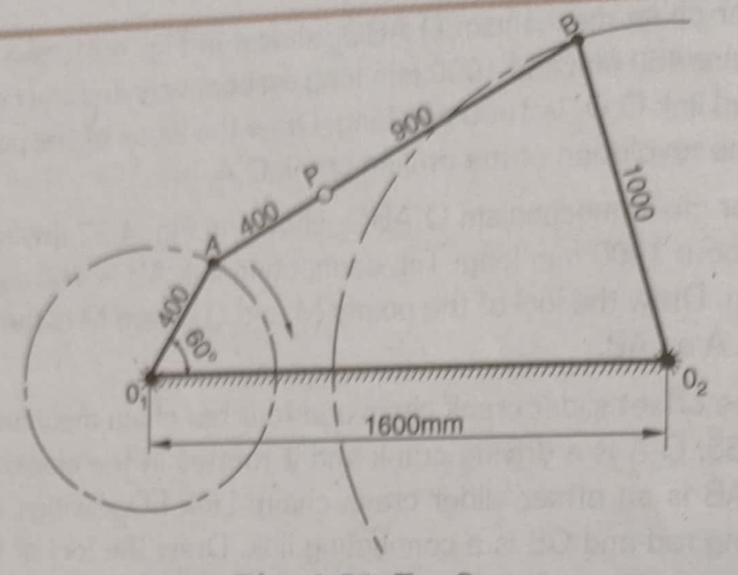


Fig. 4.31. Ex. 8

11. A modified Watt's straight line motion mechanism is shown in Fig. 4.34. Driving and driven cranks O_1A and O_2B are both 1125 mm long and swing about O_1 and O_2 respectively. The connecting link AB is 750 mm long. Draw the locus of the mid point M of AB.

12. The crank OA, of the mechanism shown in the Fig. 4.35, rotates clockwise at a uniform speed. During one revolution of the crank OA, the slider E at C slides to D and back to C at a uniform speed. Draw the locus of the point B for one revolution of the crank OA about O.

Take OA = 250 mm, AB = BC = 750 mm, CD = 500 mm.

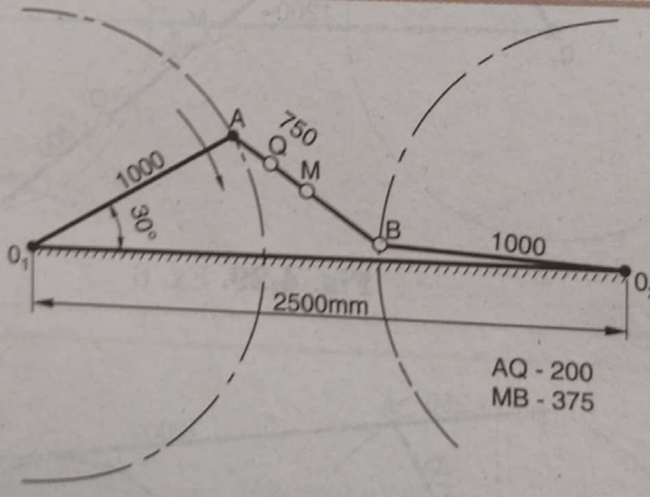


Fig. 4.32. Ex. 9

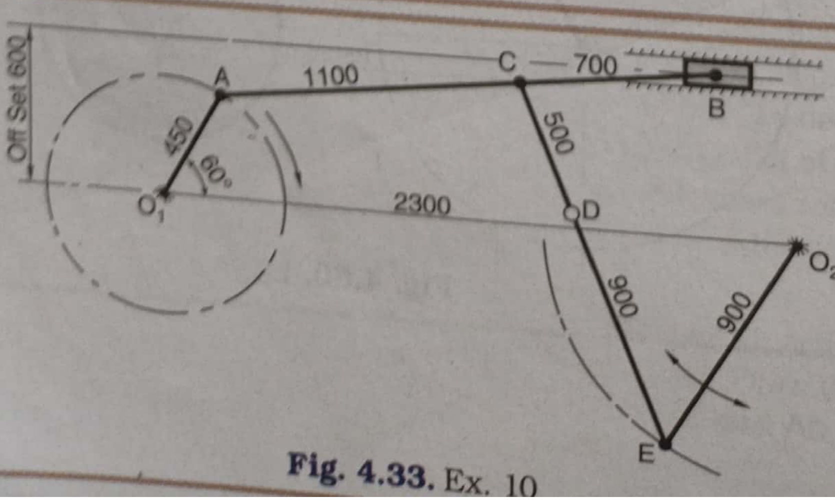


Fig. 4.33. Ex. 10

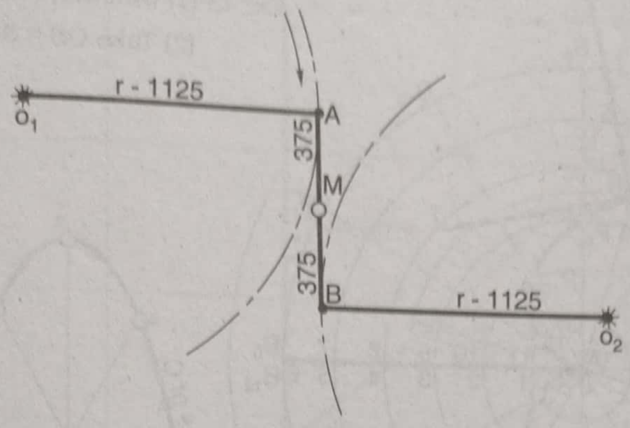


Fig. 4.34. Ex. 11

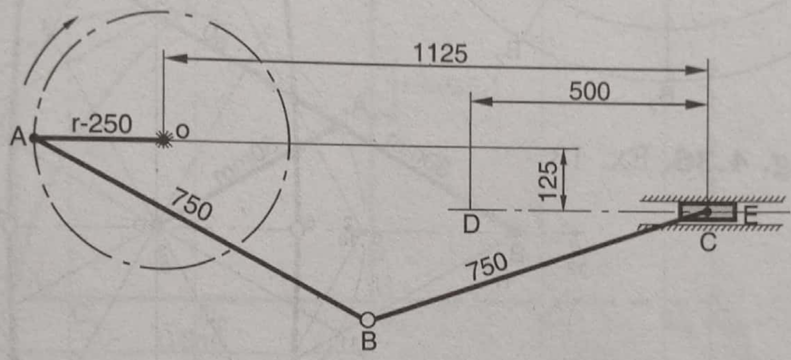


Fig. 4.35. Ex. 12

13. A straight line AB, of 60 mm length, rotates clockwise about its end A for one complete revolution and during this period a point P moves along the straight line from A to B and returns back to the point A. If rotary motion of the straight line about point A and linear motion of point P along AB are both uniform, draw the path of point P. Name the curve. Draw the normal and tangent to the curve at a point 35 mm from the point A. [Fig. 4.36] [Solved Ex. 13]

[Mumbai University, December 1994]

14. Fig. 4.37 shows the arrangement of a simple crank mechanism. This mechanism consists of two rods with pin joints at A and O. OA is a crank. A moves along a circle with O as fixed axis. B is constrained to move along a horizontal straight line. Trace the complete locus of C, for one complete revolution of OA. [Solved Ex. 14]

(1) OA = AB = 30 mm (2) AC = 50 mm. [Pune University]

Ex. 13 (1) (Answer) : Archemedian Spiral.

(2) Take OS = 35 mm.

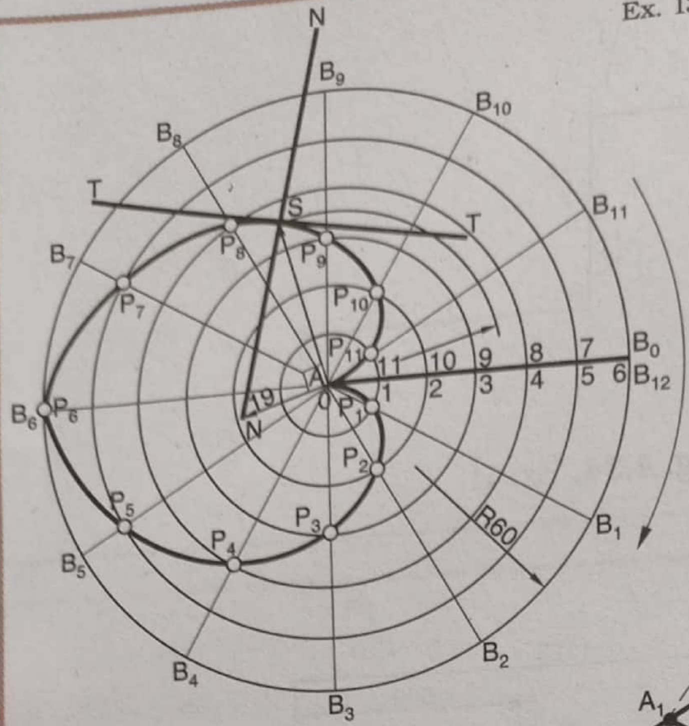


Fig. 4.36. Ex. 13

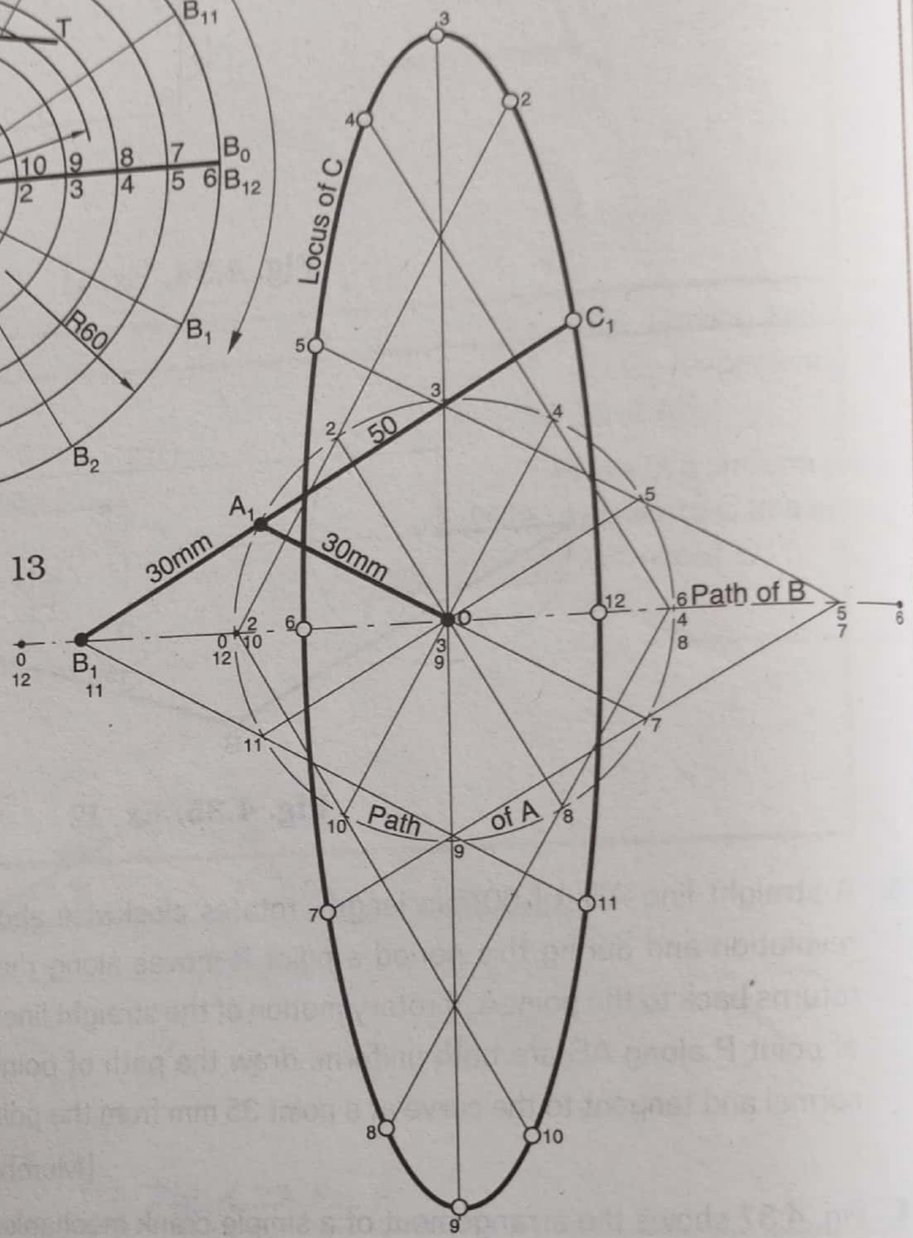


Fig. 4.37. Ex. 14

15. A rectangular door ABCD has its vertical edge AB 2 m long and a horizontal edge BC 0.8 m long. It is rotated about the hinged vertical edge AB as the axis and at the same time, a fly x moves from point C towards D and another fly Y moves from A towards D. By the time, the door rotates through 180°, both the flies reach point D. Using suitable scale, trace the paths of the flies in elevation and plan if the motions of the flies and the door are uniform. Name the curve traced out by the flies. Assume the door to be parallel to V. P. in initial position and the thickness of the door equal to that of your line.

[Fig. 4.38] [Solved Ex. 15]

[Mumbai University, December 1995]

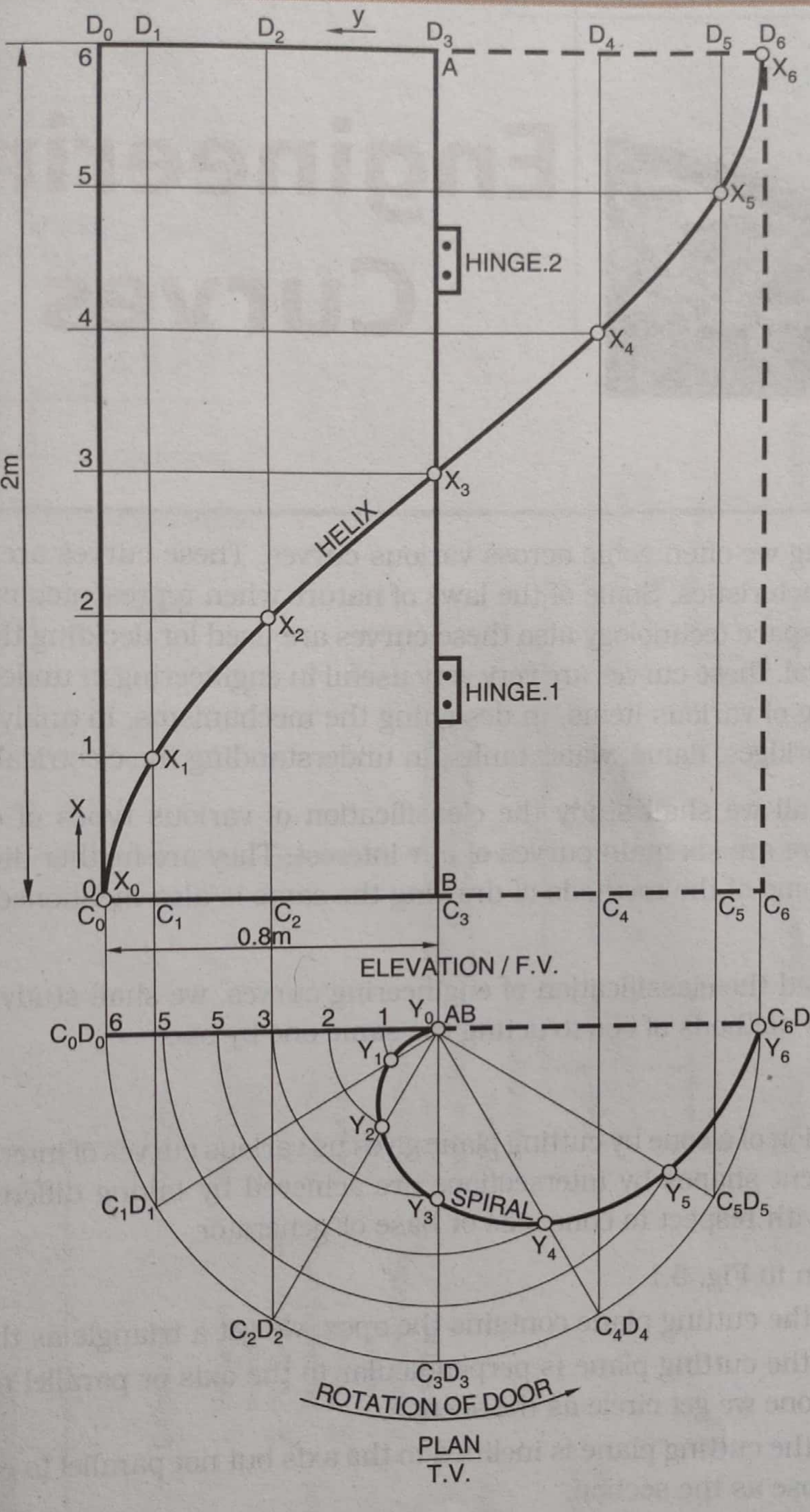
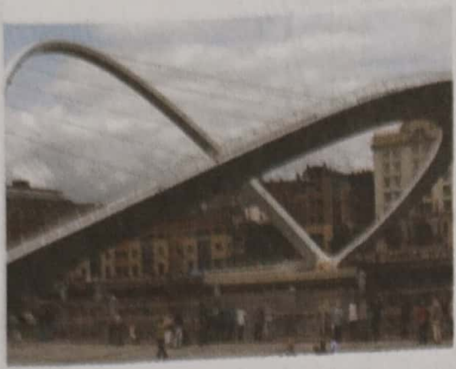

D0 D1 D2 y D3 D4 D5 D6
6
A
X6
5
X5
HINGE.2
4
X4
2m
3
X3
HELIX
2
X2
HINGE.1
1
X1
X
0 X0
B
C0 C1 C2 C3 C4 C5 C6
0.8m
ELEVATION / F.V.
C0D0 6 5 5 3 2 1 Y0 AB
C6D6
Y1
Y6
Y2
Y5
C1D1
Y3
SPIRAL
Y4
C5D5
C2D2
C4D4
C3D3
ROTATION OF DOOR
PLAN
T.V.

Fig. 4.38. Ex. 15

CHAPTER - 5



Engineering Curves

In engineering we often come across various curves. These curves are useful by the nature and characteristics. Some of the laws of nature when represented on graph give u these curves. In space technology also these curves are used for deciding the path of spac vehicles. In general, these curves are very very useful in engineering in understanding law in manufacturing of various items, in designing the mechanisms, in analysis of forces, i construction of bridges, dams, water tanks, in understanding the electrical power, etc.

Now first of all we shall study the classification of various types of curves used i engineering. There are six main curves of our interest. They are further divided as show in chart form. Some of the methods of drawing the same is also mentioned. For chart s next page.

Having studied the classification of engineering curves, we shall study its definition uses and various methods of constructing the same one by one.

I. CONICS

The intersection of a cone by cutting plane gives us various curves of intersections know as conics. Different shapes by intersections are achieved by taking different positions cutting planes, with respect to cone axis or base or generator.

This is shown in Fig. 5.1

(A) When the cutting plane contains the apex, we get a triangle as the section.

(B) When the cutting plane is perpendicular to the axis or parallel to the base in right cone we get circle as the section.

(C) When the cutting plane is inclined to the axis but not parallel to generator we g an ellipse as the section.

(D) When the cutting plane is inclined to the axis and parallel to one of the generato of the cone we get a parabola as the section.

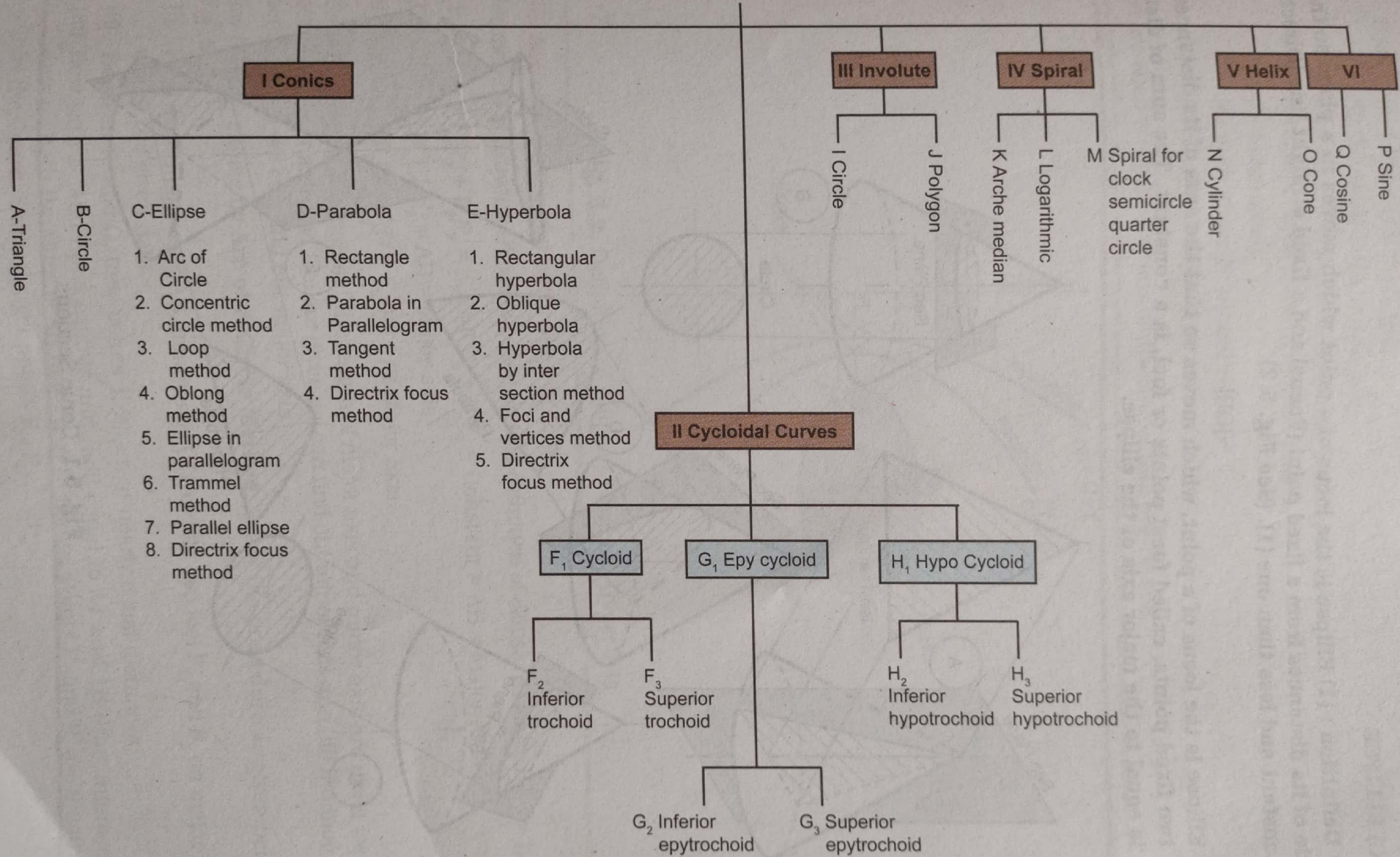
(E) When the cutting plane is parallel to the axis we get a hyperbola as the sectio

[OR]

(E) When the inclined cutting plane to the base of cone cuts both the portions double cone, we get a hyperbola as the section.

1.A and 1.B triangle and circle respectively are very common and hence not discusse

ENGINEERING CURVES



I. (C) ELLIPSE

Definition : (1) Ellipse is the locus of a point which moves in a plane so that t ratio of its distances from a fixed point (Focus) and a fixed straight line (Directrix) a constant and less than one (1). (See Fig. 5.2)

[OR]

(2) **Ellipse is the locus of a point, which moves so that the sum of its distances fr two fixed points, called focal points or foci, is a constant. The sum of distanc is equal to the major axis of the ellipse.**

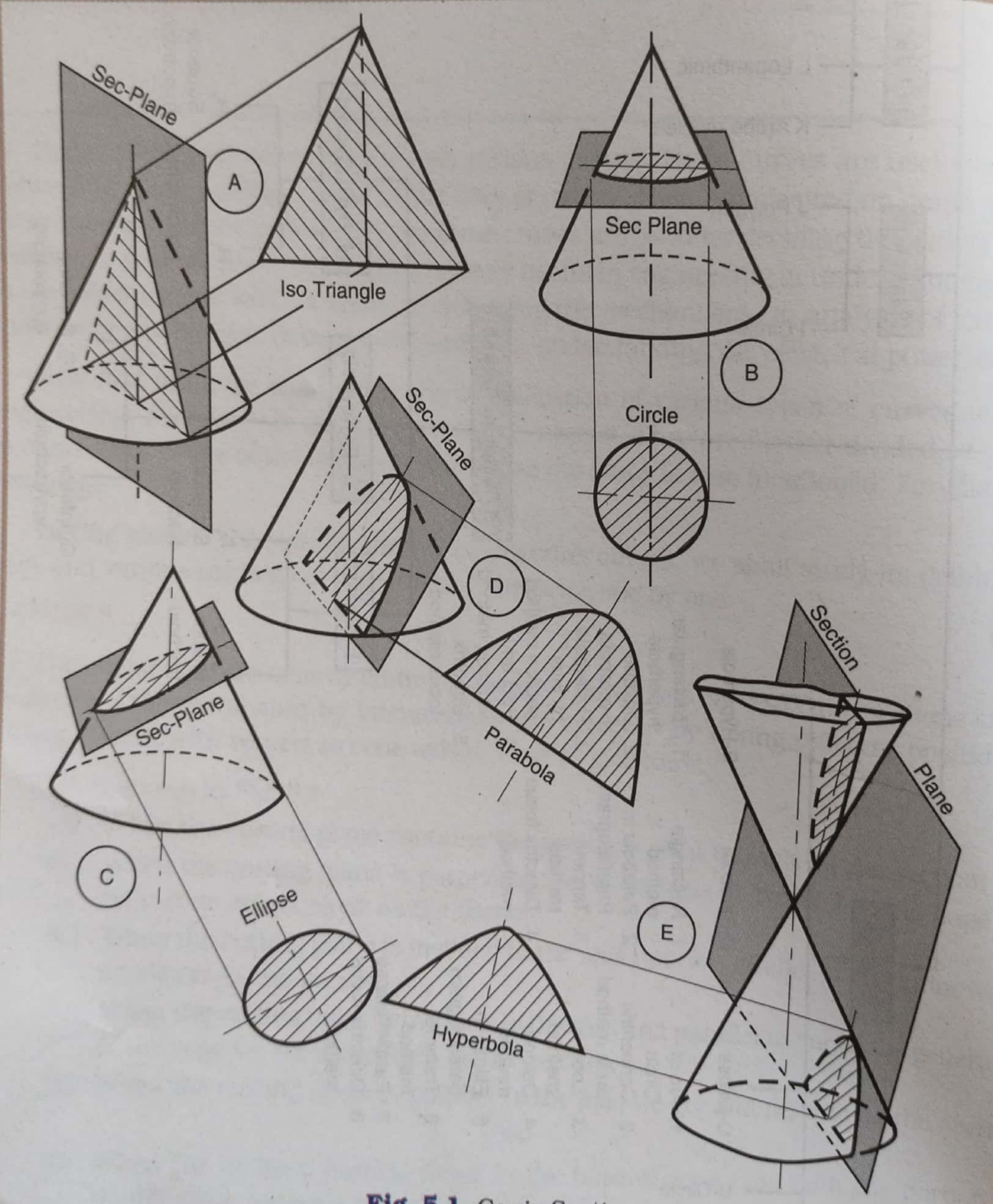


Fig. 5.1. Conic Sections

Uses : (1) Shape of a man-hole
(2) Shape of tank in a tanker
(3) Flanges of pipes, glands and stuffing boxes
(4) Shape used in bridges and arches
(5) Monuments
(6) Path of earth around the sun
(7) Shape of trays etc.

Now we shall study 8 methods of drawing ellipse one by one.

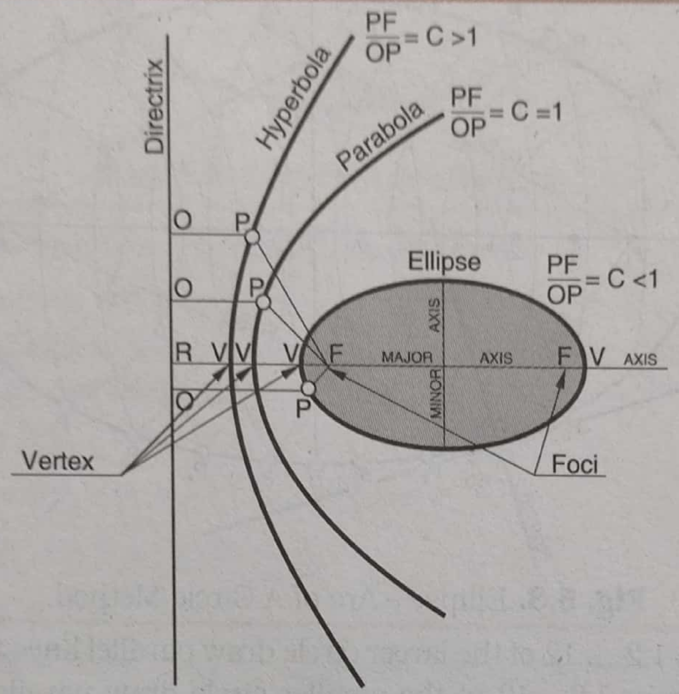


Fig. 5.2. Definition of Ellipse - Parabola-Hyperbola

1 (C) (i). Arc of A Circle Method : See Fig. 5.3

This method of drawing ellipse is based on 2nd definition of ellipse. In figure 5.3 we have,

$F_1P_1 + P_1F_2 = F_1P_2 + P_2F_2 = ...= F_1P_n + P_nF_2$ = Constant = AB = Major Axis

$F_1C + CF_2 = F_1D + DF_2$ = AB = Major axis

Further, $F_1C = F_2C = F_1D = F_2D$ = 1/2 Major axis

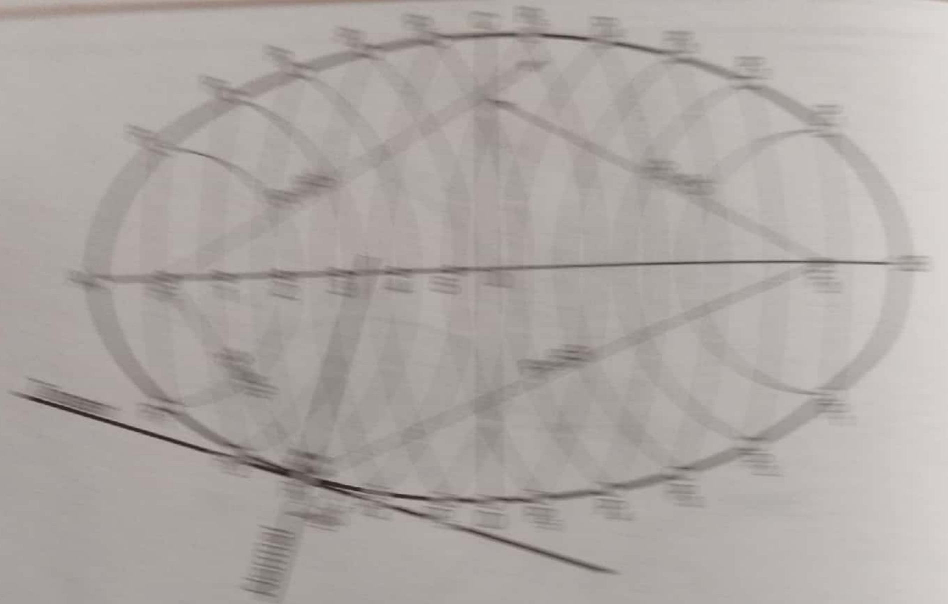
From the above relations, (i) if the length of major axis and minor axis are given we can find foci (ii). If major axis and foci are given we can find out minor axis and (iii) if minor axis and foci are given we can find out major axis.

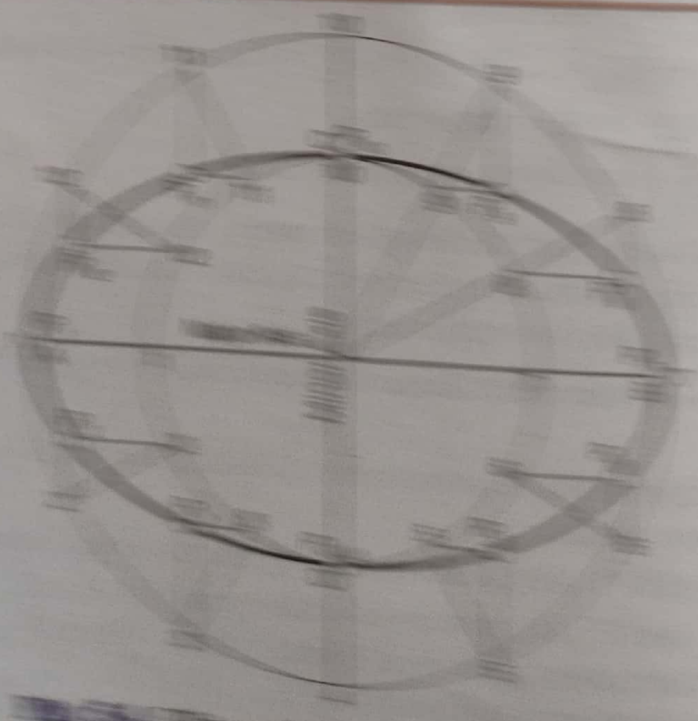
Now to draw ellipse by **Arc of A Circle Method,** follow the procedure as given below : (See Fig. 5.3)

(i) First draw major axis AB, minor axis CD and fix two foci F_1 and F_2 on major axis as per the given data.

(ii) Between F_1 and O, take points 1, 2,....n at nearly equal distances.

(iii) Now with F_1 and F_2 as centres and radii equal to A1 and 1B draw intersecting arcs on both the sides of AB to get four P_1 points. Points P_3 and P_5 are illustrated in the figure. Similarly, get points P_2, P_3,......P_n.





(iv) Join above points of intersections by a smooth curve, including points A, B, C and D to get an ellipse.

I (C) (ii). Concentric Circle Method : See Fig. 5.4.

Follow the procedure as given below :

(i) Draw two concentric circles with major axis and minor axis as diameters and divide them into 12 equal parts as shown in Fig. 5.4 and mark 1,........12 on both the circles.

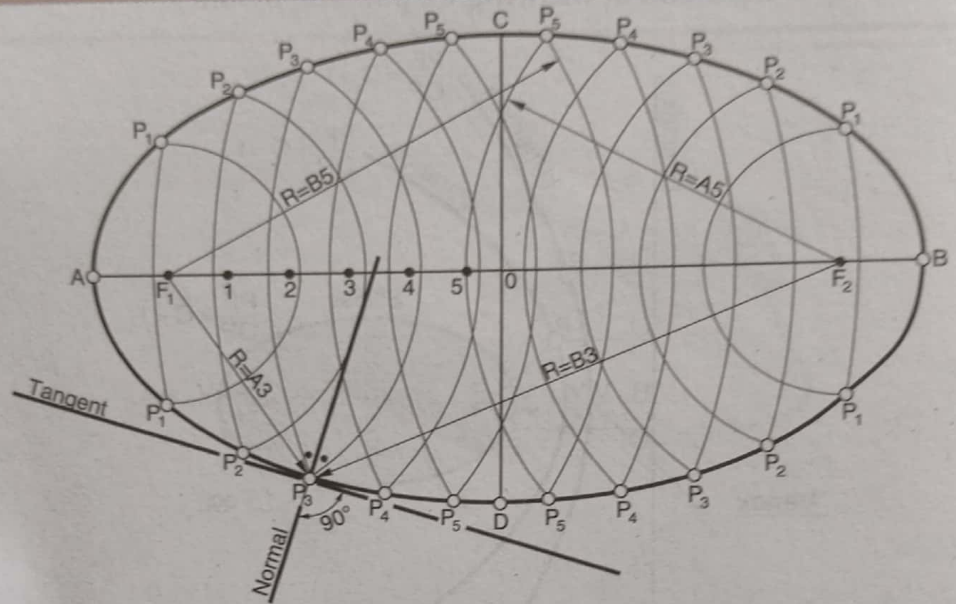


Fig. 5.3. Ellipse - Arc of A Circle Method.

(ii) From points 1,2.....12 of the larger circle draw parallel lines to minor axis (vertical lines) and from 1,2....12 of the smaller circle draw parallel lines to major axis (horizontal lines) intersecting at points P_1, P_2......P_{12} respectively.

(iii) Join all points of intersection P_1, P_2.......P_{12} in sequence by a smooth curve to get an ellipse.

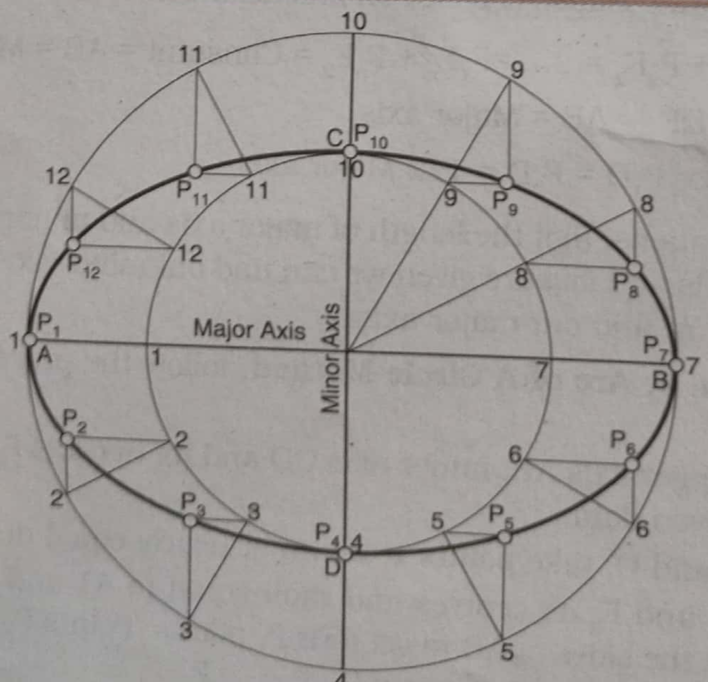


Fig. 5.4. Ellipse - Concentric Circle Method

I (C) (iii). Loop Method : See Fig. 5.5.

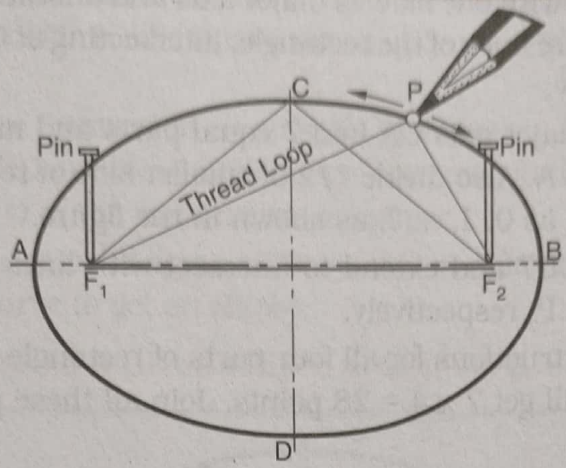


Fig. 5.5. Ellipse - Loop Method.

This method is also based on 2nd definition of ellipse.

Follow the procedure as given below :

(i) Take a closed loop of thread having L peripheral length, where L = Length of major axis + distance between two foci

$= AB + F_1F_2$

$= F_1C + CF_2 + F_1F_2 = F_1D + DF_2 + F_1F_2$

$= 2\,(AF_1 + F_1F_2) = 2\,(BF_2 + F_1F_2)$

$= 2\,AF_2 = 2BF_1$

(ii) Fix two pins at foci F_1 and F_2, as shown and arrange the loop of thread around pins as shown.

(iii) Insert pencil point inside the loop and move it, keeping the thread of loop always tight, then the pencil will draw an ellipse.

I (C) (iv). Oblong Method : See Fig. 5.6

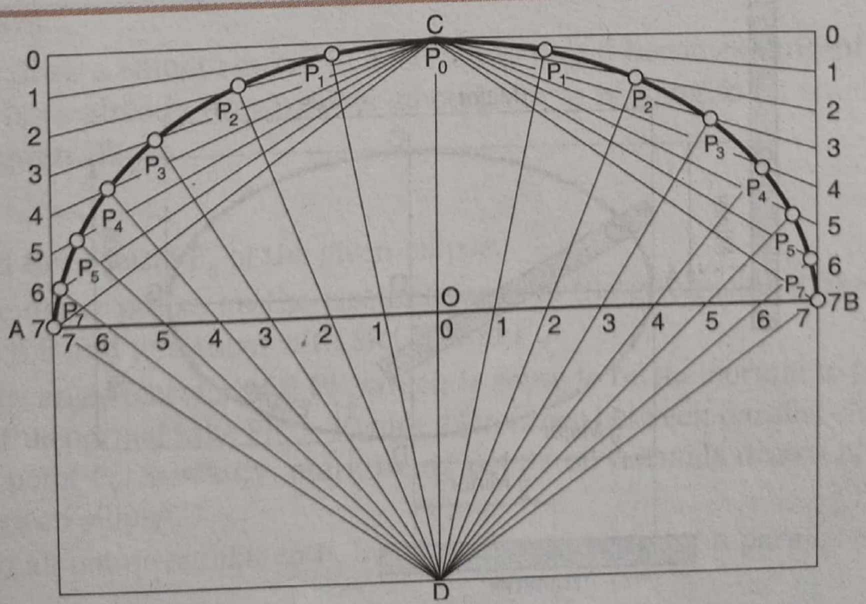


Fig. 5.6. Ellipse - Oblong Method.

Follow the procedure as given below :

(i) Draw rectangle with one side as major axis and another side as minor axis. Draw AB and CD centre lines of the rectangle, intersecting at O, as major axis and minor axis respectively.

(ii) Divide 1/2 of major axis OA into 7 equal parts and mark them as 0, 1, 2,......7 from O towards A. Also divide 1/2 of smaller side of rectangle into 7 equal parts and mark them as 0, 1,......7, as shown in the figure.

(iii) Join DO, D1,....D7 and extend to intersect with lines joining CO, C1,.......C7 at points P_0, P_1,.....P_7 respectively.

(iv) Do similar constructions for all four parts of rectangle and get points P_0, P_1,...... P_7. In all you will get 7 x 4 = 28 points. Join all these points by a smooth curve to get ellipse.

I (C) (v). Ellipse in Parallelogram : See Fig. 5.7

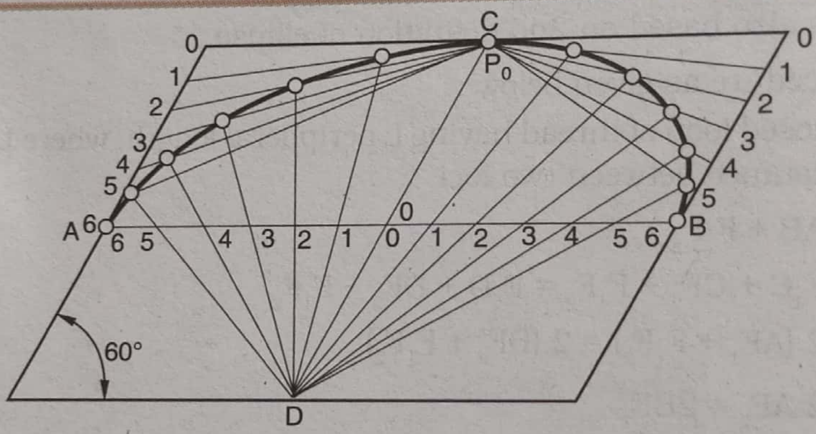


Fig. 5.7. Ellipse in A Parallelogram - Oblong Method.

(i) Draw A parallelogram first of given two sides (84 x 60, here) at given angle (60°,here) and follow the same procedure as followed in the oblong method.

I (C) (vi). Trammel Method : See Fig. 5.8.

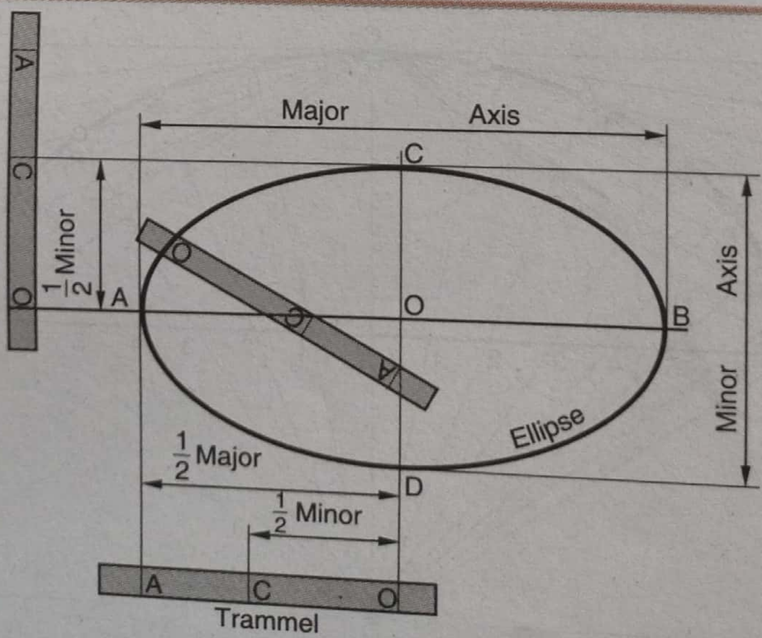


Fig. 5.8. Ellipse - Trammel Method.

(i) First draw two axes, major AB and minor CD, bisecting at right angle at point O.

(ii) Take trammel of hard board or hard paper and mark on it AO equal to (1/2) major axis and OC equal to (1/2) minor axis, as shown in Fig.5.8. Trammel is now ready for use.

(iii) Now arrange the trammel in all possible ways keeping the point A of the trammel on minor axis CD and the point C of the trammel on the major axis AB and mark points against O of the trammel on the paper.

(iv) Join the points, marked against the point O in different positions of the trammel, by a smooth curve to get an ellipse.

I (C) (vii). Parallel Ellipse to A Given Ellipse (Outside) : See Fig. 5.9.

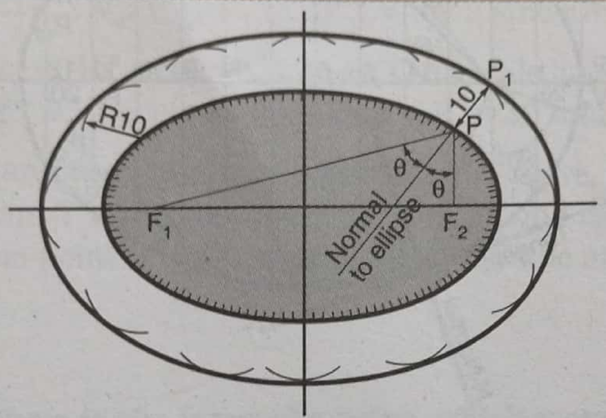


Fig. 5.9. Ellipse - Parallel to A Given Ellipse.

Follow the procedure as given below :

Method 1

(i) First draw the given ellipse.

(ii) Take many points, on the circumference of the given ellipse, as the centres and the radius equal to the distance between parallel ellipses, draw many arcs of circles, as shown in Fig. 5.9. If we require parallel ellipse inside, draw arcs inside (not shown).

(iii) Now draw a smooth curve in such a way that it becomes tangent to the arcs that you have already drawn. This smooth curve is going to be an ellipse parallel to the given ellipse.

Method 2

(i) Find foci F_1 and F_2 of the given ellipse.

(ii) Take many points on the circumference of the given ellipse like P, as shown in Fig. 5.9 and join them with foci F_1 and F_2.

(iii) Draw angle bisector of F_1PF_2 which is going to be the normal to the curve ellipse. On this normal take PP_1 = 10 mm = (distance between parallel ellipses) and mark the point P_1. Similarly, mark many points on normals drawn at different points on given ellipse.

(iv) Join all points similar to P_1 by a smooth curve to get a parallel ellipse.

I (C) (viii). Directrix - Focus Method : See Fig. 5.10

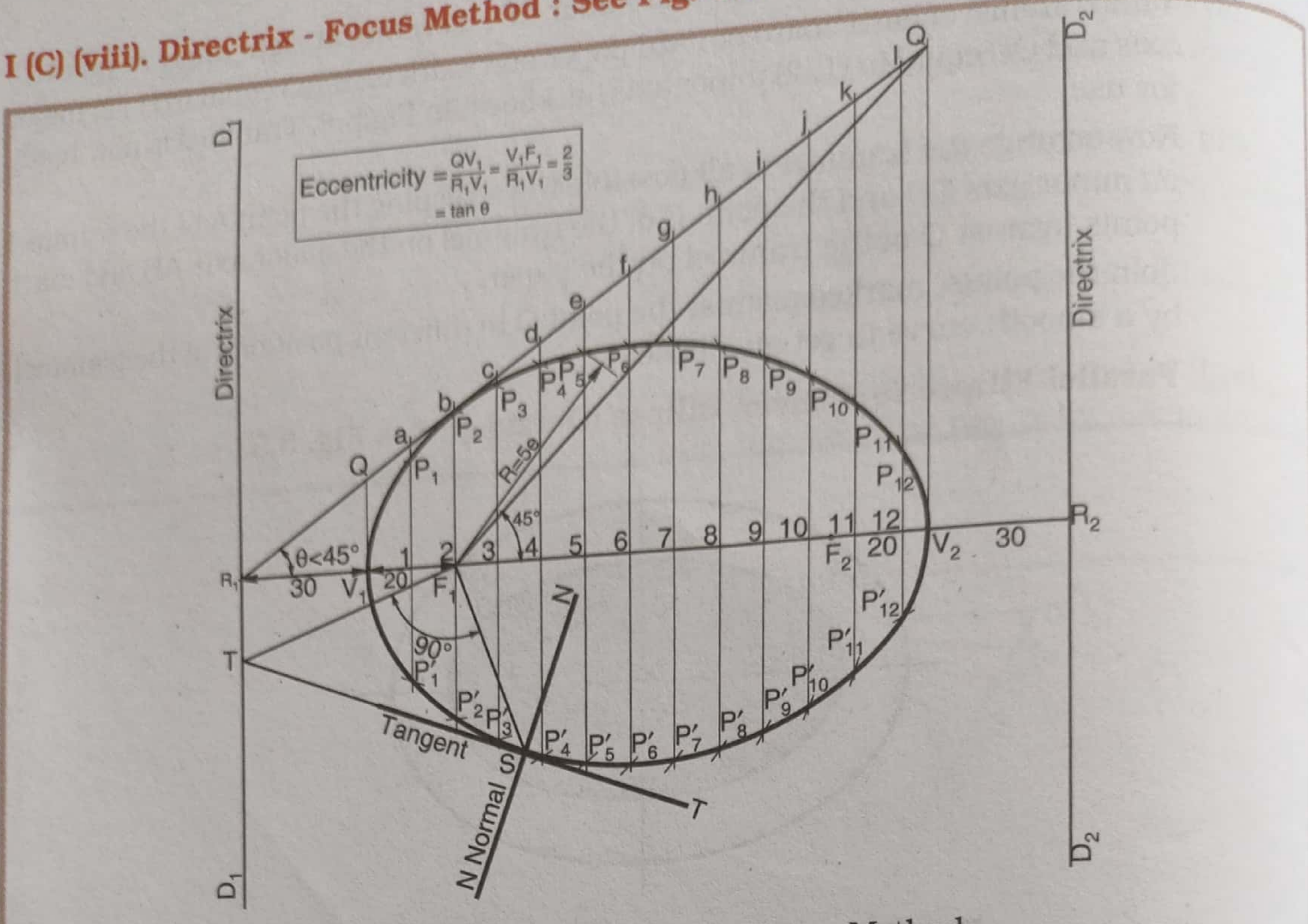


Fig. 5.10. Ellipse - Directrix Focus Method.

This method is based on the 1st definition of an ellipse. (See Fig. 5.2)

When a point moves in a plane keeping the ratio (known as eccentricity ratio) of its distances from focus to that of directrix equal to constant and less than 1 (one) then locus of the point is an **Ellipse.**

$$\text{Eccentricity Ratio} = \frac{PF}{OP} = \text{Constant} < 1. \text{ (See Fig. 5.2)}$$

If the constant ratio is one then its locus is a **Parabola.**

$$\text{Eccentricity Ratio} = \frac{PF}{OP} = \text{Constant} = 1. \text{ (See Fig. 5.2)}$$

If the constant ratio is more than one, then its locus is a **Hyperbola.**

$$\text{Eccentricity Ratio} = \frac{PF}{OP} = \text{Constant} > 1.$$

To construct an ellipse by the above method, follow the procedure as given below and see Fig. 5.10. Given the distance between the focus F_1 and directrix D_1D_1 as 50 mm and eccentricity ratio as 2/3.

(i) First draw directrix D_1D_1 and mark the focus F_1 at a distance 50 mm from R_1 on an axis line R_1R_2 drawn perpendicular to D_1D_1.

(ii) Divide F_1R_1 into ratio 2/3 by the point V_1 (Vertex) such that

$$\frac{F_1V_1}{V_1R_1} = \frac{2}{3} = \text{Eccentricity Ratio} = \frac{20 \text{ mm}}{30 \text{ mm}}$$

(iii) To construct a scale for ratio 2/3, draw $V_1Q = V_1F_1$ at V_1 at right angle to axis R_1F_1. Join R_1Q and extend it. Between V_1 and R_2 take points 1, 2,......12 at suitable equal distances and draw right angle lines to the axis 1a, 2b,......121 cutting R_1Q extended line at points a, b, ...1 respectively. Now the scale is ready for use. $\angle F_1R_1Q$ will be less than 45° for an ellipse. Tan θ = E.R. = 2/3

(iv) Now with F_1 as the centre and radii equal to 1a, 2b, ...121 draw arcs to intersect with lines 1a, 2b,......12 1 on both the sides of axis at points (P_1 - P'_1), (P_2 - P'_2)...... (P_{12} - P'_{12}) respectively.

(v) Join V_1, P_1, P_2,....P_{12}, V_2, P'_{12}, P'_{11}......P'_1, V_1 by a smooth curve to get an ellipse.

At focus F_1 draw a line, at 45° to the axis, to get Q on scale line as shown. From Q draw perpendicular to axis to get V_2. Locate R_2 by taking V_2R_2 = 30 mm.

Now to draw tangent and normal at any point S on the curve, join S with focus F_1 and draw right angle SF_1T. Point T will be on directrix. Join TS and extend it. TST line will be tangent to the curve at the point S. Draw right angle to TT line at S to get normal NN, as shown in fig. 5.10.

I - (D). PARABOLA

Definition : The parabola is the locus of a point, which moves in a plane so that its distances from a fixed point (Focus) and a fixed straight line (Directrix) are always equal.

[OR]

Ratio (known as Eccentricity) of its distances from focus to that of directrix is constant and equal to one (1). (See Fig. 5.2)

Uses : (1) Motor car head lamp reflector. (See Fig. 5.11)
(2) Sound reflector and detector
(3) Bridges and arches construction
(4) Shape of cooling towers
(5) Path of particle thrown at any angle with earth, etc.

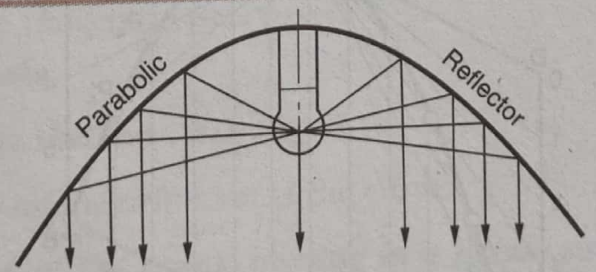


Fig. 5.11. Parabolic Reflector

Now we shall study 4 methods of constructing parabola one by one.

I (D) (i). Rectangle Method : See Fig. 5.12

Given the base length AB and the axis length OV. Axis OV is perpendicular to the base AB. Follow the procedure as given below and see Fig. 5.12.

1. Draw base AB and draw axis OV at right angle to AB at the mid point O of AB.
2. Complete the rectangle ABCD, as shown in the figure.

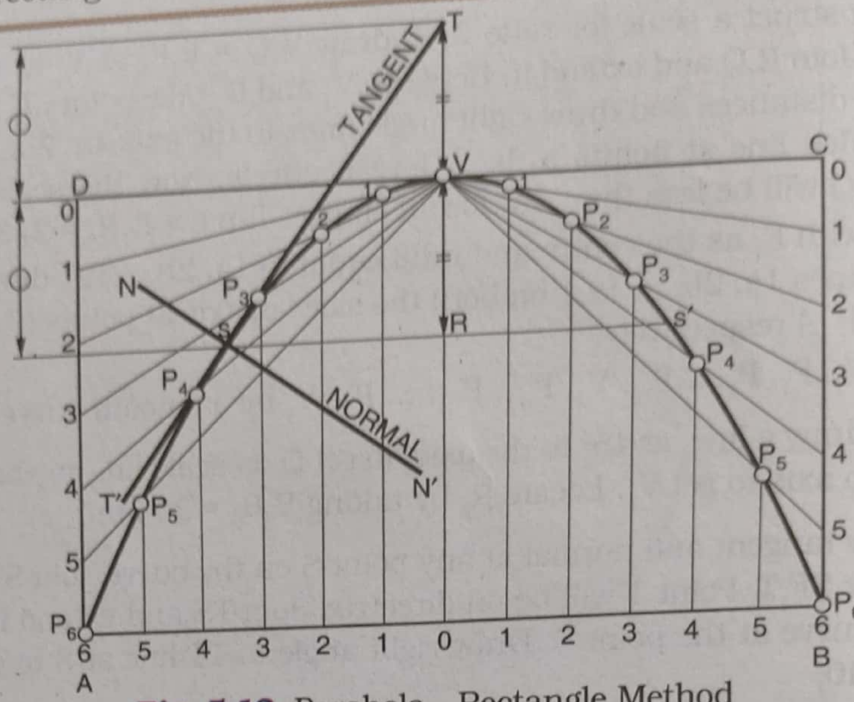


Fig. 5.12. Parabola - Rectangle Method

3. Divide OA and OB into 6 equal parts and mark them as 0, 1,....6. Similarly, divide CB and DA also into six equal parts and mark them as 0, 1,....6, as shown in the figure.
4. Draw parallel lines to the axis OV from 1, 2,....6 of AB to intersect with lines V1, V2,... V6 at points P_1, P_2,.....P_6 respectively.
5. Join all points by a smooth curve to get a parabola.

I (D) (ii). Parabola in Parallelogram : See Fig. 5.13.

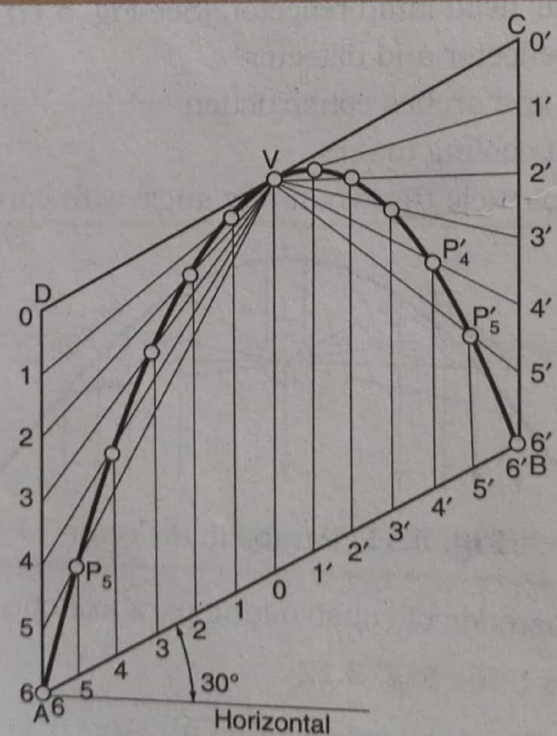


Fig. 5.13. Parabola in A Parallelogram.

This method is similar to the rectangle method. Here, instead of drawing a rectangle we have to draw a parallelogram.

I (D) (iii). Tangent Method : See Fig. 5.14.

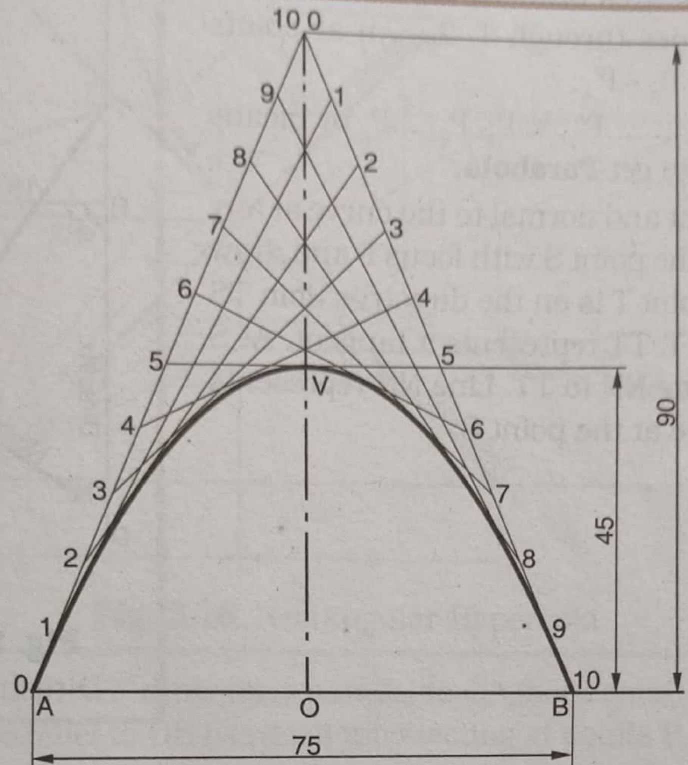


Fig. 5.14. Parabola - Tangent Method.

Given the base AB, 75 mm and axis OV, 45 mm, follow the procedure as given below and see Fig. 5.14.

1. Draw isosceles triangle OAB with base AB = 75 mm and height OO = 2 x length of axis = 2 x 45 = 90 mm.
2. Divide lines OB and AO into 10 equal parts and mark them as 0, 1, 2,....10, as shown in Fig. 5.14.
3. Join 11, 22, 33,.......10 - 10 and draw a smooth curve in such a way that all lines 11, 22, ...10 - 10 become tangent to the curve.

   This curve is a **Parabola.**

I (D) (iv). Directrix Focus Method : See Fig. 5.15.

This method is based on the definition of the curve.

Parabola is the locus of the point, moving in a plane, such that the ratio of its distances from the focus to that of directrix is constant and equal to one. Given the distance between focus F and the directrix DD. i.e. RF = 36 mm.

Follow the procedure as given below and see Fig. 5.15.

1. Draw directrix DD and axis line RF at right angle to DD. Take RF = 36 mm and find its mid point V.

2. On the axis, take points 1, 2,.....n and draw right angle lines to the axis.
3. Now with focus F as the centre and radii equal to R1, R2,..... Rn draw arcs on both the sides of axis to intersect with lines through 1, 2,......n at points P_1 - P'_1, P_2 - P'_2,.......P_n - P'_n.
4. Join points P'_n, P'_{n-1} ,........P'_1, V, P_1, P_2,....P_n by means of a smooth curve to get **Parabola.**
5. Now to draw tangent and normal to the curve at any given point S, join the point S with focus F and draw ÐSFT = 90°. The point T is on the directrix. Join TS and extend it upto T. TT represents a tangent. At S draw right angle line NN to TT. Line NN represents normal to the curve at the point S.

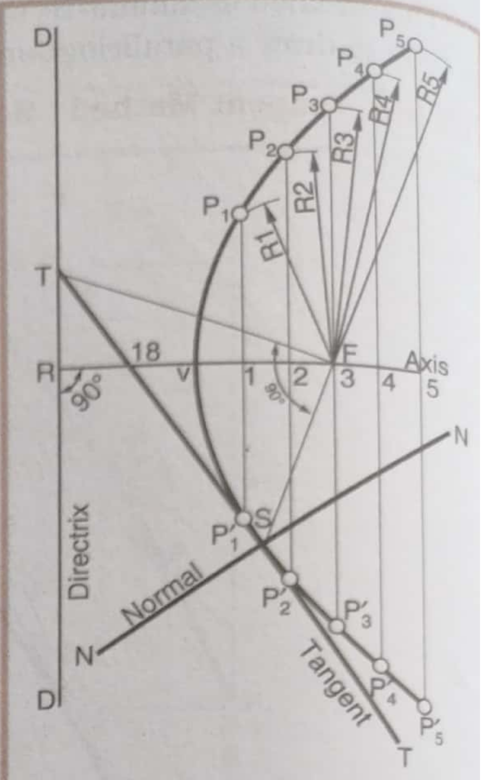


Fig. 5.15. Parabola - Directrix Focus Method.

I (E). HYPERBOLA

Definition : It is the locus of a point which moves in a plane so that the ratio of its distances from a fixed point (Focus) and a fixed straight line (Directrix) is constant and greater than one (1).

[OR]

It is the curve generated by a point, moving in a plane, so that the difference of its distances from the two fixed points called Foci is a constant and is equal to the distance between two vertices.

Uses : (1) Nature of graph of Boyle's law
(2) Shape of overhead water tanks
(3) Shape of cooling towers etc.

I (E) (i). Rectangular Hyperbola : See Fig. 5.16.

Draw a rectangular hyperbola passing through the point P0. Co-ordinates x and y of the point P_0 are given.

Follow the procedure as given below and see Fig. 5.16.

1. First draw two axes OA and OB and mark point P_0 with the given co-ordinates x and y.
2. Through P_0 draw CD and EF lines parallel to OA and OB respectively.
3. Draw lines 01'1, 02'2, 03'3,....07'7 from 0 intersecting EF and CD lines at points 1', 2',....7' and 1, 2,.....7 respectively.

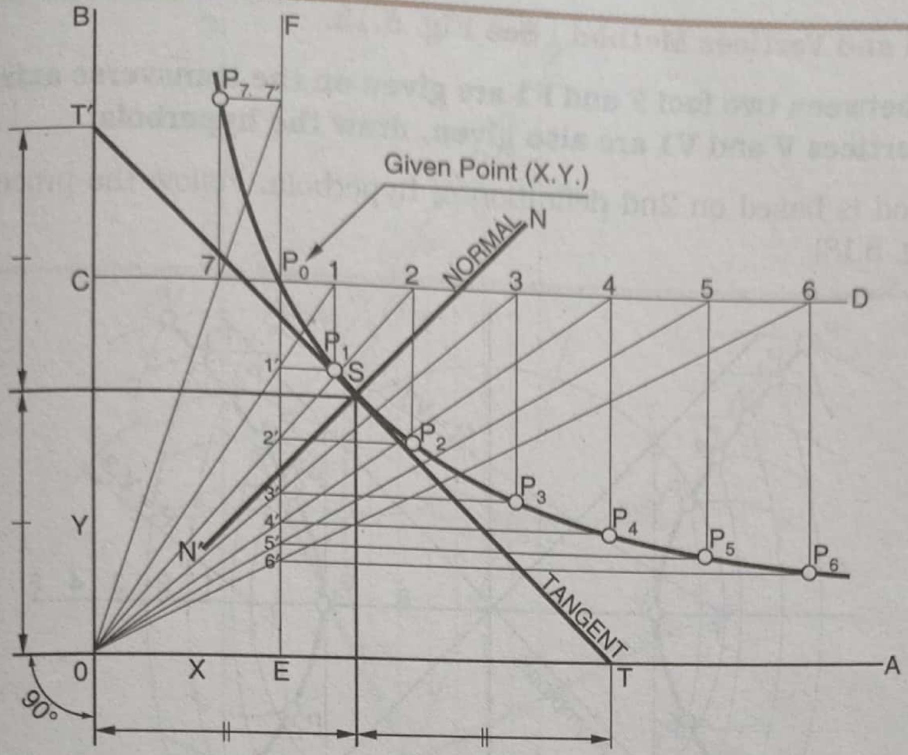


Fig. 5.16. Rectangular Hyperbola

4. Through points 1', 2',......7' draw lines parallel to OA (horizontal) and through points 1, 2,....7 draw lines parallel to OB (vertical) intersecting at points P_1,P_2,.....P_7 respectively.
5. Draw a smooth curve passing through P_7, P_0, P_1, P_2,....P_6 to get a **Hyperbola.**

I (E) (ii). Oblique Hyperbola : See Fig. 5.17.

Draw oblique hyperbola passing through point P_0, x and y co-ordinates of point P_0 are given. X and Y axes are at 75°.

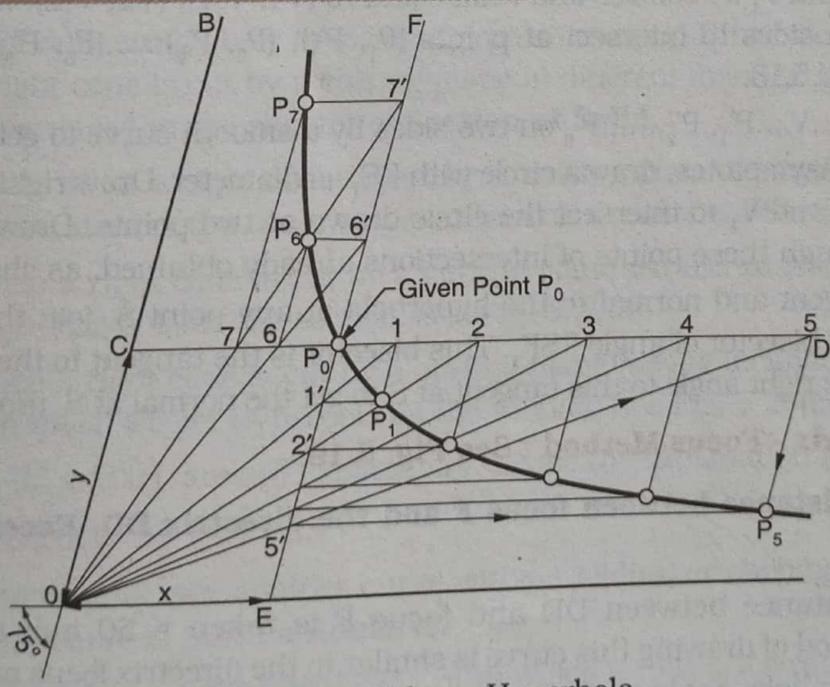


Fig. 5.17. Oblique Hyperbola

Procedure of construction is same as the rectangular hyperbola. (See Fig. 5.17)

I (E) (iii). Foci and Vertices Method : See Fig. 5.18.

Distance between two foci F and F1 are given on the transverse axis and distance 2a between vertices V and V1 are also given, draw the hyperbola.

This method is based on 2nd definition of hyperbola. Follow the procedure as given below. (See Fig. 5.18)

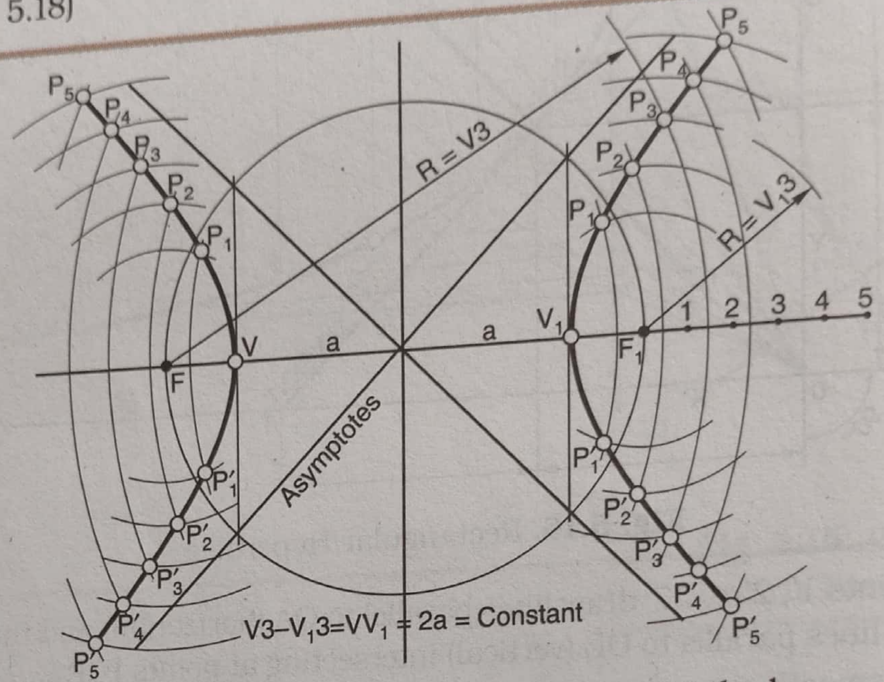


Fig. 5.18. Hyperbola - Focus Vertices Method.

1. First draw transverse axis and mark on it two foci F and F_1 and two vertices V and V_1 as shown.
2. On the axis mark points 1, 2,.....5 beyond F_1 at nearly equal distances.
3. Now with F and F_1 as centres and radii equal to (V1, $V_1$1), (V2, $V_1$2), ...(V5, $V_1$5) draw arcs on both sides to intersect at points (P_1, P'_1), (P_2, P'_2),......(P_5, P'_5) respectively, as shown in Fig. 5.18.
4. Join P_5, P_4,.....V....P'_1, P'_2,......P'_5 on two sides by a smooth curve to get two hyperbolas.
5. Now to get 2 Asymptotes, draw a circle with FF_1 as diameter. Draw right angle to the axis at vertices V and V_1 to intersect the circle drawn at two points. Draw two asymptotes passing through these points of intersections already obtained, as shown in Fig. 5.18.
6. To draw tangent and normal to the hyperbola at any point S, join the point S with F and F_1. Draw bisector of angle FSF_1. This bisector is the tangent to the hyperbola at the point S. Draw right angle to this tangent at S to get the normal at S. (Not shown).

I (E) (iv). Directrix - Focus Method : See Fig. 5.19.

Given the distance between focus F and the directrix DD. Eccentricity ratio is also given.

Here the distance between DD and focus F is taken = 50 mm and eccentricity ratio = 3/2. Method of drawing this curve is similar to the directrix focus method, for ellipse as well as for parabola. Scale angle θ will be more than 45°. Tanθ = E.R. = 3/2.

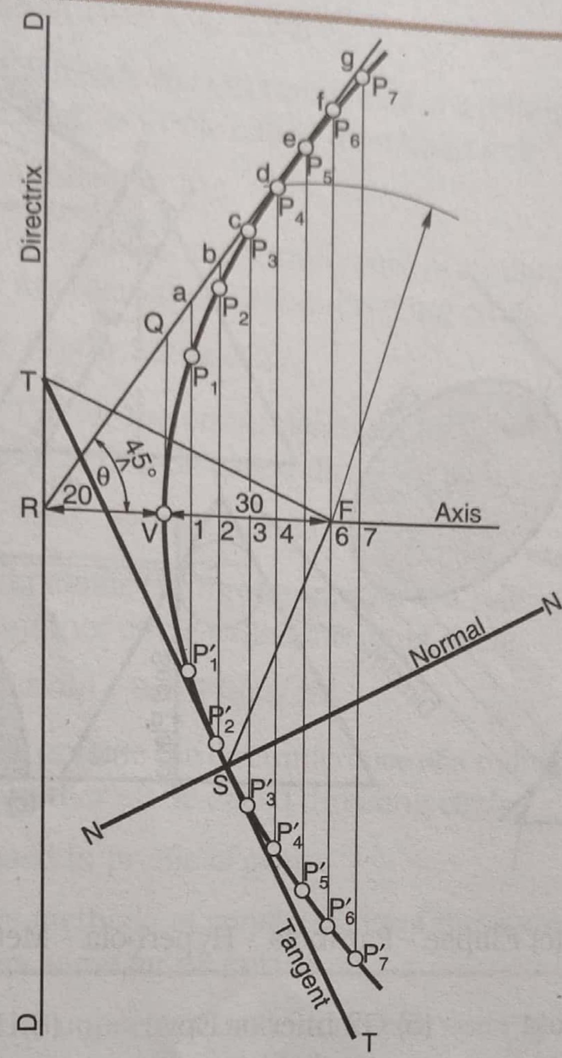


Fig. 5.19. Hyperbola - Directrix Focus Method.

Ellipse - Parabola - Hyperbola by Method of Intersection of A Plane and A Right Cone :-

Here we shall only study, how to locate the position of (1) Directrix (2) Vertex and (3) Focus, when a right cone is cut by a cutting plane in different manner. Method of drawing entire curve is discussed in the chapter of **Section of Solid.**

We know that, when a right cone is cut by a cutting plane inclined to the base and cutting all the generators of the cone, we get Ellipse as the section. See Fig. 5.20 (a).

Similarly when a right cone is cut by a cutting plane parallel to one of the generators of the cone, we get Parabola as the section. See Fig. 5.20(b).

Similarly when a right cone is cut by a cutting plane having inclination more with the base than for parabola, we get Hyperbola as the section. See Fig. 5.20(c).

Fig. 5.20(a), (b) and (c) are self explanatory, hence the explanation is not given.

II CYCLOIDAL GROUP OF CURVES

When one curve rolls over another curve without sliding or slipping, the path of any point of the rolling curve is called a **Roulette.**

When rolling curve is a circle and the curve on which it rolls is a straight line or a circle, we get cycloidal group of curves. There are nine different curves in this group. They are -

(1) F1 Cycloid (4) G1 Epycycloid (7) H1 Hypocycloid

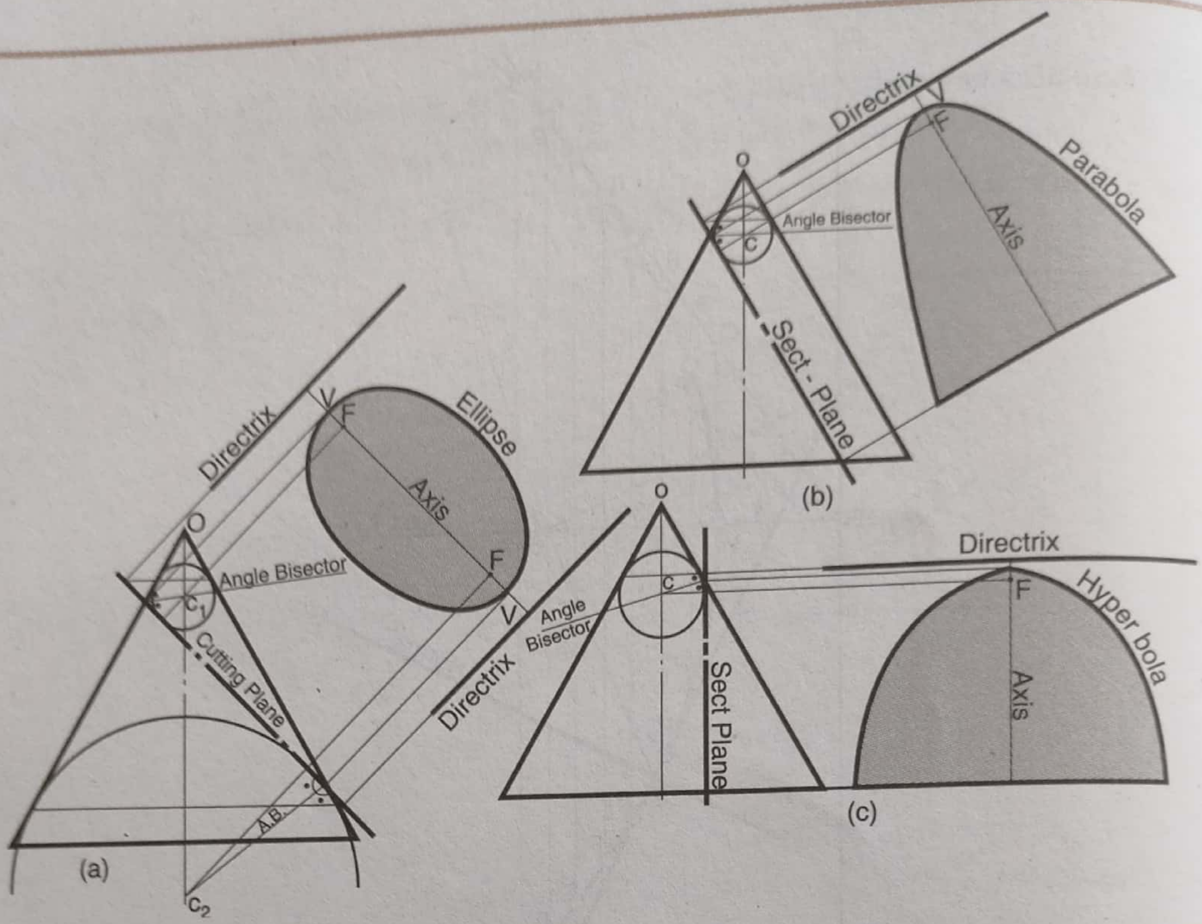


Fig. 5.20. (a), (b) & (c) Ellipse - Parabola - Hyperbola - Method of Intersection.

(2) F2 Inferior Trochoid

(3) F3 Superior Trochoid

(5) G2 Inferior Epy trochoid

(6) G3 Superior Epy trochoid

(8) H2 Inferior Hypotrochoid

(9) H3 Superior Hypotrochoid

Now we shall study the definition of each curve one by one.

II (F1) Cycloid : See Fig. 5.22 to understand definition.

Cycloid is a locus of a point (P) on the circumference of a rolling circle (generator), which rolls without slipping or sliding along a fixed straight line or a directing line or a director.

(F2) Inferior Trochoid : See Fig. 5.23 or 5.24.

Inferior trochoid is a locus of a point (Q) inside the circumference of a rolling circle which rolls without slipping or sliding along a fixed straight line.

II (F3) Superior Trochoid : See Fig. 5.23 or 5.25.

It is a locus of a point (R) outside the circumference of a rolling circle, which rolls without slipping or sliding along a fixed straight line.

II (G1) Epycycloid : See Fig. 5.26 or 5.27

It is a locus of a point (P) on the circumference of a rolling circle, which rolls without sliding or slipping outside another circle called directing circle.

II (G2) Inferior Epytrochoid : See Fig. 5.27.

It is a locus of a point (Q) inside the circumference of a rolling circle, which rolls without sliding or slipping outside another circle called directing circle.

II (G3) Superior Epytrochoid : See Fig. 5.27.

It is a locus of a point (R) outside the circumference of a rolling circle, which rolls without sliding or slipping outside another circle called directing circle.

II (H1) Hypocycloid : See Fig. 5.26 or 5.27.

It is a locus of a point (M) on the circumference of a rolling circle, which rolls without sliding or slipping inside another circle called directing circle.

II (H2) Inferior Hypotrochoid : See Fig. 5.27.

It is a locus of a point (L) inside the circumference of a rolling circle, which rolls without sliding or slipping inside another circle called directing circle.

II (H3) Superior Hypotrochoid : See Fig. 5.27.

It is a locus of a point (N) outside the circumference of a rolling circle, which rolls without sliding or slipping inside another circle called directing circle.

Uses : These curves are used in profile of gears.

Now we shall study the methods of construction of the above curves. Basic concept of construction is more or less same for all curves.

To understand one of the important basic concepts, see Fig. 5.21.

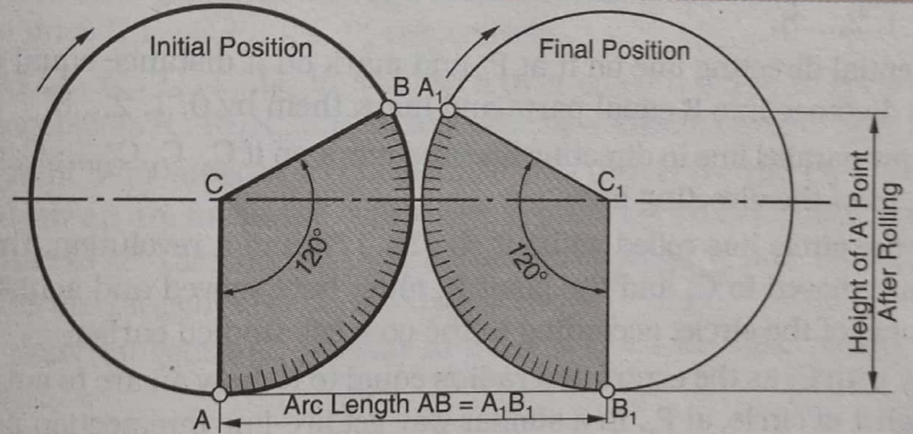


Fig. 5.21. Basic concept of cycloidal curves

Circle which rolls without slipping or sliding on a straight line (AB_1) is shown in initial (C) and final positions (C_1) after 120° rotation of the circle. When a circle rolls from position C to C_1, a distance equal to the arc AB = AB_1, point B moves down to B_1, while the point A moves upto A_1. The horizontal or parallel to the directing line from B of the initial position of the circle decides the height of the point (A) in the new position of a circle. This concept will be utilised in finding the positions of a point (P) after rotation of circle by 1/8th or 1/12th or multiple of its rotation either on straight lines or on arcs of circles.

II (F1) Cycloid : See Fig. 5.22

Problem 1 : Diameter of the rolling or the generating circle is D. Construct the cycloid. Draw tangent and normal at any point of curve. Initial position of point P is at the point of contact between generating circle and the directing line.

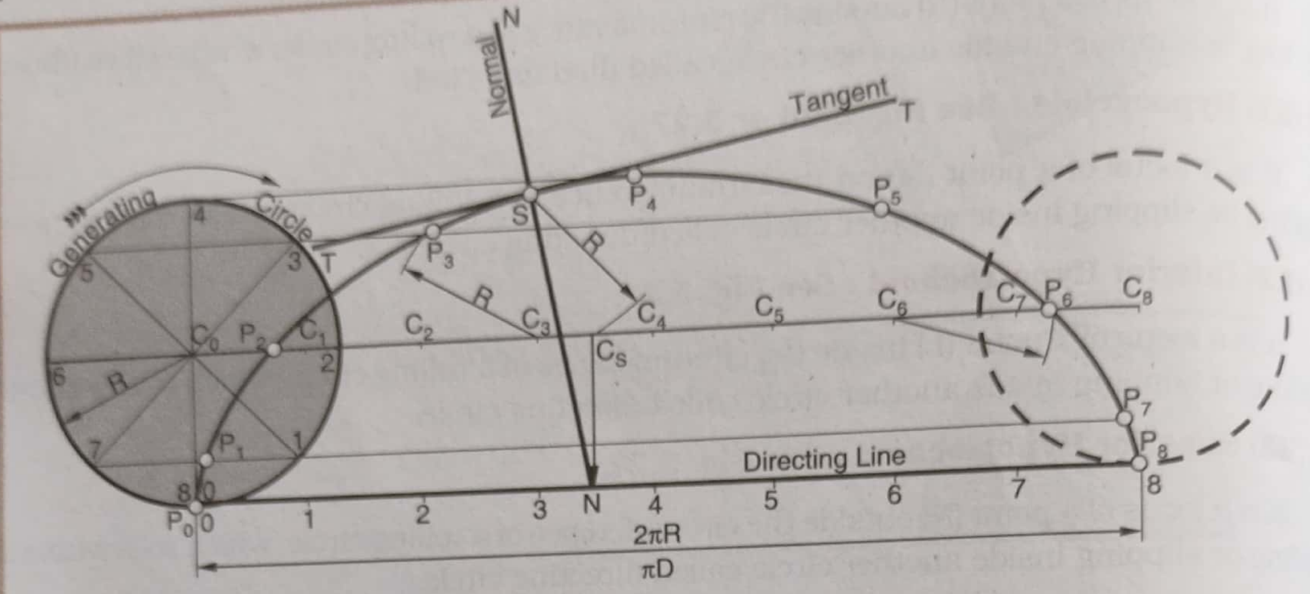


Fig. 5.22. Cycloid.

Follow the procedure as given below :

1. Draw a circle ⊙ (c_0, R) and mark P_0 on the circumference, the initial position of P, a shown in figure. Divide this circle into 8 equal parts and mark on the circle 0, 1, ... in opposite direction to rotation. Draw horizontal lines or parallel lines to directing line through 0, 1, 2,....8.
2. Draw tangential directing line on it at P_0 and mark on it distance equal to πD from P_0 Divide this distance into 8 equal parts and mark them by 0, 1, 2,...8.
3. From C_0 draw parallel line to directing line and mark on it C_0, C_1,C_2,.....C_8 correspondin to 0, 1, 2,....8 of the directing line.
4. Now when the circle has rolled without slip by 1/8th of a revolution, the centre poin C_0 must have moved to C_1 and the point P_0 must have moved and achieved the heigh of point 1(one) of the circle, according to the concept studied earlier.
5. Accordingly with C_1 as the centre and radius equal to R draw an arc to cut the horizonta line, through 1 of circle, at P_1. In a similar way get arc-line intersection points P_2, P_3... P_8 by drawing arcs with C_2, C_3,.....C_8 as centres and radius equal to R to intersect wit lines through 2, 3,....8 of circle respectively.
6. Join P_0, P_1, P_2,....P_8 by means of a smooth curve to get a cycloid.
7. Take any point S on the curve. Now with S as the centre and radius equal to R dra an arc to cut centre line C_0C_s at some point C_s. Find the point N on the directing lin corresponding to C_s. Join NS and draw perpendicular to it at S. The line NSN is norm and the line TST is tangent to the curve.

II (F2) and II (F3) Inferior and Superior Trochoids : See Fig. 5.23.

There are two methods of drawing trochoids.

Method 1 : Given the diameter of the rolling or generating circle as 50 mm. Given points P, Q and R are on the circumference, 5 mm inside the circumference, and 10 mm outside the circumference respectively on rolling circle. This circle is rolling without sliping/sliding on a fixed straight line. Draw the loci of points P, Q and R for one revolution of rolling circle. Draw tangent and normal to it at any point.

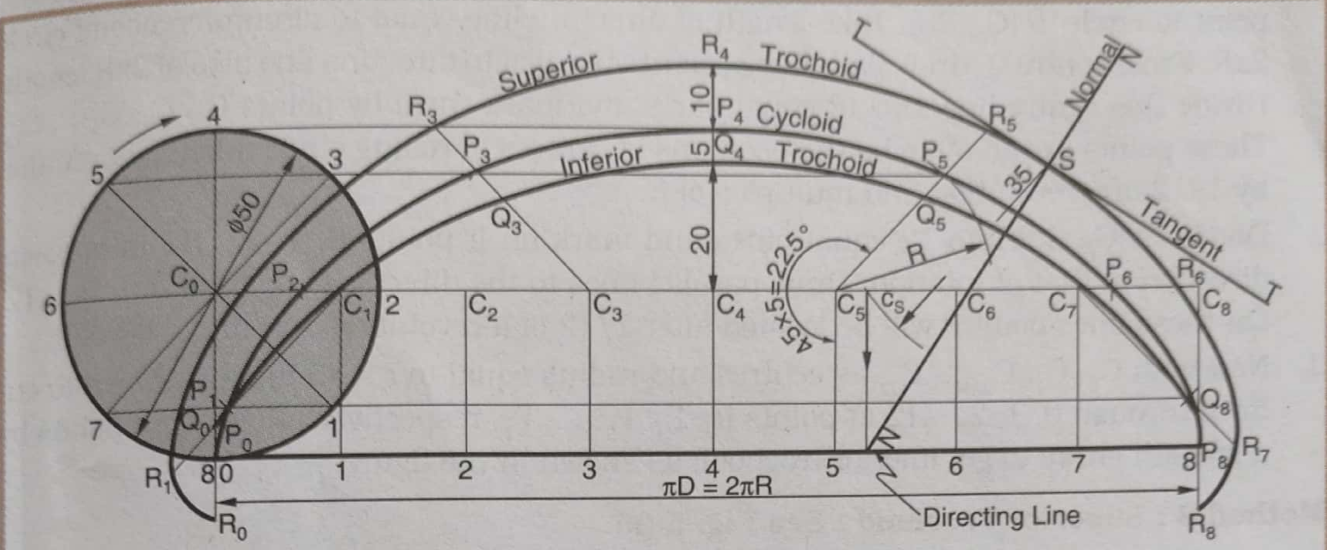


Fig. 5.23. Cycloid - Inferior & Superior Trochoids. Method - 1.

For solution see Fig. 5.23 and follow the procedure as given below :

1. Do the same construction as done for cycloid and obtain points C_0, C_1, C_2,....C_8 an P_0, P_1, P_2,.....P_8, as shown in Fig. 5.23.
2. Join C_0P_0 and mark Q_0 on it and R_0 on the extension of it by taking P_0Q_0 = 5 mm and P_0R_0 = 10mm. Take Q_0 inside and R_0 outside. Similarly, find points (Q_1, R_1), (Q_2, R_2)..... (Q_8, R_8) on lines C_1P_1, C_2P_2,....C_8P_8 respectively.
3. Join points Q_0, Q_1, Q_2,.....Q_8 in sequence by a smooth curve to get inferior trochoid. Similarly join points R_0, R_1, R_2,....R_8 in sequence by a smooth curve to get superior trochoid.
4. Take any point S on superior trochoid. With S as the centre and radius equal to 35 mm (10 + 25) draw an arc to cut the centre line at point C_s. From C_s draw right angle on the directing line to get the point N. Join NS and draw at S the line TST right angle to NS. TST and NS are tangent and normal respectively.
5. Similarly draw tangent and normal to inferior trochoid. Take radius 20 mm (25-5), instead of 35.

Method 2 : Inferior Trochoid. See Fig. 5.24.

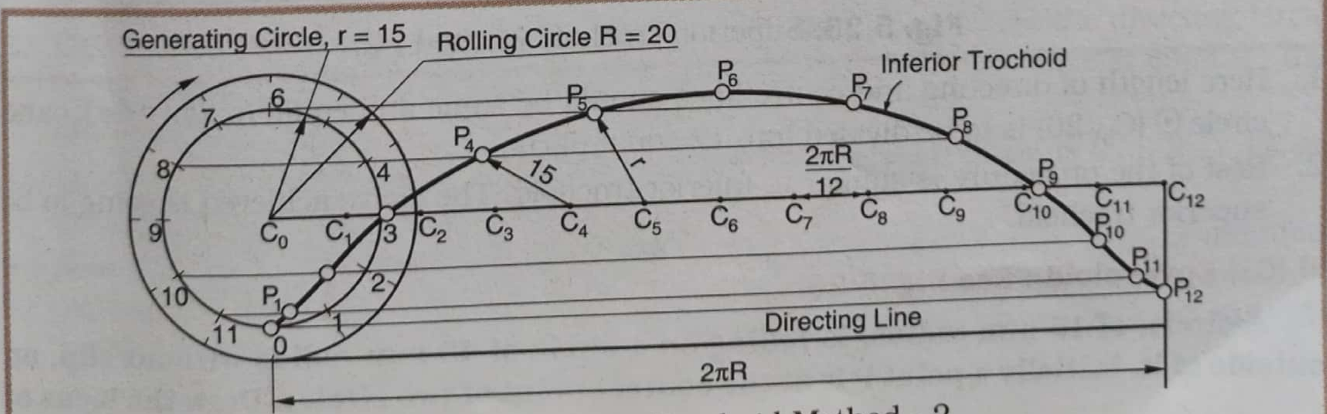


Fig. 5.24. Inferior Trochoid Method - 2.

A circle, of 20 mm radius, is rolling on a straight line without slip. A point P is inside the circle, 15 mm from the centre is initially at the bottom most position. Draw the locus of point P for one revolution and name the curve.

For solution follow the procedure as given below and see Fig. 5.24.

1. Draw two circles ⊙ (C_0, 20) and ⊙ (C_0, 15). Draw tangential directing line at bottom most point to circle ⊙ (C_0, 20). Take length of directing line equal to circumference of circle 2πR. From centre C_0 draw path line of centre parallel to directing line also of 2πR length. Divide this centre line into 12 equal parts and mark them by points C_0, C_1, C_2,....C_{12}. These points are going to be the positions of centres of rolling circle during its rotation by 1/12 of a revolution and multiples of it.
2. Divide ⊙ (C_0, 15) into 12 equal parts and mark on it points 0, 1, 2,....12 in opposite direction to that of rotation. Draw parallel lines to the directing line from 0, 1, 2,....12. On these lines point P will be located after 1/12 of a revolution and multiples of it.
3. Now with C_0, C_1, C_2,.....C_{12} as centres and radius equal to r = 15 mm draw arcs to cut lines through 0, 1, 2,....12 at points P_0, P_1, P_2,......P_{12} respectively. Join these points by a smooth curve to get inferior trochoid, as shown in the figure.

Method 2 : Superior Trochoid : See Fig. 5.25.

A circle of 15 mm radius is rolling on a straight line without slip. A point P is outside the circle 20 mm from the centre and is initially at the bottom most position. Draw the locus of the point P for one revolution and name the curve.

For solution follow the procedure as given below and see Fig. 5.25.

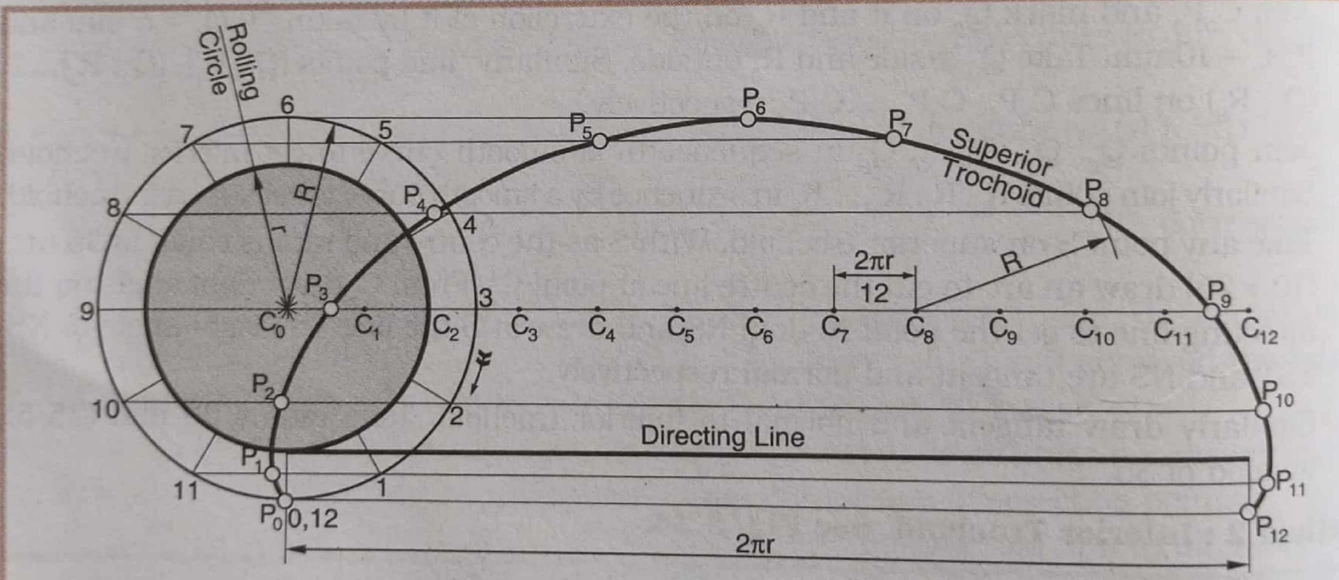


Fig. 5.25. Superior Trochoid Method - 2.

1. Here length of directing and centre lines should be same and equal to 2πr = 2π15 and circle ⊙ (C_0, 20) is to be divided into 12 equal parts.
2. Rest of the procedure is similar to inferior trochoid. The curve achieved is going to be superior trochoid.

II (G1) Epycycloid : See Fig. 5.26.

A circle, of 15 mm radius, is rolling on a circle of 45 mm radius without slip, on outside of it. Initially a point P is at the contact point of two circles. Draw the locus of

the point P for one revolution of the rolling circle. Name the curve and draw tangent and normal to the curve at any point S.

For solution see Fig. 5.26 and follow the procedure as given below :

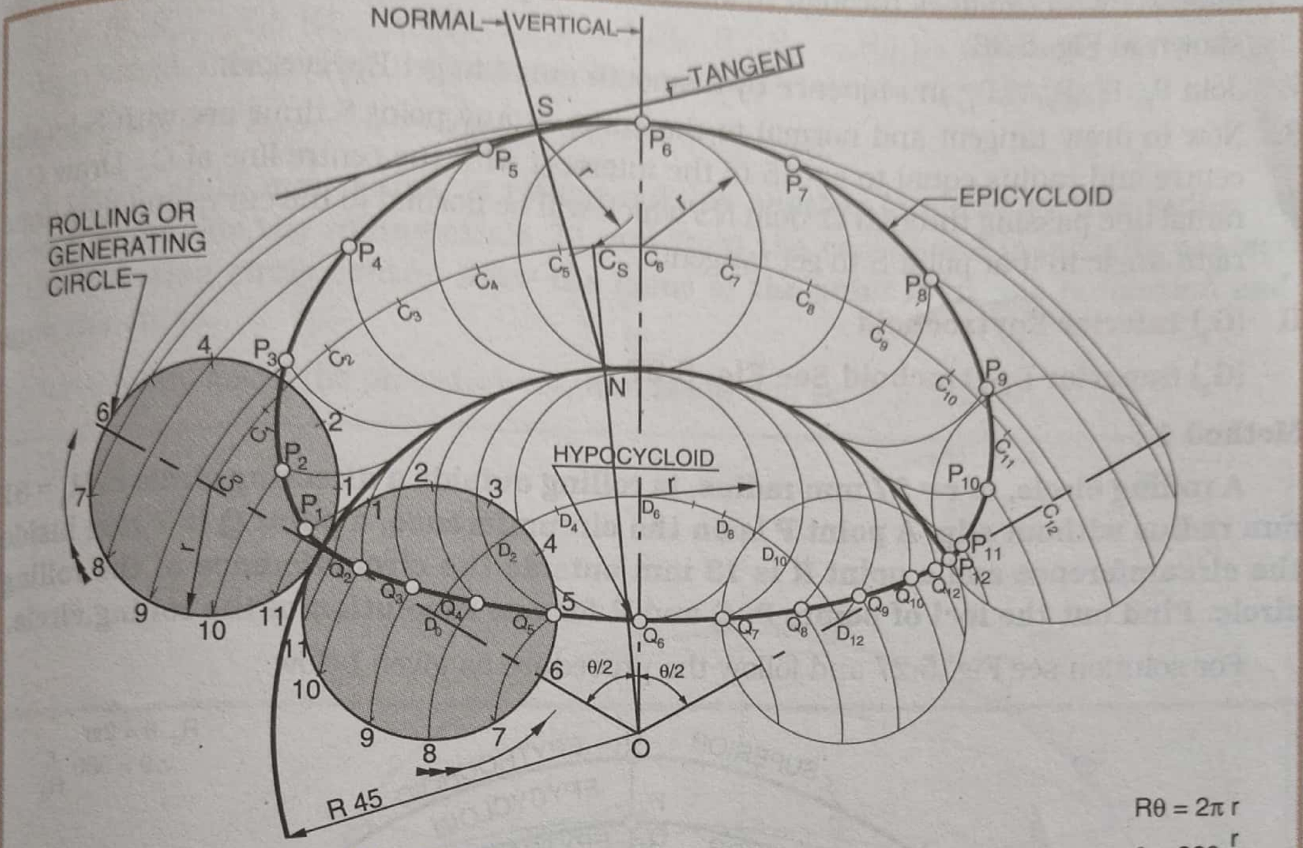


Fig. 5.26. Epycycloid and Hypocycloid.

Theory :

When the rolling circle of radius (r) rolls by one revolution it will advance on directing circle of radius (R) a distance equal to $2\pi r$.

This arc length $2\pi r$ of directing circle will subtend an angle θ at its centre so that

$2\pi r = R.\theta$.

$$\therefore \quad \theta = 360 \times \frac{r}{R} = 360 \times \frac{15}{45} = 120°$$

1. First of all draw an arc of a circle ⊙ (O, R = 45). At the point O draw angle of 120° (equal, on two sides of vertical through O).
2. On the left limb of angle mark point C_0 at a distance 15 mm from the directing circle on outside of it. Draw rolling circle ⊙ (C_0, 15) touching directing circle at point P_0, the starting position of P.
3. Divide rolling circle ⊙ (C_0, 15) into 12 equal parts and mark them as 0, 1, 2,.....12, in opposite direction to that of rotation.
4. Now with O as the centre and radii equal to O1, O2,....O-12 draw arcs of required sufficient length to represent the position of the point P after rotation of rolling circle by 1/12 of a revolution and multiples of it. Also draw arc with O as centre and OC_0 (45+15) as the radius to get path line of the centre.

5. Divide angle of 120° into 12 equal parts and draw small radial lines through ... in the figure, to intersect with the centre line at C_0, C_1, C_2,.......C_{12}.
6. Now with C_0, C_1, C_2,.......C_{12} as centres and radius equal to r = 15 mm draw arcs to intersect with arc lines through 0, 1, 2,.......12 at points P_0, P_1, P_2,.....P_{12} respectively as shown in Fig. 5.26.
7. Join P_0, P_1, P_2,.....P_{12} in sequence by a smooth curve to get Epycycloid.
8. Now to draw tangent and normal to the curve at any point S draw arc with S as the centre and radius equal to r = 15 to the intersect with the centre line at C_s. Draw C_s radial line passing through O. Join NS which will be normal to the curve and now draw right angle to it at point S to get tangent.

II (G_2) Inferior Epytrochoid

(G_3) Superior Epytrochoid See Fig. 5.27.

Method 1 :

A rolling circle, of r = 27 mm radius, is rolling outside a directing circle of R_d = 8... mm radius without slip. A point P is on the circumference, a point Q is 7 mm inside the circumference and a point R is 13 mm outside the circumference of the rolling circle. Find out the loci of points P, Q and R for one revolution of the rolling circle.

For solution see Fig. 5.27 and follow the procedure as given below :

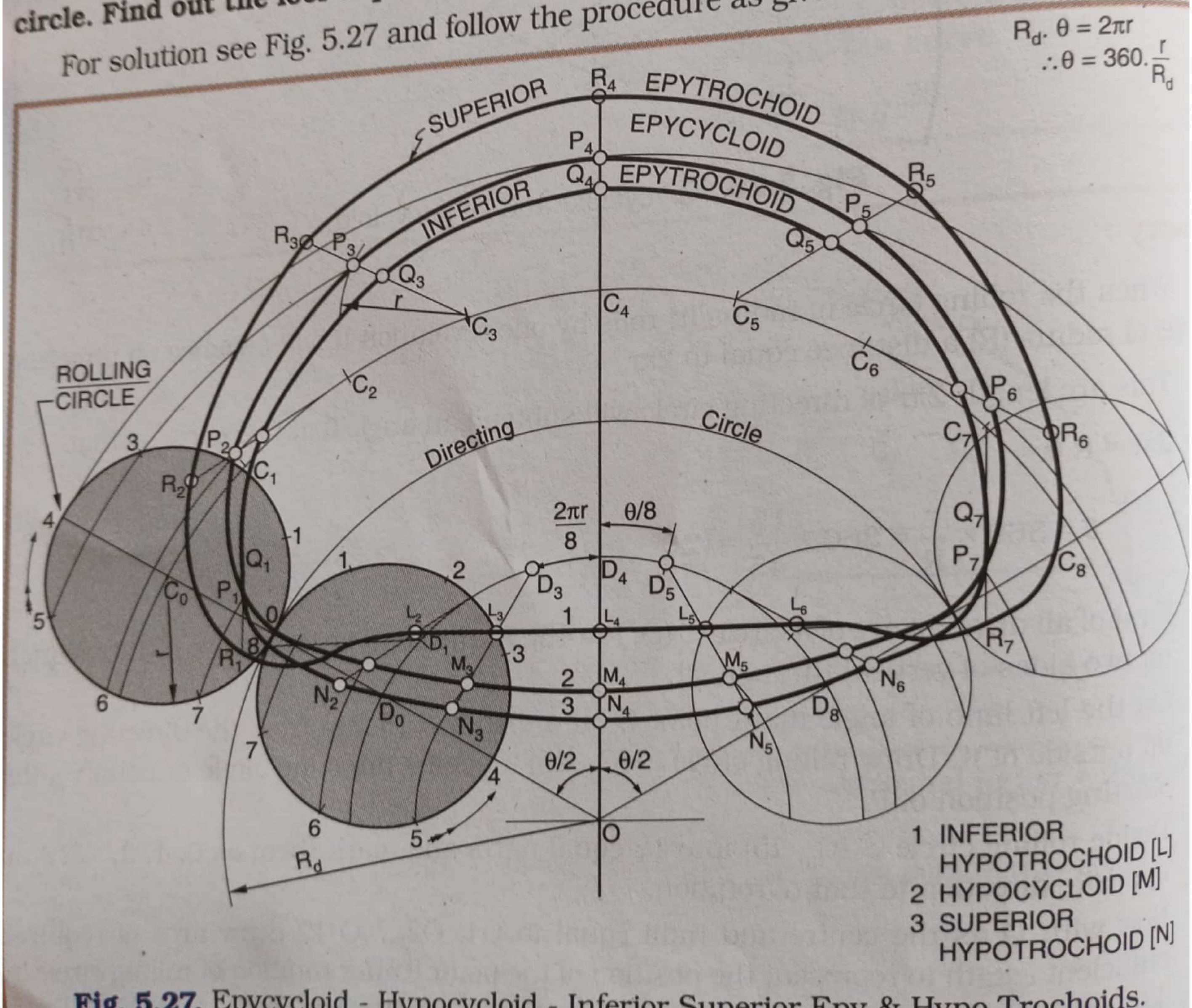


Fig. 5.27. Epycycloid - Hypocycloid - Inferior Superior Epy & Hypo Trochoids.

1. First follow the same procedure as followed for epycycloid and get points P_0 - C_0, P_1 - C_1,....P_8 - C_8.
2. Now join C_0P_0 and mark on it Q_0 and R_0 taking C_0Q_0 = 20 (27 - 7) and C_0R_0 = 40 (27 + 13). Similarly on C_1P_1, C_2P_2,.......C_8P_8 mark points (Q_1,R_1), (Q_2, R_2),.....(Q_8,R_8) respectively.
3. Join (P_0,P_1,......P_8), (Q_0, Q_1, Q_2,....Q_8) and (R_0, R_1, R_2,......R_8) by a smooth curve to get Epycycloid, Inferior Epytrochoid and Superior Epytrochoid respectively.

Method 2 : II (G_3) Superior Epytrochoid. See Fig. 5.28.

A circle, of 14 mm radius, is rolling outside another circle of 42 mm radius. A point P is outside the rolling circle 20 mm from the centre and is initially nearest to the directing circle centre. Draw the locus of the point P for one revolution and name the curve.

For solution follow the procedure as given below and see Fig. 5.28.

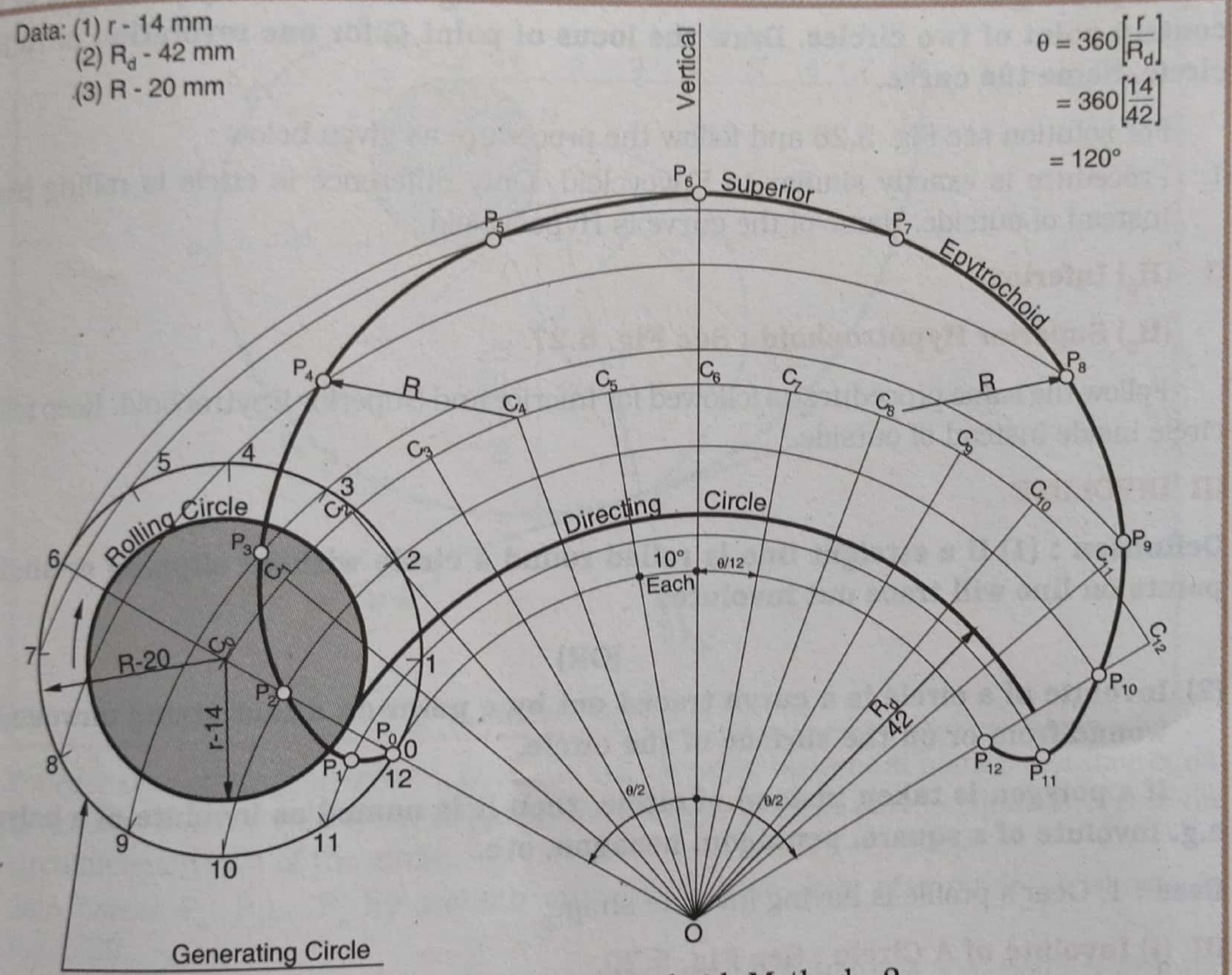


Fig. 5.28. Superior Epytrochoid. Method - 2.

1. First find out angle θ by following formula

$$\theta = 360 \times \frac{r}{R_d} = 360 \times \frac{14}{42} = 120°$$

Now draw θ/2 = 60° on both sides of vertical through O.

2. On the left limb of angle θ mark point C_0 at a distance 56(42 + 14)mm from O and draw

circle ⊙ (C_0, 14). Draw also circle ⊙ (C_0, 20) and divide it into 12 equal parts and mar points 0, 1, 2,....12 in the opposite direction to that of rotation.

3. Now draw arcs of circles with O as centre and radii equal to O-O, 01, 02,....012. O these arcs point P will be located after 1/12 revolution and multiple of it.
4. Draw arc with O as centre and radius equal to O-C_0. Divide angle θ into 12 equal par and get 12 centres C_0, C_1, C_2,....C_{12} on the centre line.
5. Now with C_1, C_2,....C_{12} as centres and radius equal to R = 20 mm draw arcs to cut ar of circle through 1, 2,....12 at points P_1, P_2,....P_{12} respectively. Arcs from centres C_4 ar C_8 are shown in Fig. 5.28 to get P_4 and P_8 respectively.
6. Join P_0, P_1, P_2,....P_{12} in sequence by a smooth curve to get Superior Epytrochoid.

II (H1) Hypocycloid : See Fig. 5.26.

A circle, of r = 15 mm radius, is rolling inside a circle of R = 45 mm radius witho slip. A point Q is on the circumference of the rolling circle. Initially point Q is at t contact point of two circles. Draw the locus of point Q for one revolution of rolli circle. Name the curve.

For solution see Fig. 5.26 and follow the procedure as given below :

1. Procedure is exactly similar to Epycycloid. Only difference is circle is rolling ins instead of outside. Name of the curve is Hypocycloid.

II (H_2) Inferior

(H_3) Superior Hypotrochoid : See Fig. 5.27.

Follow the same procedure as followed for Inferior and Superior Epytrochoid. Keep roll circle inside instead of outside.

III INVOLUTE

Definition : (1) If a straight line is rolled round a circle without slipping or slidi points on line will trace out involutes

[OR]

(2) Involute of a circle is a curve traced out by a point on a taut string unwound wound from or on the surface of the circle.

If a polygon is taken instead of circle, then it is named as involute of a poly e.g. Involute of a square, pentagon, hexagon, etc.

Uses : 1. Gear's profile is having involute shape.

III (I) Involute of A Circle : See Fig. 5.29.

A string is unwound from a circle of 20 mm diameter. Draw the locus of end string P for unwounding the string's one turn. String is kept tight during unwoundi Draw tangent and normal to the curve at any point.

For solution see Fig. 5.29 and follow the procedure as given below :

Principle :

When the string is unwound, the length of the string goes on increasing by the amount equal to the arc length of the circle from which the string is unwound. Similarly, the length goes on decreasing during wounding operation. Further as the string is kept tight during unwounding or wounding operation it will remain tangential to the circle.

1. Draw a circle of 20 mm diameter and divide it into 8 equal parts and mark them as 0, 1, 2,....8.
2. Draw tangents to the circle at points 1, 2, ...8 in the direction of position of string during unwounding operation.
3. On tangents at points 1, 2,....8 take length equal to arc length 01, 02, 03,....08 to mark points P_1, P_2, P_3,....P_8 respectively.

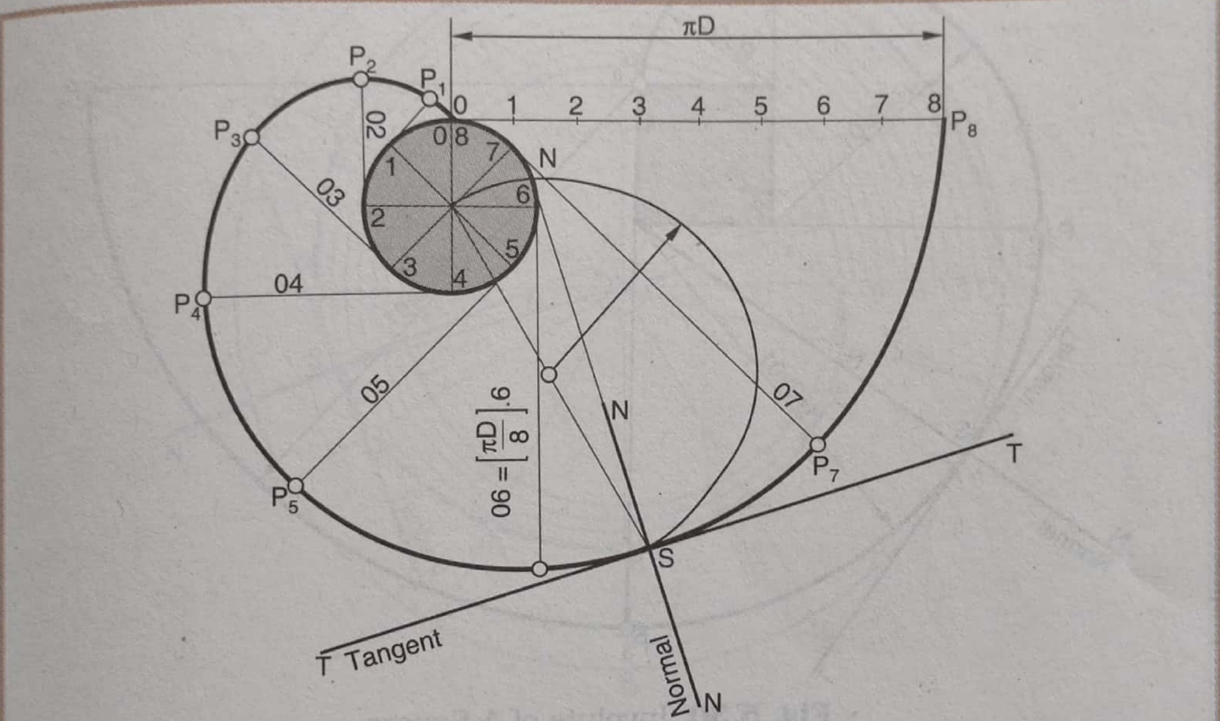


Fig. 5.29. Involute of A Circle.

For ease of taking arc length 01, 02,....08 etc. draw on tangent at point 8 distance equal to πD and divide it into 8 equal parts. Here length of each part is equal to 1/8 of the circumference (πD) of the circle.

4. Join points P_0, P_1,....P_8 by smooth curve to get involute of a circle as shown in Fig. 5.29.
5. To draw tangent and normal to the curve at any point S of curve, join point S with the centre of the circle. With this line as diameter draw a semicircle cutting the circle of involute at point N. Join N with S to get normal and draw right angle to this normal at point S to get tangent TT.

III (J) Involute of A Polygon : See Fig. 5.30.

A string is unwound from a square of 25 mm side. Draw the locus of end P of string for unwounding the string's one turn. String is kept tight during unwounding operation. Draw tangent and normal to the curve at any point.

Principle :

When the string is unwound from the surface of a square or any polygon, it turn on the corner of the square or polygon till it comes in line with the next surface. I Fig. 5.30 from P_0 to P_1 position the string turns on corner 1. At P_1 the string is i line with surface 12. With this principle follow the procedure as given below and se Fig. 5.30.

1. Draw a square of 25 mm side and mark corners 0, 1, 2,....4. Extend lines 21, 32, 4 and 14 by suitable amount.
2. Draw quarter circles with centres 1, 2, 3 and 4 and radii (1 × 01), (2 × 01), (3 × 01) an (4 × 01) to cut previously drawn lines at points P_1, P_2, P_3 and P_4 as shown in figure.

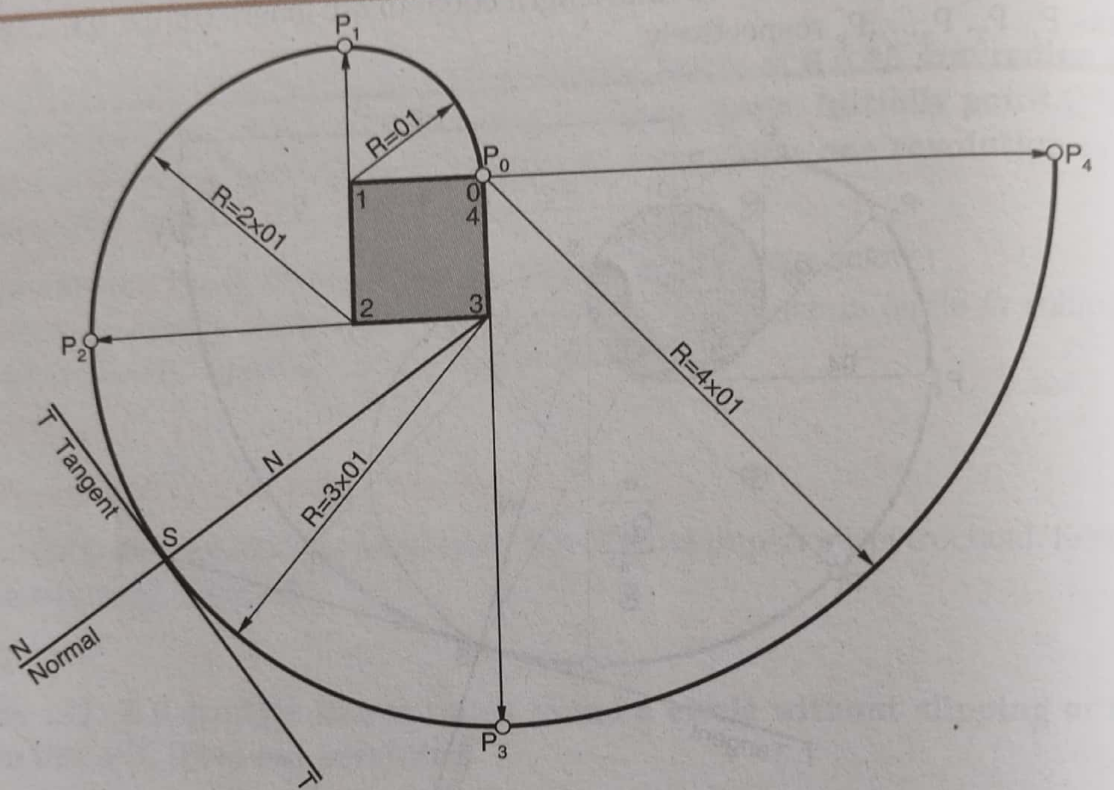


Fig. 5.30. Involute of A Square.

3. Combination of these four quarter circles P_0P_1, P_1P_2, P_2P_3 and P_3P_4 is nothing but a involute of a square.
4. To draw tangent and normal to the curve at any point S, join S with centre of th corresponding circle, which will be normal to the curve. Draw right angle to it at S t get tangent.

IV SPIRAL

Definition :-

Archemedian Spiral : It is a curve generated by a point moving uniformly alon a straight line, while the line swings or rotates about a fixed point with uniform angul velocity.

[OR]

It is a curve traced out by a point which moves uniformly both about the centr and at the same time away or towards the centre.

Logarithmic Spiral : It is the curve generated by the end of radius vector rotating about the centre so that the ratio of the lengths of the consecutive radius vectors for equal angular movements is constant.

Uses :
1. Shape of springs of watch mechanism, toys etc
2. Scroll plate of lathe chuck
3. Clamping devices of jigs and fixtures
4. Profile of cams for automation

IV (K) Archemedian Spiral : See Fig. 5.31.

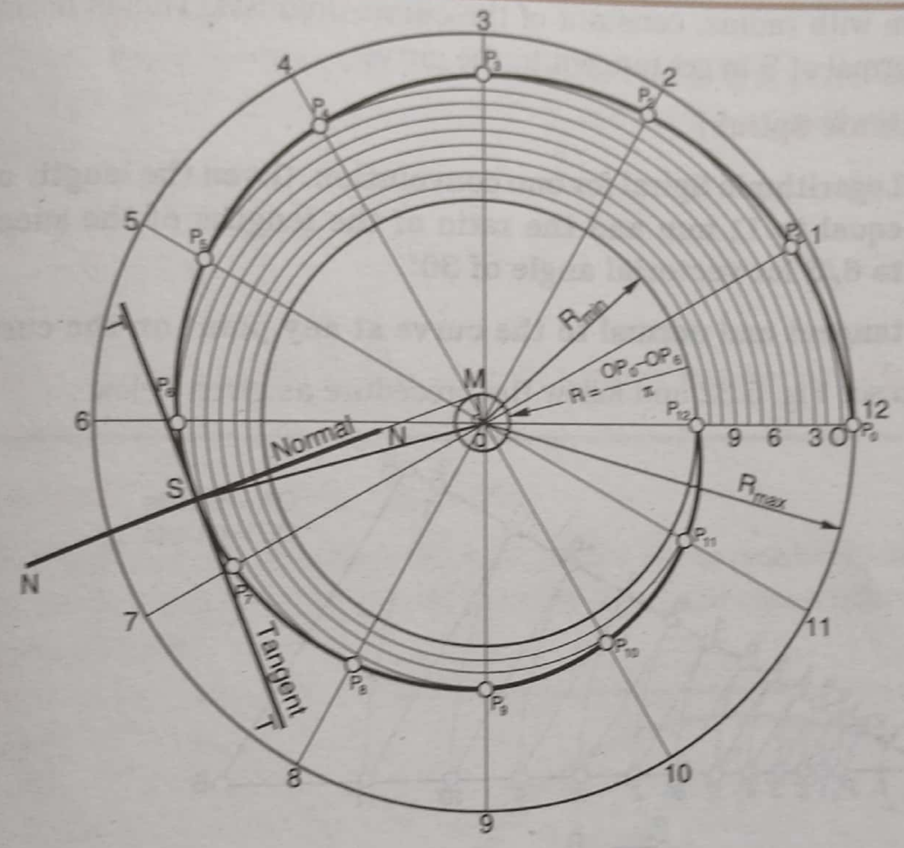


Fig. 5.31. Archemedian Spiral.

Problem 2 : Construct an Archemedian Spiral of one convolution, given the maximum and minimum radii as 55 mm and 31 mm respectively. Draw tangent and normal to the curve at any point.

See Fig. 5.31 and follow the procedure as given below :

1. With pole O as centre and radii equal to R_{max} = 55 mm and R_{min} = 31 mm, draw two circles.
2. Divide 360° at pole O into 12 equal parts and draw radial lines 00, 01, 02,....012.
3. Divide 24 mm [(55 - 31) or ($R_{max} - R_{min}$)] length also into same 12 equal parts as shown in Fig. 5.31.
4. On lines 01, 02,....0.12 mark points P_1, P_2,.....P_{12} by successively decreasing the length of radius vector by one division each time. For decreasing radius vector by one division, draw arcs of circles with O as centre and radii equal to 01, 02,....012 respectively.
5. Join points P_0, P_1, P_2,....P_{12} by smooth curve to get **Archemedian Spiral.**

6. Now to draw tangent and normal to the curve at any point S draw a circle with pole O as centre and radius equal to constant of the curve.

$$\text{Constant of the curve} = \frac{\text{Difference in lengths of any two radius vectors}}{\text{Angle between the corresponding radius vectors in radian}}$$

$$= \frac{OP_0 - OP_{12}}{2\pi} = \frac{55-31}{2\pi}$$

$$= 3.82 \text{ mm}$$

Join point S with pole O. Draw right angle at O with this SO line to get a point M on the circle drawn with radius, constant of the curve. Join SM. This is normal. Draw right angle to this normal at S to get tangent to the curve.

IV (L) Logarithmic Spiral :

Construct Logarithmic Spiral for one convolution. Given the length of the shortest radius vector equal to 11 mm and the ratio of the lengths of the successive radius vectors equal to 6/5 for vectorial angle of 30°.

Draw also tangent and normal to the curve at any point on the curve.

For solution see Fig. 5.32 and follow the procedure as given below :

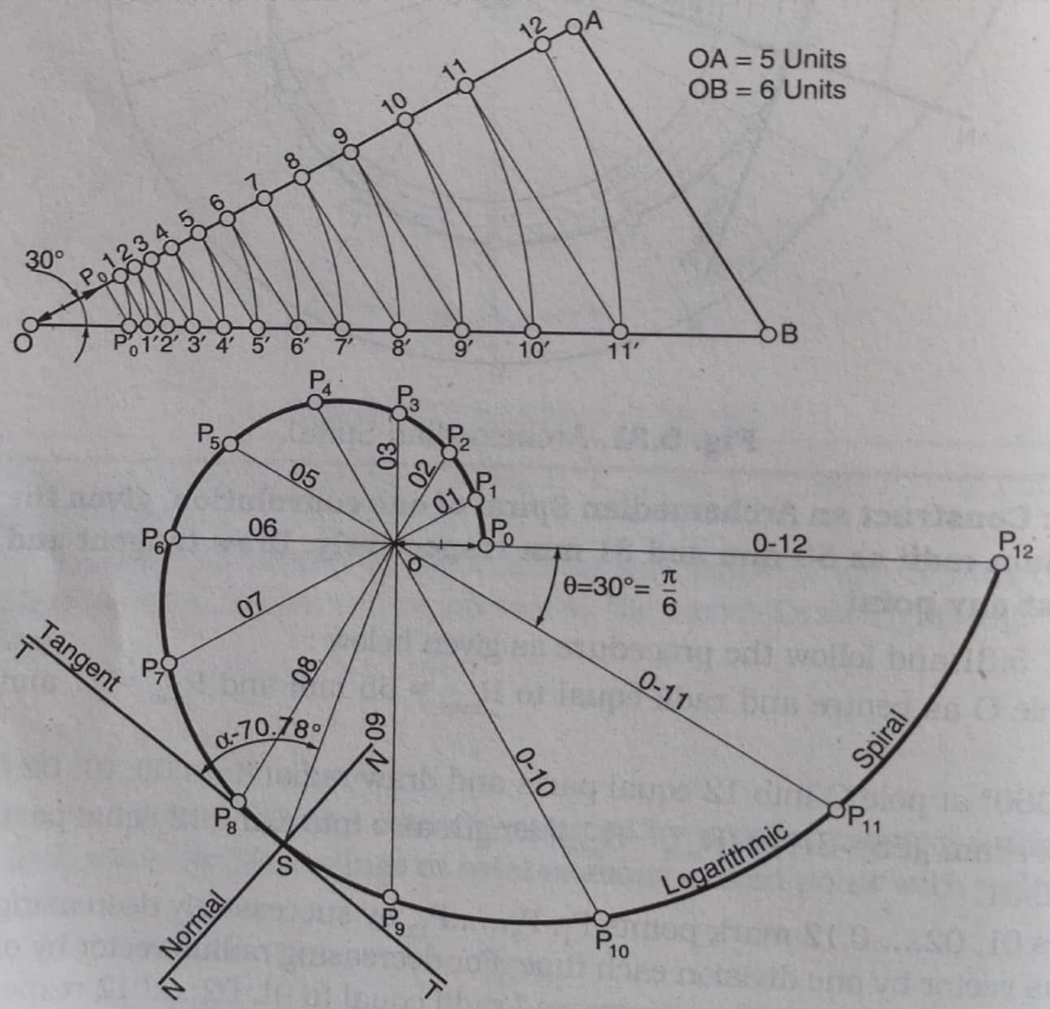


Fig. 5.32. Logarithmic Spiral.

1. To find the lengths of successive radius vectors w.r.t shortest radius vector of 11 mm, without undergoing calculation, construct the scale for radius vector as shown in Fig. 5.32 following the procedure as given below :
   (a) Draw two lines OA and OB at 30° angle. On line OA take OA = 5 units and on line OB take OB = 6 units. On line OA take OP_0 = 11 mm. Draw $P_0P_0¢$ parallel to AB.
   (b) Now with O as centre and $OP¢_0$ as radius draw an arc intersecting at 1 with OA.
   (c) From 1 draw line 11' parallel to AB.
   (d) Continue this procedure till we get 12 points on OA. Scale is ready for use.
2. Take any point O and draw at O, 12 radius vectors at 30° of lengths OP_0, 01, 02,....012 from scale and get points P_0, P_1, P_2,....P_{12} respectively. Join these points in sequence by smooth curve to get **Logarithmic Spiral.**
3. To draw tangent and normal to any point S on the curve, first join S with O. Now with OS line at S draw α = 70.78° to get tangent T - T and draw perpendicular at S to tangent to get normal N - N.

$$\tan \alpha = \frac{\log_{10}^{e}}{\frac{1}{\pi/6}\log_{10}\frac{6}{5}} \qquad \therefore\ \alpha = 70.78$$

IV (M) Spiral for Clock Spring - Spiral of Semi Circles and Spiral of Quarter Circles.

Spiral of Semi Circles : See Fig. 5.33

Spiral of Quarter Circles : See Fig. 5.34.

This spiral curve is a combination of semi circles of successively increasing size as shown in Fig. 5.33. It is easy to understand from the figure the constructional features and so explanation is not given.

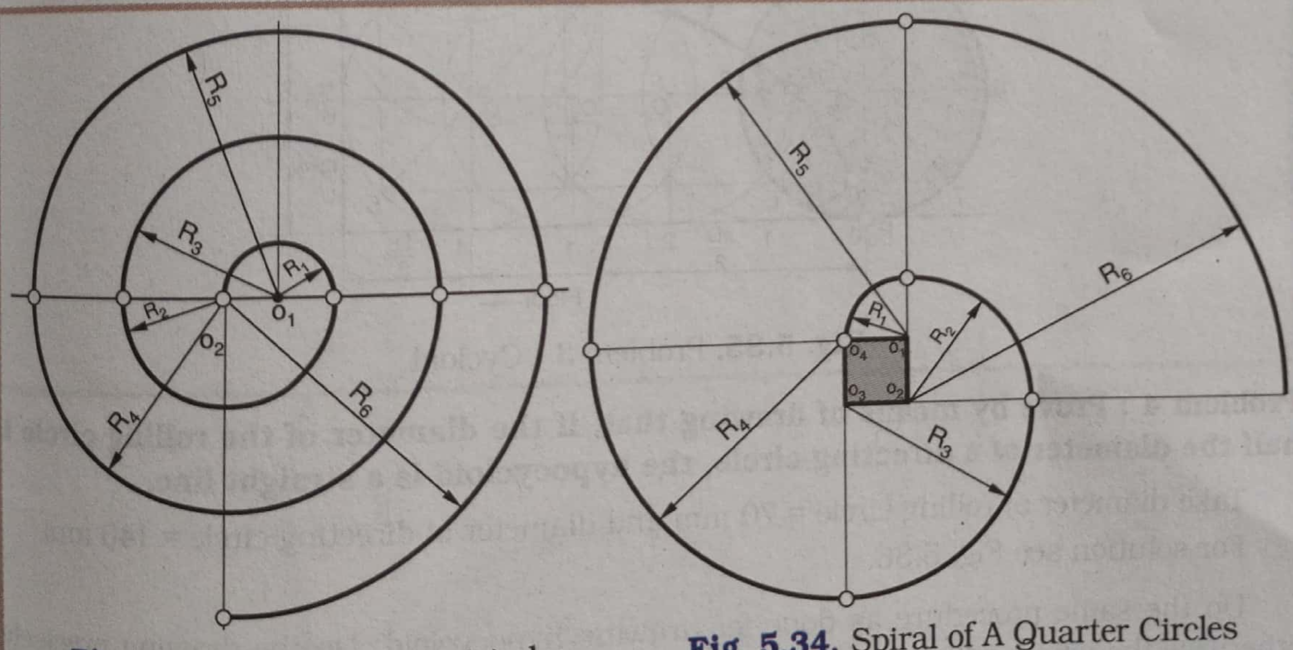


Fig. 5.33. Spiral of A Semicircles

Fig. 5.34. Spiral of A Quarter Circles

It is a combination of quarter circles as shown in Fig.5.34. Fig.5.34 is self explanatory and so explanation is not given.

Problem 3 : A circle, of diameter D, rolls without slip on a horizontal surface (floor) by 1/2 revolution and then it rolls up a vertical surface (wall) by another 1/2 revolution. Initially the point P is at the bottom of circle touching the floor. Draw the path of the point P.

Take diameter of circle = 40 mm

Initially distance of centre of circle from the wall Ω 83 mm [Half circumference + D/2]

For solution see Fig. 5.35 and follow the procedure as given below :

1. Draw two lines one horizontal and another vertical to represent floor and wall respectively.
2. Draw ⊙ (C_0, 20) keeping distance of C_0 from the wall equal to 83 mm and just touching the floor. Mark position P_0 of point P at the bottom of the circle.
3. For first 1/2 revolution just do the procedure of drawing cycloid till the point P, reaches the top position (P_4), as shown in Fig. 5.35.
4. Now the circle will roll on the wall and for that initial position of Point P is P_4. Continue the same procedure as cycloid and get points P_5, P_6, P_7 and P_8. The curves P_4 to P_6 and P_6 to P_8 are nothing but last 1/4 part of a cycloid and first 1/4 part of a cycloid respectively. Join P_0, P_1,....P_4....P_6....P_8 by smooth curve to get the path of the point P.

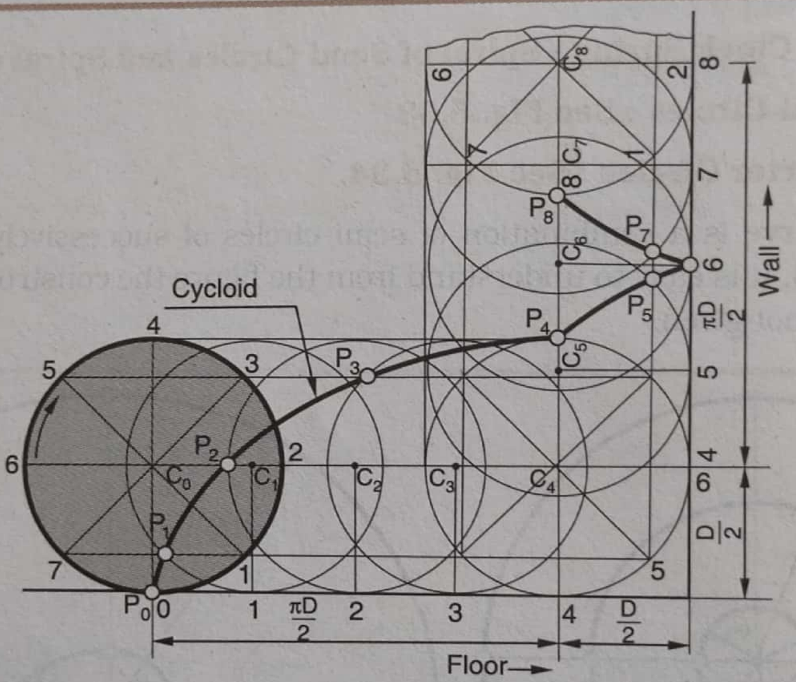


Fig. 5.35. Problem 3 - Cycloid

Problem 4 : Prove by means of drawing that, if the diameter of the rolling circle is half the diameter of a directing circle, the hypocycloid is a straight line.

Take diameter of rolling circle = 70 mm and diameter of directing circle = 140 mm. For solution see Fig. 5.36.

Do the same procedure as done for drawing hypocycloid. Do the drawing precisely, otherwise the points will deviate a little bit from the straight line and you will be confused while joining. Fig. 5.36 is self explanatory.

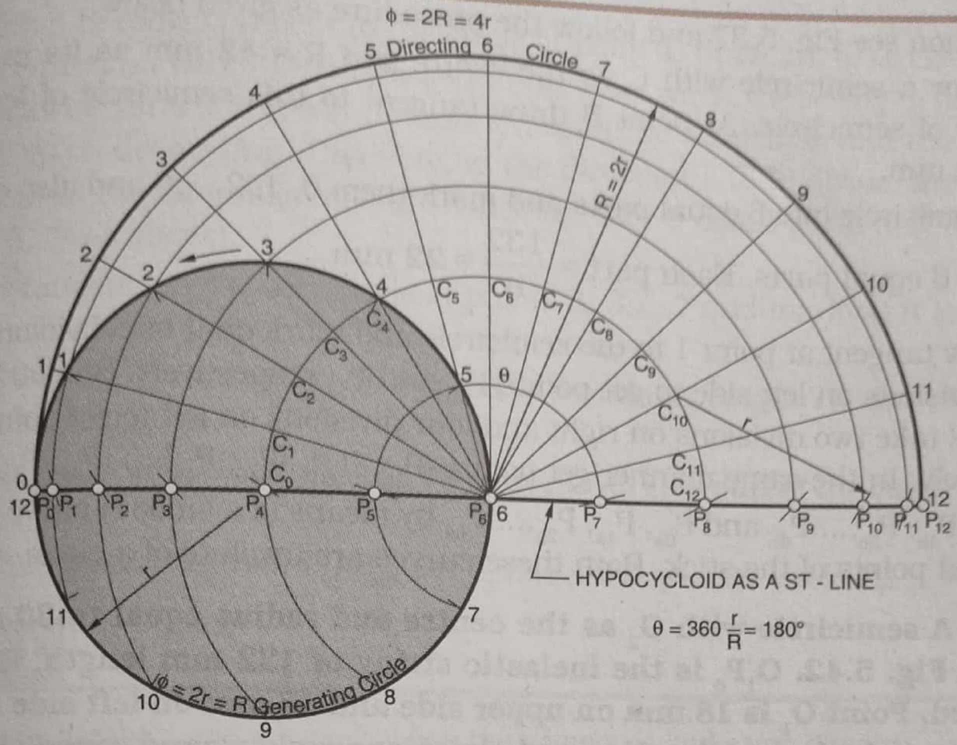


Fig. 5.36. Problem 4 - Hypocycloid Straight Line

Problem 5 : A stick, of length equal to the circumference of a semicircle, is initially tangent to the semicircle on the right side of it. This stick now rolls over the circumference of a semicircle without sliding till it becomes tangent on the left side of the semicircle. Draw the loci of two end points of this stick. Name the curve. Take R = 42 mm.

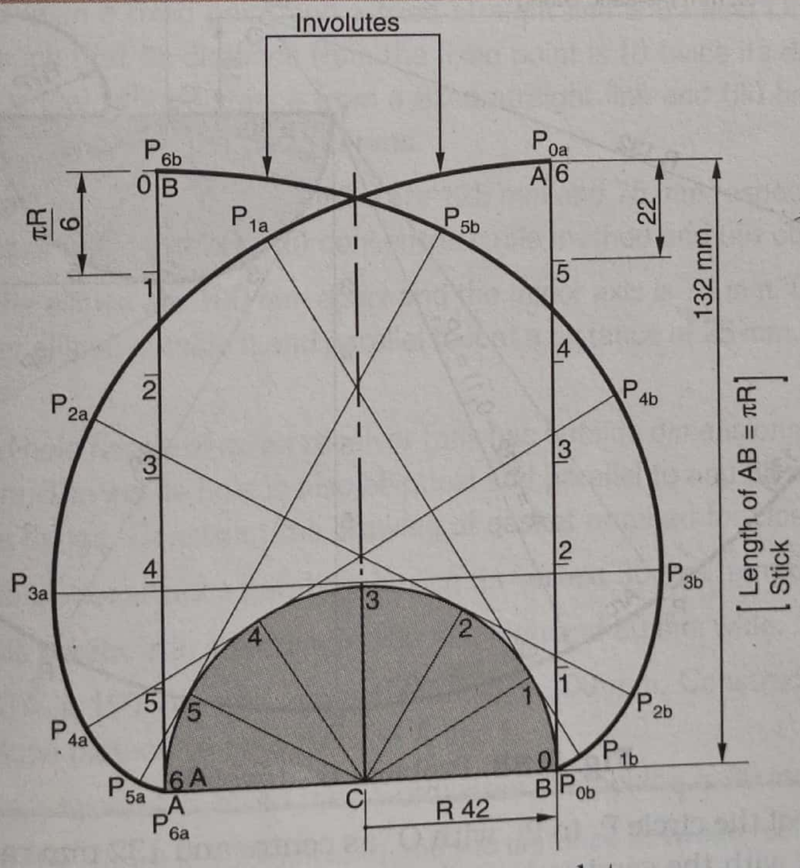


Fig. 5.37. Problem 5 - Involute.

For solution see Fig. 5.37 and follow the procedure as given below:

1. First draw a semicircle with C as the centre and R = 42 mm as its radius. AB is diameter of semicircle. At point B draw tangent to this semicircle of length equal $\pi R = 132$ mm.
2. Divide semicircle into 6 equal parts and mark them 0, 1, 2,....6 and also divide tange at B into 6 equal parts. Each part = $\frac{132}{6} = 22$ mm.
3. Now draw tangent at point 1 to the semicircle and mark on it one division on right si and 5 divisions on left side to get points P_{1b} and P_{1a} respectively. Similarly, on tange at point 2 take two divisions on right and four divisions on left to get points P_{2b} and P respectively. In the same manner get points (P_{3b}, P_{3a}), (P_{4b}, P_{4a}),....(P_{6b}, P_{6a}).
4. Join P_{0b}, P_{1b}, P_{2b},....P_{6b} and P_{0a}, P_{1a}, P_{2a},....P_{6a} by means of a smooth curve to get the lo of two end points of the stick. Both these curves are involute of a circle.

Problem 6 : A semicircle with O_2 as the centre and radius equal to 30 mm is fixe as shown in Fig. 5.42. O_1P_0 is the inelastic string of 132 mm length. End O_1 of th string is fixed. Point O_1 is 18 mm on upper side and 18 mm on left side of O_2. Strin is turned in anticlockwise direction and simultaneously wound round the surface the semicircle. Draw the locus of the point P, the free end of the string.

For solution see Fig. 5.38 and follow the procedure as given below :

1. Draw semicircle and divide it into 6 equal parts. Mark points 1, 2,....7. as shown.

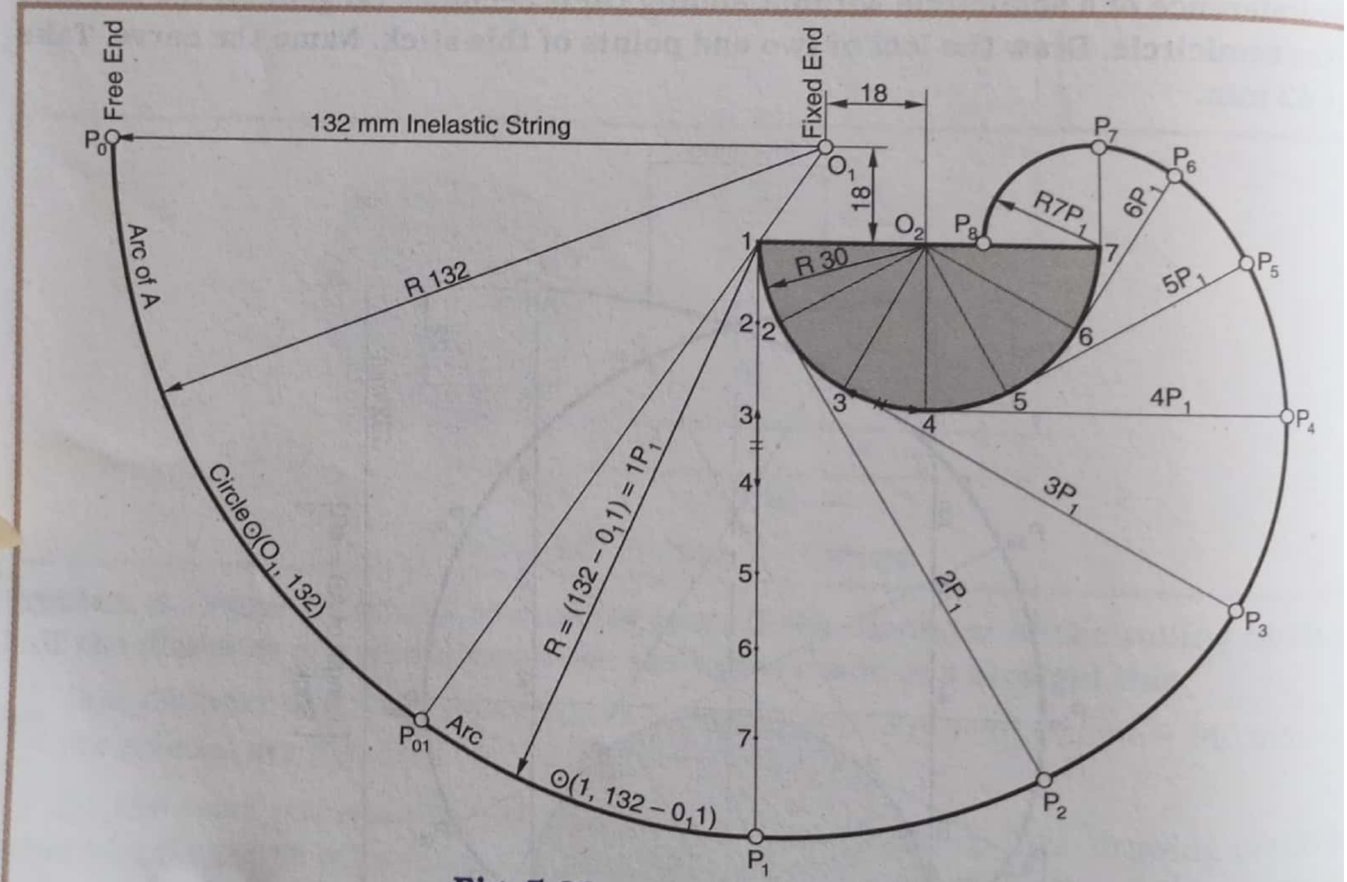


Fig. 5.38. Problem 6 - Involute

2. First draw arc of the circle P_0 to P_{01} with O_1 as centre and 132 mm radius, till the strin makes contact with the semicircle at the point 1.

3. Till the string becomes tangent to the circle at point 1, draw arc of circle P_{01} to P_1 with 1 as the centre and (132 - $O_1$1) mm as radius. Now when the string is further turned in anticlockwise direction it will be wound round the semicircle and the length of the string will go on decreasing. Therefore, in the direction 1 to P_1 divide, the length of the string equal to the circumference of semicircle, into 6 equal parts and mark them as points 2, 3,..7 as shown.
4. Now draw tangents to the semicircle at points 2, 3,....7 and mark on it length equal to $2P_1$, $3P_1$,....$7P_1$ to get points P_2, P_3,....P_7 respectively.
5. Now with 7 as the centre and radius equal to remaining length $7P_1$ draw quarter circle, as shown in the figure.
6. Join points P_1, P_2,....P_7 by means of a smooth curve. This part of the curve is an involute of a circle.
7. Total path of point P is from P_0, P_{01}, P_1, P_2,....P_7 and quarter circle P_7P_8.

EXERCISE

1. Draw ellipse, parabola and a hyperbola on the same axis and same directrix. Take distance of focus from the directrix equal to 50 mm and eccentricity for the ellipse, parabola and hyperbola as 2/3, 1 and 3/2 respectively. Plot at least 8 points. Take suitable point on each curve and draw tangent and normal to the curve at that point.
2. The vertex of a hyperbola is 30 mm from its directrix and the eccentricity ratio of the hyperbola is 5/3. Draw the hyperbola curve and draw tangent and normal to the curve at a point 65 mm from the directrix.
3. The distance between a fixed point and a fixed straight line is 60 mm. Draw the locus of the moving point P such that its distance from the fixed point is (i) twice its distance from a fixed straight line, (ii) equal to its distance from a fixed straight line and (iii) half its distance from the fixed straight line. Name the three curves.
4. The major axis and minor axis of the ellipse are 125 mm and 75 mm respectively. Construct an ellipse by (i) arcs of circle method, (ii) concentric circle method and (iii) oblong method.
5. Focal points of the ellipse are 100 mm apart and the minor axis is 75 mm. Draw the ellipse and construct another ellipse outside it and parallel to it at a distance of 25 mm. Find also the length of the major axis.
6. An elliptical hand hole flange of an air receiver tank has outside dimensions, major axis 100 and minor axis 70 mm. The inside hole is also elliptical and parallel to and 25 mm away from outer peryphery of the flange. Construct the drawing of gasket required for closing the hole.
7. Using locus method, construct a parabola having its vertext 30 mm from the focus.
8. Draw vertical axis parabola in a rectangle 100 mm high and 80 mm wide.
9. Horizontal line OA is 150 mm and vertical line OB is 100 mm. Construct a parabola by the tangent or envelope method to pass through A and B.
10. Draw a rectangular hyperbola, given the co-ordinates of a point x = 30 mm and y = 120 mm.
11. The major axis of the ellipse is 100 mm long and the distance between foci is 60 mm. Draw the ellipse by oblong method. Find the length of the minor axis.

12. Two fixed points F_1 and F_2 are 90 mm apart. Construct the locus of the point P moving in th same plane of F_1 and F_2 in such a way that the sum of its distances from the fixed points F_1 an F_2 is always the same and equal to 120 mm. Give name to the curve.

13. Construct an ellipse in a parallelogram 125 mm x 90 mm sides. Take included angles parallelogram as 120° and 60°.

14. PQR is a triangle having sides PQ = 100 mm, QR = 80 mm and RP = 60 mm. Construct a ellipse passing through points P, Q and R.

15. A throw of ball from boundary of a cricket ground reaches the Wicket Keeper's gloves followi the parabolic path. Maximum height achieved by the ball above the ground is 31 metres. Assum the point of throw and the point of catching position 1 metre above the ground. Radial distanc of boundary from Wicket Keeper is 75 metres. Construct the path of ball.

16. Draw a rectangle of 125 mm x 100 mm sides and construct two parabolas in it with their axe parallel to two sides of a rectangle.

17. Two asymptotes OX and OY are at 75° angle. Point P is 35 mm from OX and 50 mm from O Draw hyperbola passing through the point P.

18. Two fixed points F_1 and F_2 are 60 mm apart. Trace two curves, described out by points P_1 an P_2 moving in the same plane of F_1 and F_2 such that the difference between distances from and F_2 is always constant and equal to 30 mm.

19. Draw a cycloid for a rolling circle, of 60 mm diameter rolling along a straight line without slippin Take initial position of the tracing point at the bottom of the vertical centre line of the rollin circle. Draw tangent and normal to the curve at a point 35 mm above the directing line.

20. A circle of 49 mm diameter rolls along a straight line without slipping. Draw the curves trace by points Q, P and R located 15 mm inside the circle, on the circle and 25 mm outside the circl respectively. Take initial positions of points Q, P and R at the bottom on vertical centre line c circle. Name the curves traced.

21. A circle of 50 mm diameter rolls on the circumference of another circle of 150 mm diamete and outside it. Draw the locus of the point P on the circumference of the rolling circle for on complete revolution of it. Take initial position of point P at the contact point between two circle Name the curve and draw tangent and normal to the curve at a point 115 mm from the centr of the bigger circle.

22. In problem 21 take rolling circle inside the directing circle and draw locus of point P. Name th curve and draw tangent and normal to the curve at point 60 mm from the centre of the directin circle.

23. Show by means of drawing that when the diameter of a rolling circle is half the diameter o directing circle, the hypocycloid is a straight line.

24. A circular ring of 100 mm diameter is rolling on the fixed cylinder of 50 mm diameter keepin contact from inside. Draw the locus of point P on the circumference of circular ring initially a the top contact point.

25. A wheel of 49 mm diameter rolls downward on the vertical wall by 1/2 a revolution and then on the floor by 1/2 a revolution without slipping. Draw the locus of point P on the circumference of the wheel. Take initial position of the point P at the contact point of the wheel with the wall

26. A wheel of 50 mm diameter rolls on (i) inside and (ii) outside on another circle of 150 mm diameter. Draw and name the curves traced out by points Q and S 15 mm and 35 mm from the centre on the wheel along a straight line passing through the centre of the wheel.

27. Construct one complete turn of an involute of a
    (i) circle of 30 mm diameter.
    (ii) square of 30 mm side
    (iii) hexagon of 25 mm side

28. One end Q of inelastic string PQ, 150.5 mm long, is attached to the circumference of a half circular half hexagonal disc of 49 mm diameter. Draw the curve traced out by the other end of the string P when it is completely wound round the circumference of the disc, keeping the string always tight. Take initial position of string tangent at the mid point Q of circular portion.

29. A rod PQ 99 mm long is resting horizontally on the circumference of a circular disc of 63 mm diameter touching at the mid point. Rod PQ rolls without slipping on the circumference of a circular disc to the full extent on both the sides. Draw the loci of the points P and Q and name the curves.

33. Draw an Archemedian Spiral of 1.5 convolutions, the greatest and the least radii being 125 mm and 35 mm respectively. Draw tangent and normal to the spiral at a point 85 mm from the pole.

34. A point P moves radially outward from the centre of the circular disc to the peryphery when disc completes 2 revolutions. Radial movement of a point P and circular motion of disc is assumed uniform. Take diameter of the disc 120 mm Draw the locus of the point P and name the curve.

35. Construct logarithmic spiral for one convolution, given the length of the shortest radius equal to 15 mm and the ratio of the lengths of the successive radius vectors equal to 6/5 for vectorial angle of 30°.

36. Draw a square of 5 mm sides and then draw 4 turns of a spiral of quarter circles on it using four corners of the square as centres.

37. Taking 2 centres 5 m.m. apart on vertical line draw 4 turns of a spiral of half circles.

Problem 38 : A plot of ground in the shape of a parallelogram 15000 mm x 10000 mm, the angle between the sides being 60°. Inscribe an elliptical flower bed in it. Select a suitable scale.

Problem 39 : Three points A, B and P while lying along a horizontal line in order, have AB = 60 mm and AP = 80 mm. While A and B are fixed and P starts moving such that AP + BP remain always constant and when they form an isosceles triangle, AP = BP = 50 mm. Draw the path traced out by the point P from the commencement of its motion back to its initial position.

Problem 40 : The concrete arch for a water channel of a railway bridge is semi elliptical in shape with major axis 2.5 metres and minor axis 1.5 metres. Draw the boundary of concrete arc to suitable scale.

Problem 41 : A cricket ball is thrown and reaches a maximum height of 10 metres and falls on the ground at a distance of 30 metres from the point of projection. Determine the angle of projection. Draw the path of the cricket ball and name the curve. Assume that the point of projection is on the ground level.

Problem 42 : An autohead light reflector, for parallel rays of light, is parabolic in shape wit its bulb at the focus point. The distance of the focus from where the parabola cuts the ax is 100 mm. Draw the shape of the reflector.

Problem 43 : Motor car head lamp parabolic reflector is having an aperture (opening) 175 mm and a depth of 135 mm Draw the shape of the reflector.

Problem 44 : A stone is thrown from a building 7 metres high and its highest point of fligh just crosses a palm tree 14 metres high. Trace the path of the projectile, if the distanc between the building and the palm tree is 3.5 metres. Take a suitable scale.

Problem 45 : For a perfect gas, the relation between the pressure P and the volume V isothermal expansion is given by PV = constant. Draw the curve of isothermal expansio of an enclosed volume of gas if 0.056634 cubic metres of the gas correspond to a pressu of 0.3515 N. per sq.cm.

Problem 46 : A wheel 1.5 metres in diameter has seven spokes connecting the rim and th hub. The wheel is rotating in anticlockwise direction at 80 rpm. A particle of dust star from the centre and travels along a spoke with uniform velocity and reaches the rim aft two seconds. Trace the path of the particle. Select a scale 1 : 25

Problem 47 : Draw a logarithmic spiral for one convolution, the successive radii are of th ratio 9:8, final radius vector is 90 mm and the angle between the successive radii bei 30°. Draw the tangent at any point of the curve.

Problem 48 : A vehicle has 60 cm diameter wheels. For one complete revolution of the whee draw the locus of the point on its circumference when it passes over a segmental arche culvert of radius 3 metres. Take a scale of 1 : 20.

Problem 49 : A circus man rides a motorcycle inside a globe of diameter 4 metres. The mot cycle wheel is 0.8 meter in diameter. Draw the locus of a point spot on the circumferen of the motor cycle wheel for its one complete turn.

Miscellaneous Problems

Problem 1 : The foci of an ellipse are 110 mm apart. The minor axis is 70 mm long. Determine the length of the major axis and draw half ellipse by rectangle method and other half by concentric circles method. [8]

[Ans : Major Axis 130 mm] [B. T. E. M. S. April 2000 (Civil)]

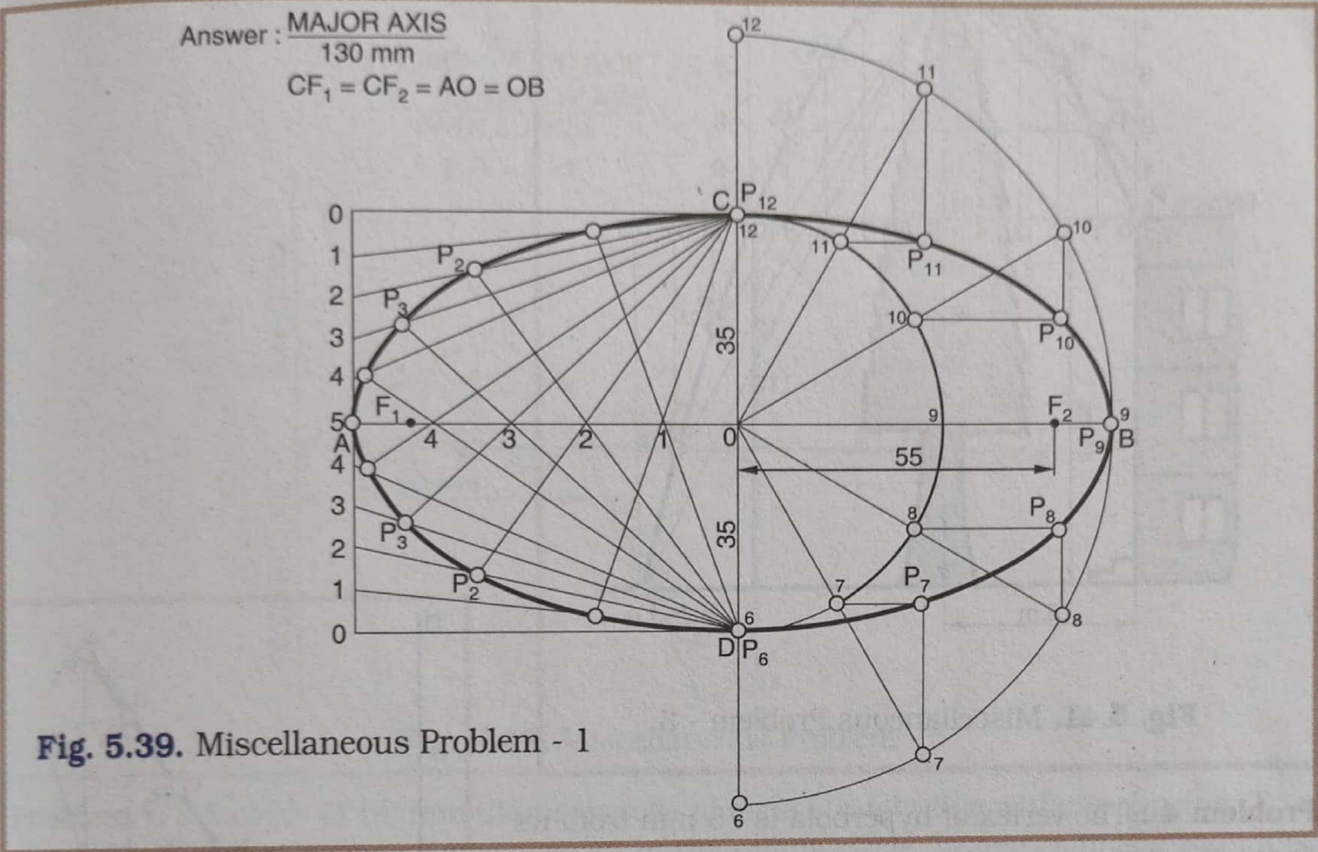


Fig. 5.39. Miscellaneous Problem - 1

Problem 2 : Inscribe an ellipse in a parallelogram having sides 100 mm and 65 mm long and included angle of 120°. Determine the major and minor axis of an ellipse. Draw the tangent and normal to the ellipse at a point 20 mm above the horizontal axis and to the left of the centre. [Mumbai University, December 1997]

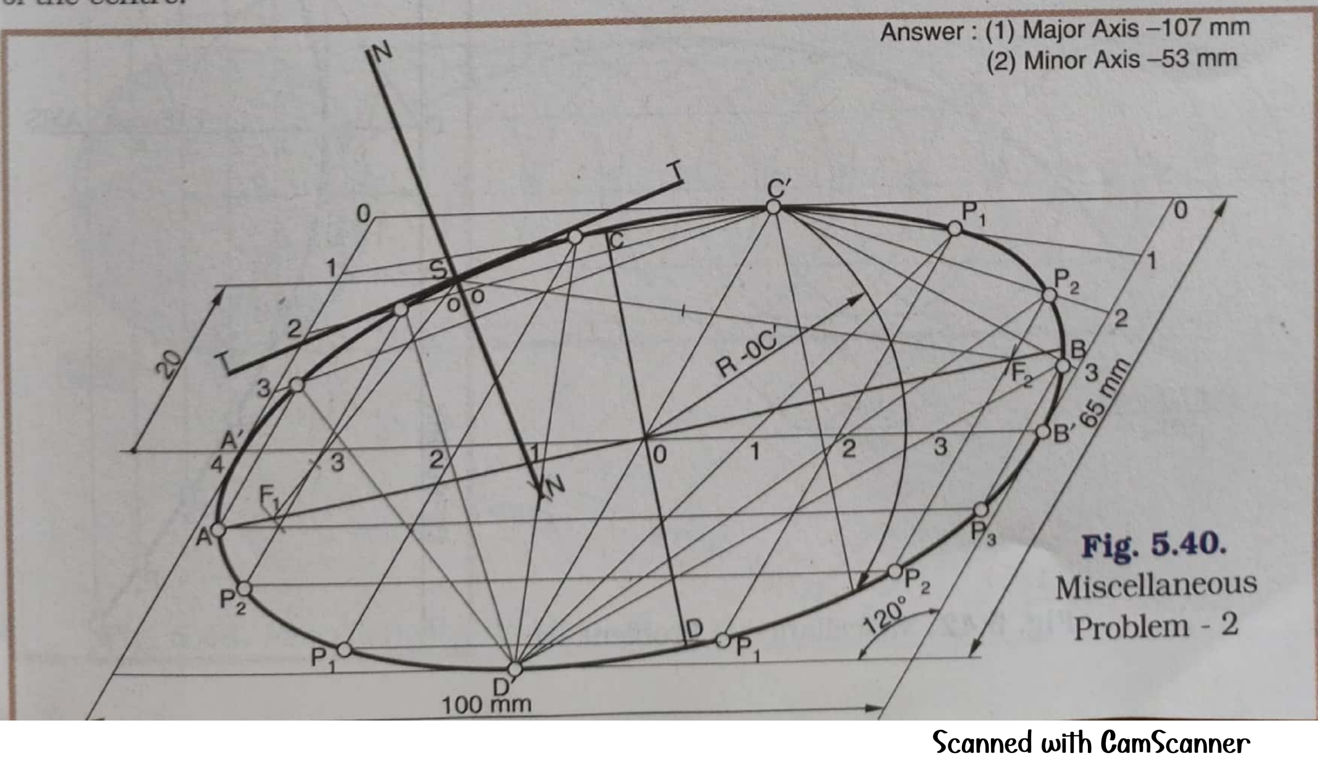


Fig. 5.40. Miscellaneous Problem - 2

Problem 3 : A boy, standing on the terrace of a building of 15 m height, throws a ball, whic has its highest flight and just crosses a tree of 25 m height. Trace the path of the ball, the distance between the building and the tree is 8 m.

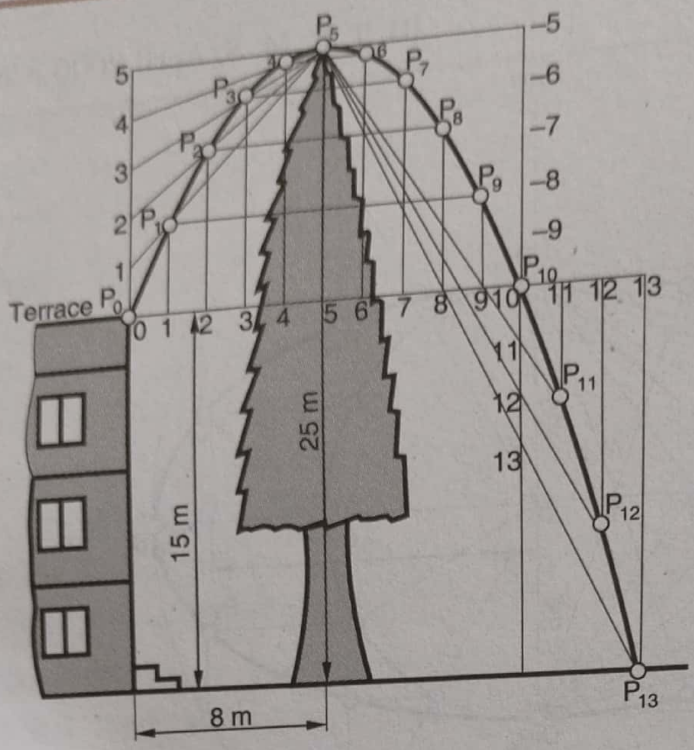


Fig. 5.41. Miscellaneous Problem - 3

Problem 4 : The vertex of hyperbola is 75 mm from its focus. Draw the curve if eccentricity is 3/2.[8]

[B. T. E. M. S. April/May 1997]

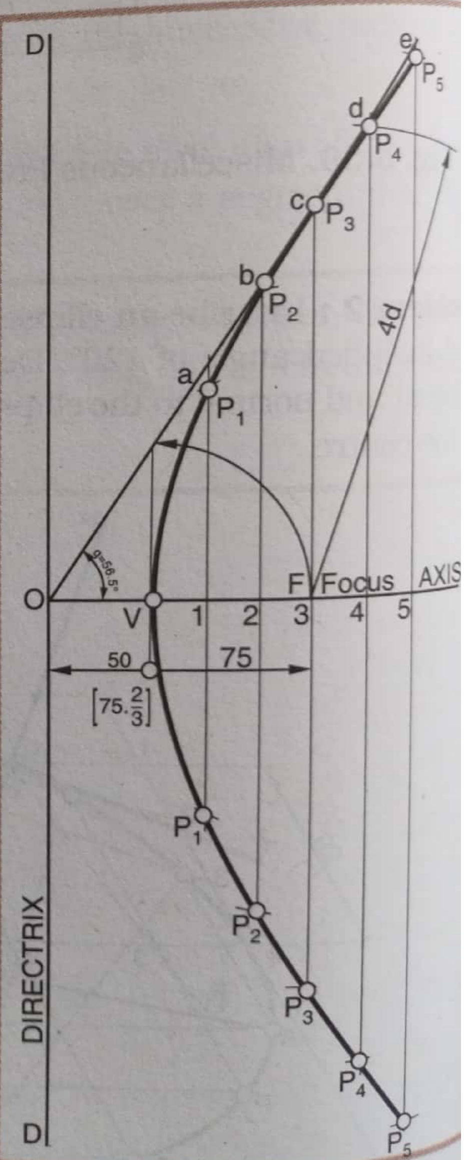


Fig. 5.42. Miscellaneous Problem - 4

Problem 5 : Draw the locus of a point P, which moves in such a way that the ratio of its distance from a fixed straight line to its distance from a fixed point is always constant and equal to 5/3. A fixed point is 60 mm away from the fixed straight line. Draw the tangent and normal to the curve at a point 70 mm from the fixed straight line. Name the curve.

[Ans : Ellipse] [Mumbai University, June 1996]

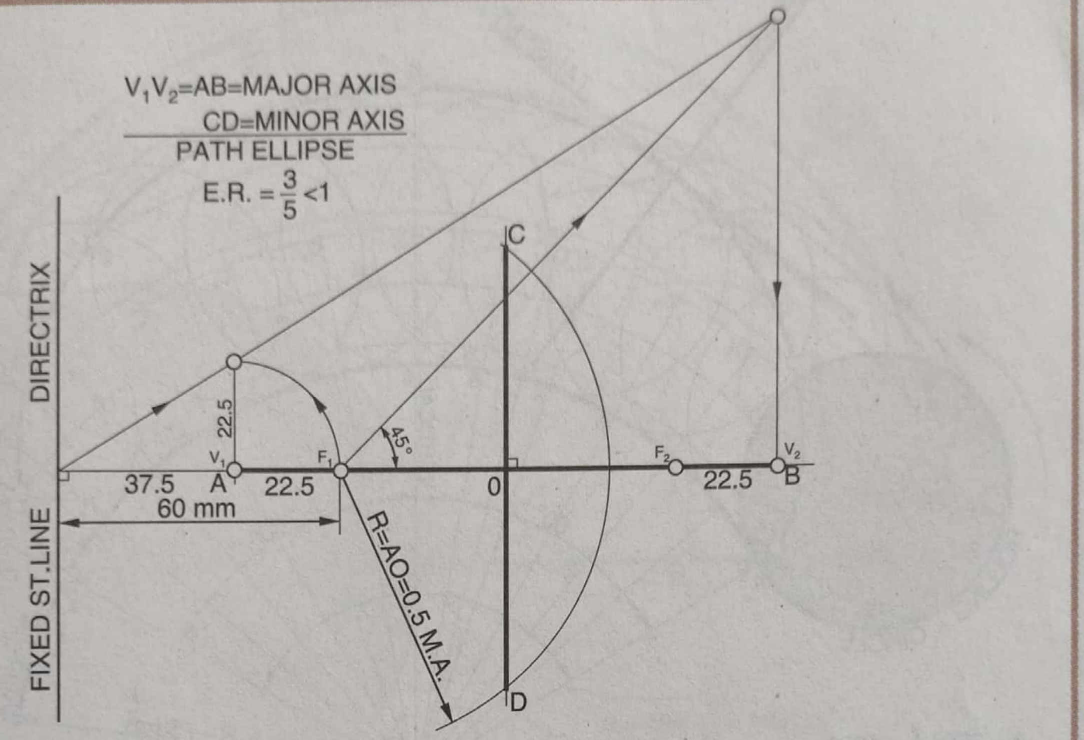


Fig. 5.43. Miscellaneous Problem - 5

Problem 6 : A circle of 50 mm diameter rolls along a straight line without slipping. Trace the path of a point on the circumference of the rolling circle for one complete revolution. Name the curve. [8]

[Ans : Cycloid] [B. T. E. M. S. April/May 1998]

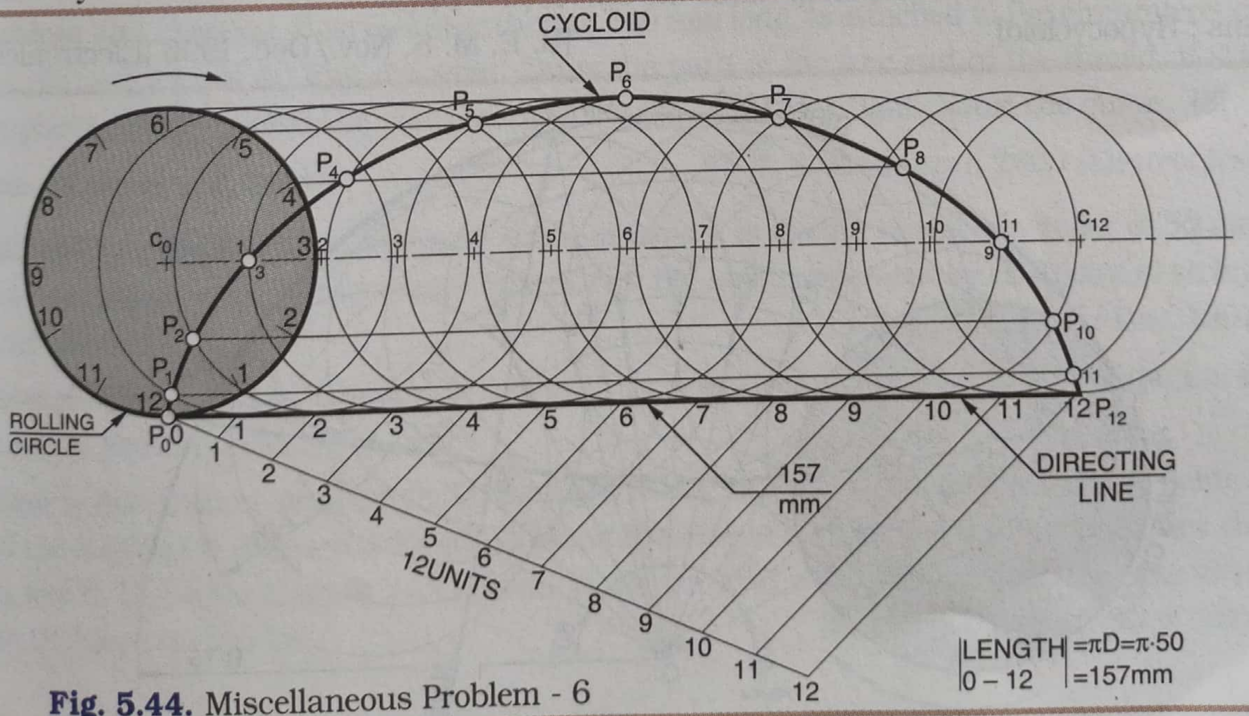


Fig. 5.44. Miscellaneous Problem - 6

Problem 7 : A circle of 50 mm diameter rolls along the circumference of another circle o 150 mm diameter from outside. Trace the path of a point P on the circumference of th rolling circle for one complete revolution. Name the curve. [7 + 1]

[B. T. E. M. S. November 1999 (Electronics)]

[Ans : Epycycloid]

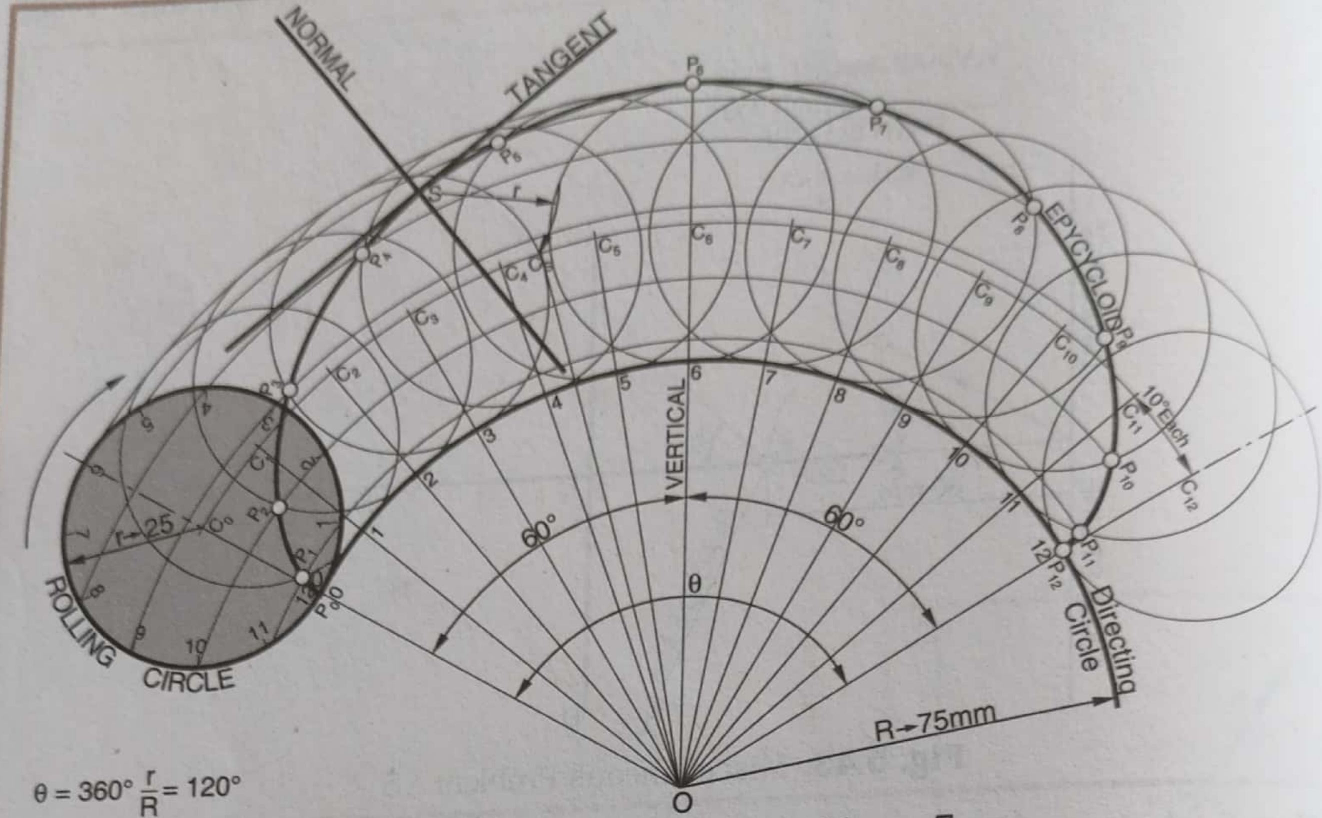


Fig. 5.45. Miscellaneous Problem - 7

Problem 8 : A circle, of 50 mm diameter, rolls along the circumference of another circle 150 mm diameter from inside. Draw the path of a point P on the circumference of the rolli circle for one complete revolution. Name the curve. [7 +1]

[B. E. M. S. Nov./Dec. 1996 (Electronics

[Ans : Hypocycloid]

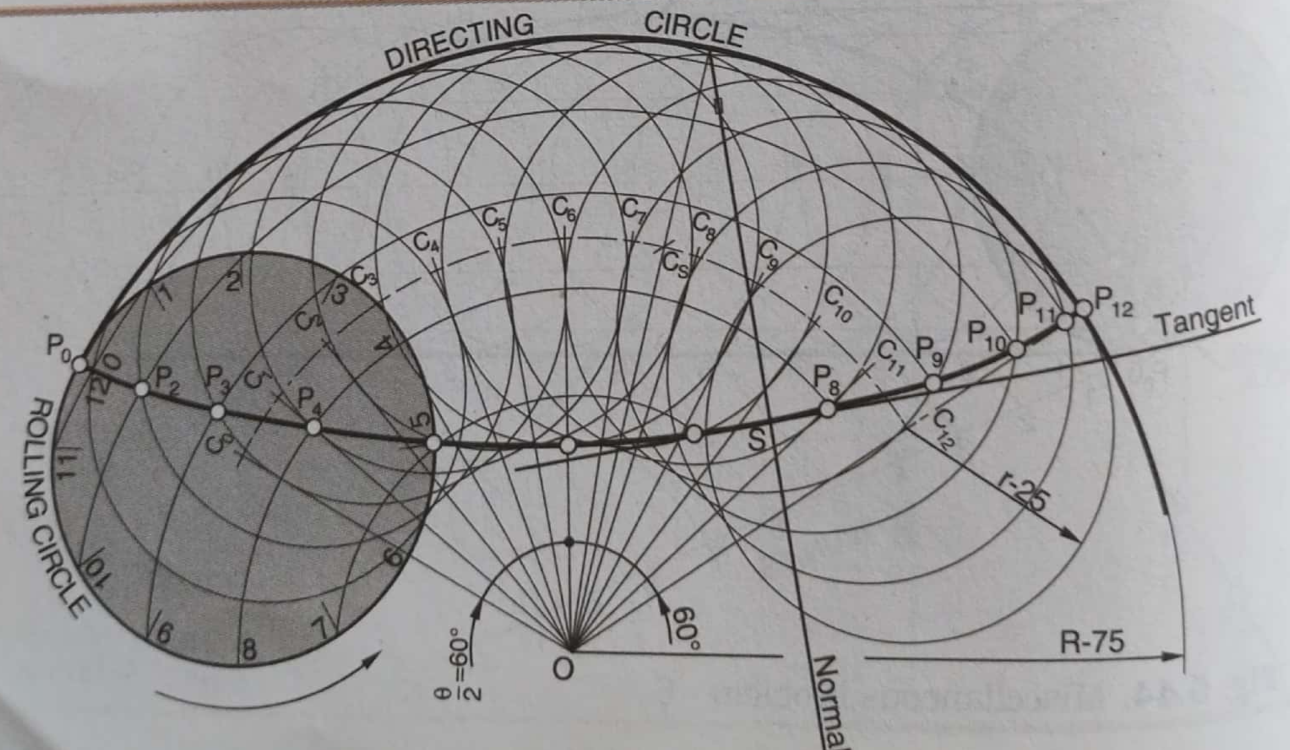


Fig. 5.46. Miscellaneous Problem - 8

Problem 9 : Show graphically that the hypocycloid is a straight line, when the diameter of the rolling circle is half that of the directing circle. Take radius of the rolling circle as 40 mm. [8]

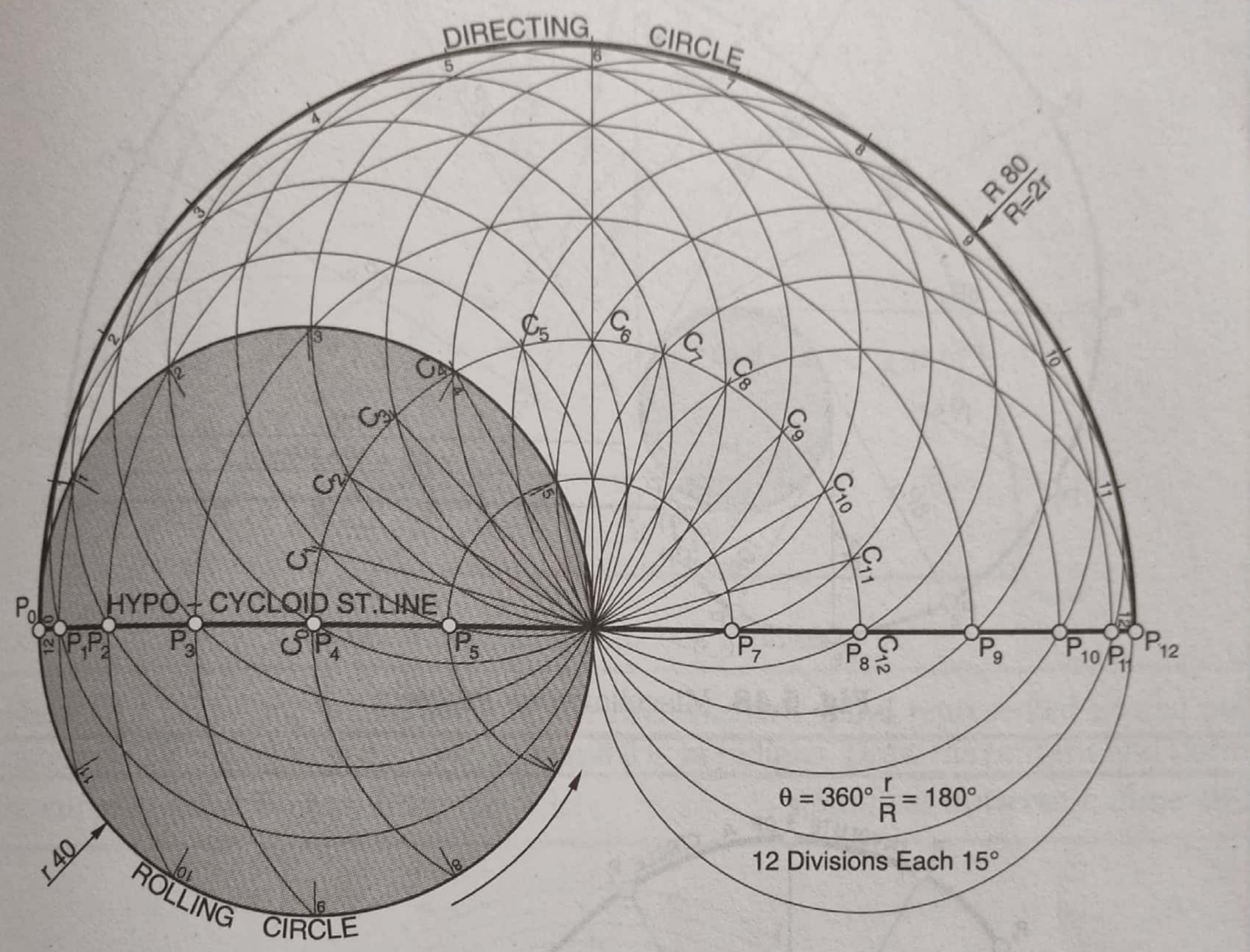


Fig. 5.47. Miscellaneous Problem - 9

Problem 10 : One end of an inelastic thread, 140 mm long, is attached to the circumference of a circular disc of 40 mm diameter. Draw the path of the free end of the thread, if it is completely wound round the disc, keeping the thread always tight. Name the curve. [8]

[Ans : Involute of A Circle] [B. T. E. M. S. April 2000 (Electronics)]

Problem 11 : An in-elastic string, of 100 mm length, is wound round a cylinder of 50 mm diameter, keeping the string always tight. Draw the curve generated by end point of string. Name the curve. [B. T. E. M. S. Nov. / Dec. 1997]

Problem 12 : A disc in the form of a semicircle and a semi-regular hexagon of thickness 10 mm is shown in the figure below.

A disc is firmly fixed at point O. An inelastic string of length 160 mm is fixed at point A and the free end B of a string is turned round the disc in anticlockwise direction. Draw the locus of B. Draw the tangent and normal to the curve at a distance of 110 mm away from pole O. Name the curve. [Mumbai University, May 1992]

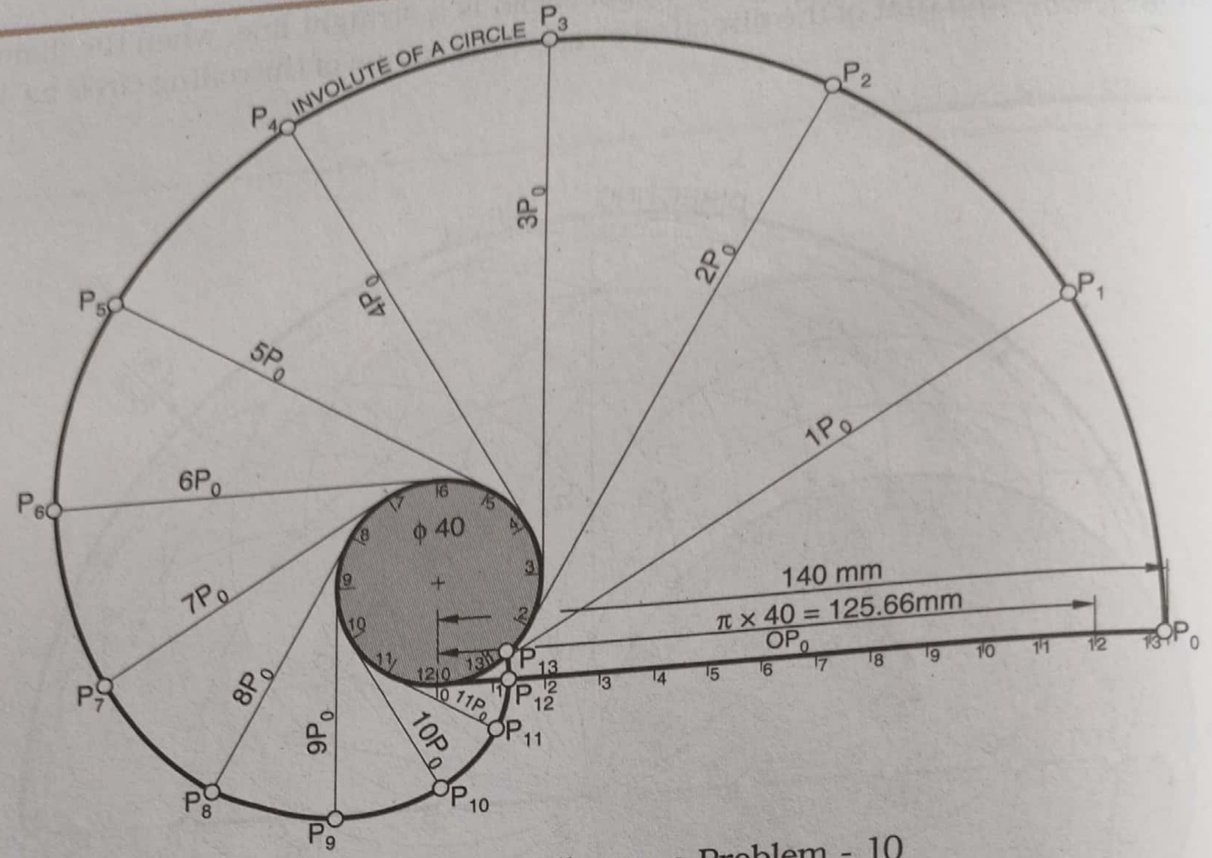


Fig. 5.48. Miscellaneous Problem - 10

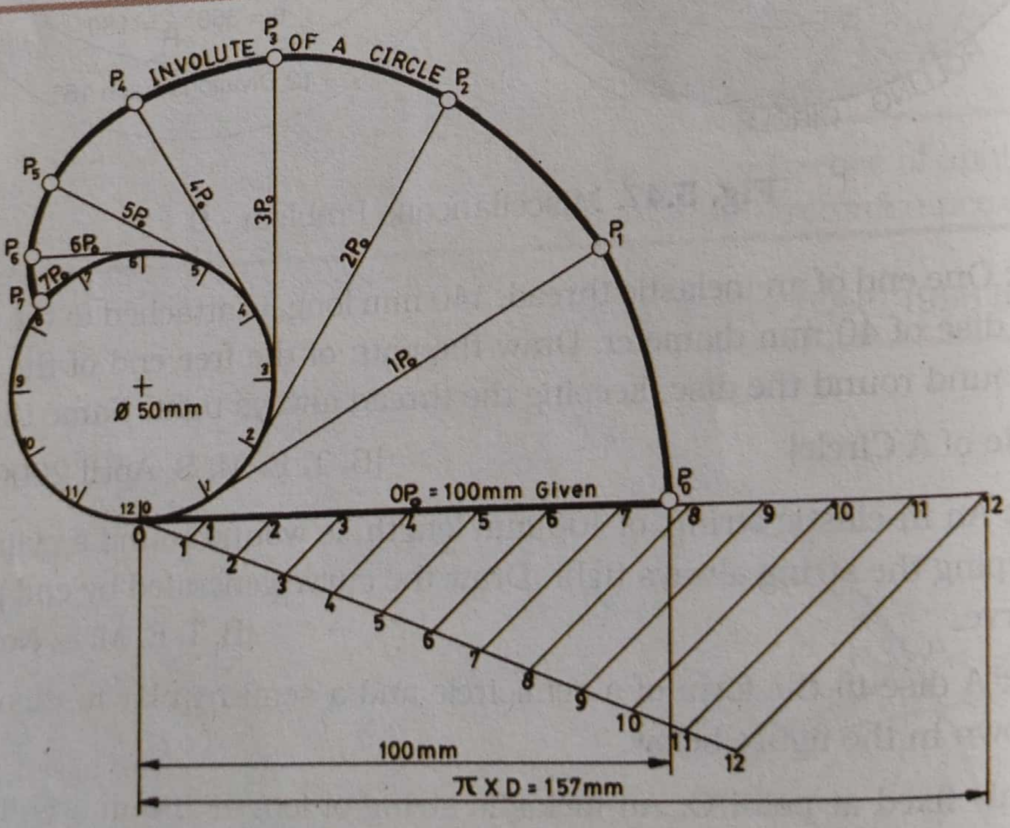


Fig. 5.49. Miscellaneous Problem - 11

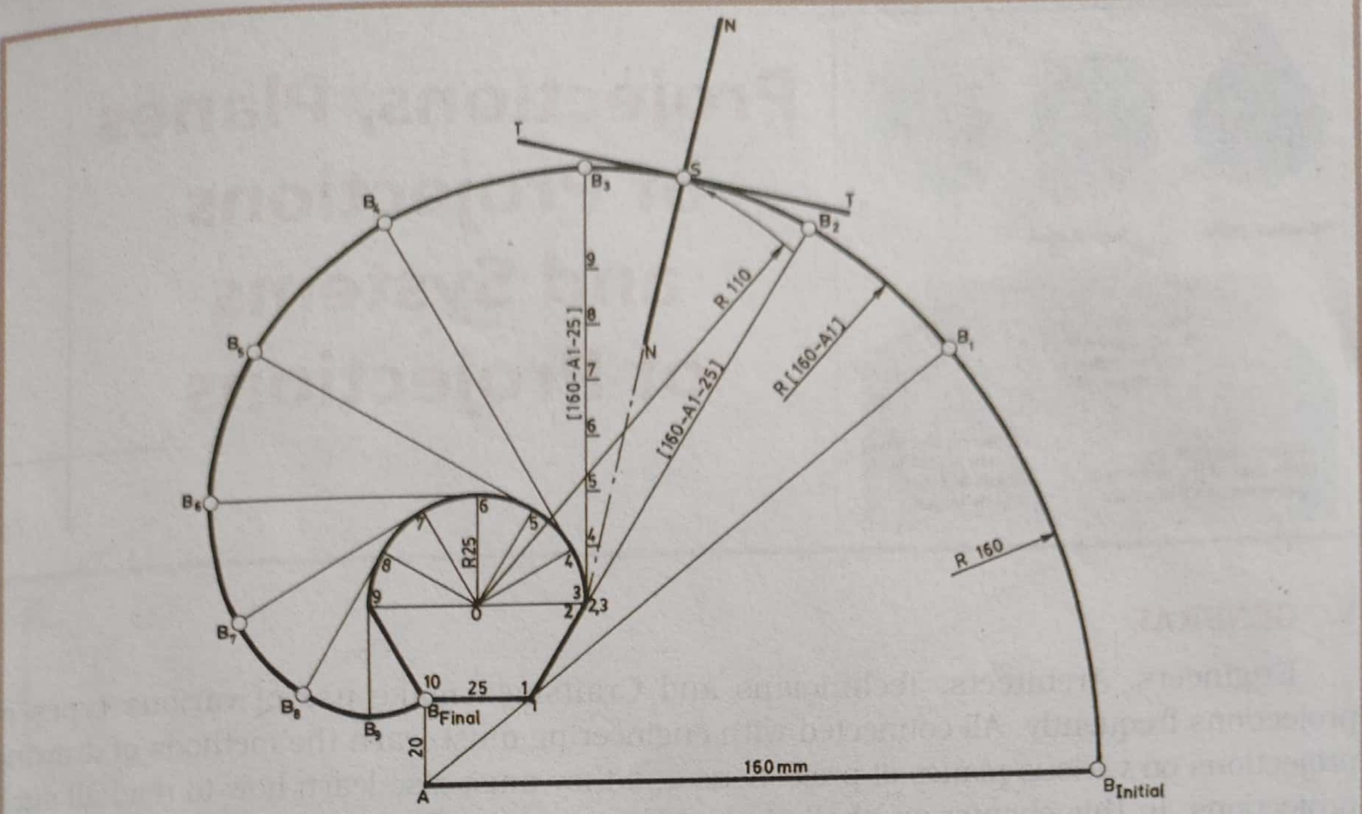


Fig. 5.50. Miscellaneous Problem - 12

Problem 13 : Draw one convolution of the Archemedian spiral represented by the polar equation r = [32 + 7.θ] where 'r' is in mm and θ is in radians. Draw the tangent and normal to the curve at point 55 mm from the pole. [Mumbai University, June 1996]

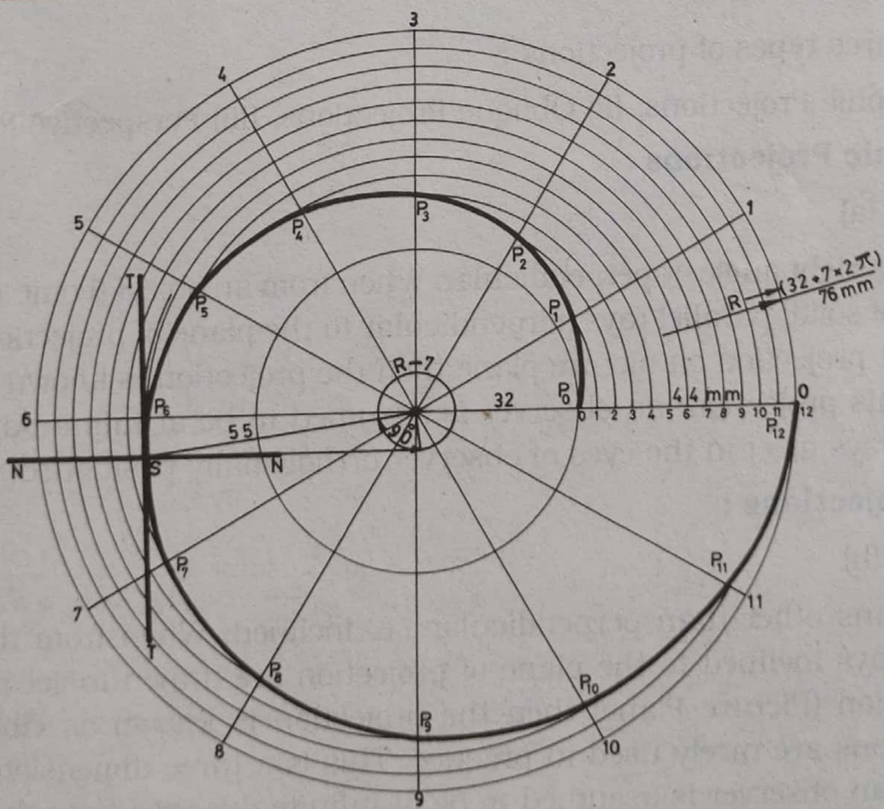
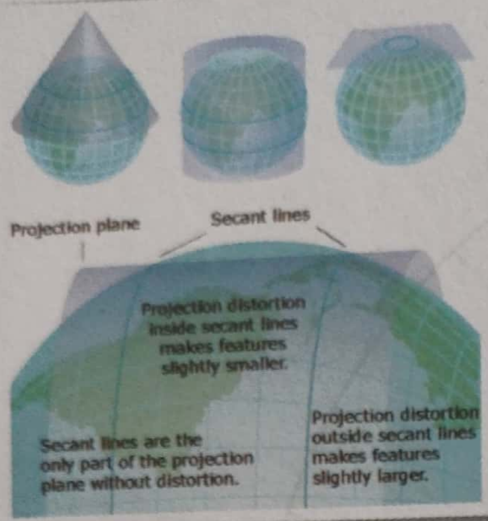


Fig. 5.51. Miscellaneous Problem - 13

CHAPTER - 6



Projections, Planes of Projections and Systems of Projections

1. GENERAL

Engineers, Architects, Technicians and Craftsmen make use of various types of projections frequently. All connected with engineering must learn the methods of drawing projections on various planes of projections and they must also learn how to read all such projections. In this chapter we shall study different types of projections, different types of planes of projections and systems of projections.

Initilly students should not bother about systems of projections. Once the clear understanding is developed about projections and planes of projections, study of systems of projections will be carried out.

2. PROJECTION

There are three types of projections :-

(i) Orthographic Projections, (ii) Oblique Projections, (iii) Perspective Projections.

(i) Orthographic Projections :

See Fig. 6.1 (a)

Ortho means right angle or perpendicular. When from an object (Point, ends of a straight line or corners of solid) parallel rays perpendicular to the plane of projection (Picture Plane) are drawn to get projection on picture plane then the projection is known as Orthographic Projection. In this projection, an observer is assumed to be at infinite distance from the object. Parallel rays meet in the eyes of observer orthogonally positioned at infinity.

(ii) Oblique Projections :

See Fig. 6.1 (b)

Oblique means other than perpendicular i.e. inclined. When from the corners of an object parallel rays inclined to the plane of projection are drawn to get projection on the plane of projection (Picture Plane) then the projection is known as Oblique Projection. Oblique projections are rarely used in practice. This is a three dimensional projection. In this projection, an observer is assumed to be at infinite distance from the object. Parallel rays meet in the eyes of observer, obliquely positioned at infinity.

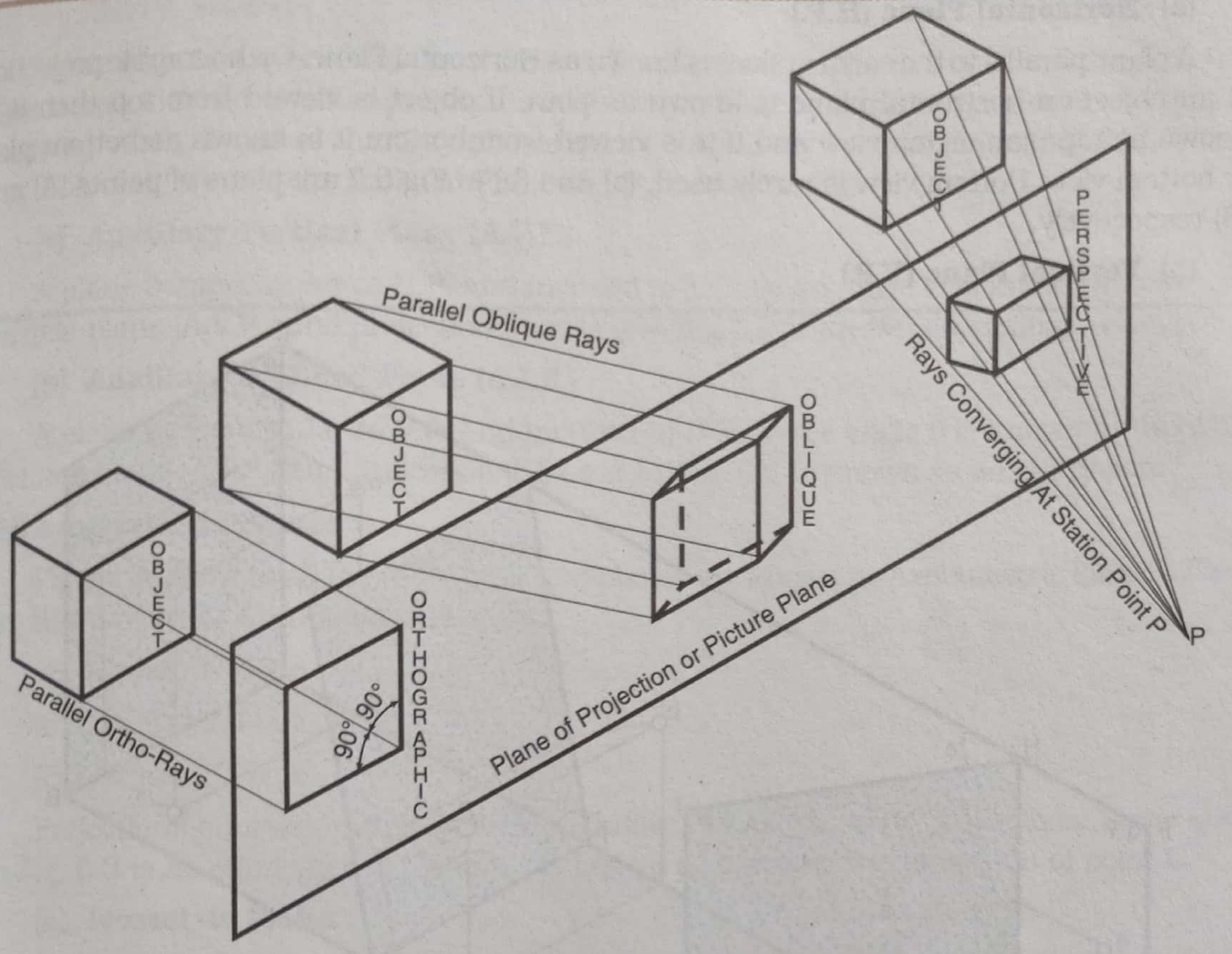


Fig. 6.1. (a) (b) & (c) Principles of Orthographic, Oblique and Perspective Projections

(iii) Perspective Projections :

See Fig. 6.1 (c)

In perspective projection rays from corners of an object converge to a finite point, where the eye of the observer is assumed to be located. If the plane of projection is between the eye and the object the projection will be smaller. If the plane of projection is on another side of the object than the projection will be larger than the object. Perspective projection gives real three dimensional picture of the object, which our eye is observing.

3. PLANES OF PROJECTIONS AND CORRESPONDING ORTHOGRAPHIC PROJECTIONS

Planes of projections or picture planes used in Engineering Drawing are :-

(i) Principle Planes. (H.P., V.P. & P.P. or A.V.P.)

(ii) Auxiliary Planes. (A.I.P. and A.V.P.) (Discussed in chapter no. 10).

(iii) Axoniometric Planes (Discussed in chapter on Isometric)

(i) Principle Planes : See Fig. 6.2

(a) Horizontal Plane. (H.P.) (ABCD in Fig 6.2)

(b) Vertical Plane. (V.P.) (CDFE in Fig. 6.2)

(c) Profile Plane (P.P.) (BCEG in Fig. 6.2)

(a) Horizontal Plane (H.P.)

A plane parallel to the earth or floor is known as Horizontal Plane. Orthographic projectio of an object on horizontal plane is known as plan. If object is viewed from top then it i known as top plan or top view and if it is viewed from bottom it is known as bottom pla or bottom view. Bottom view is rarely used. (a) and (b) in Fig.6.2 are plans of points (A) an (B) respectively.

(b) Vertical Plane (V.P.)

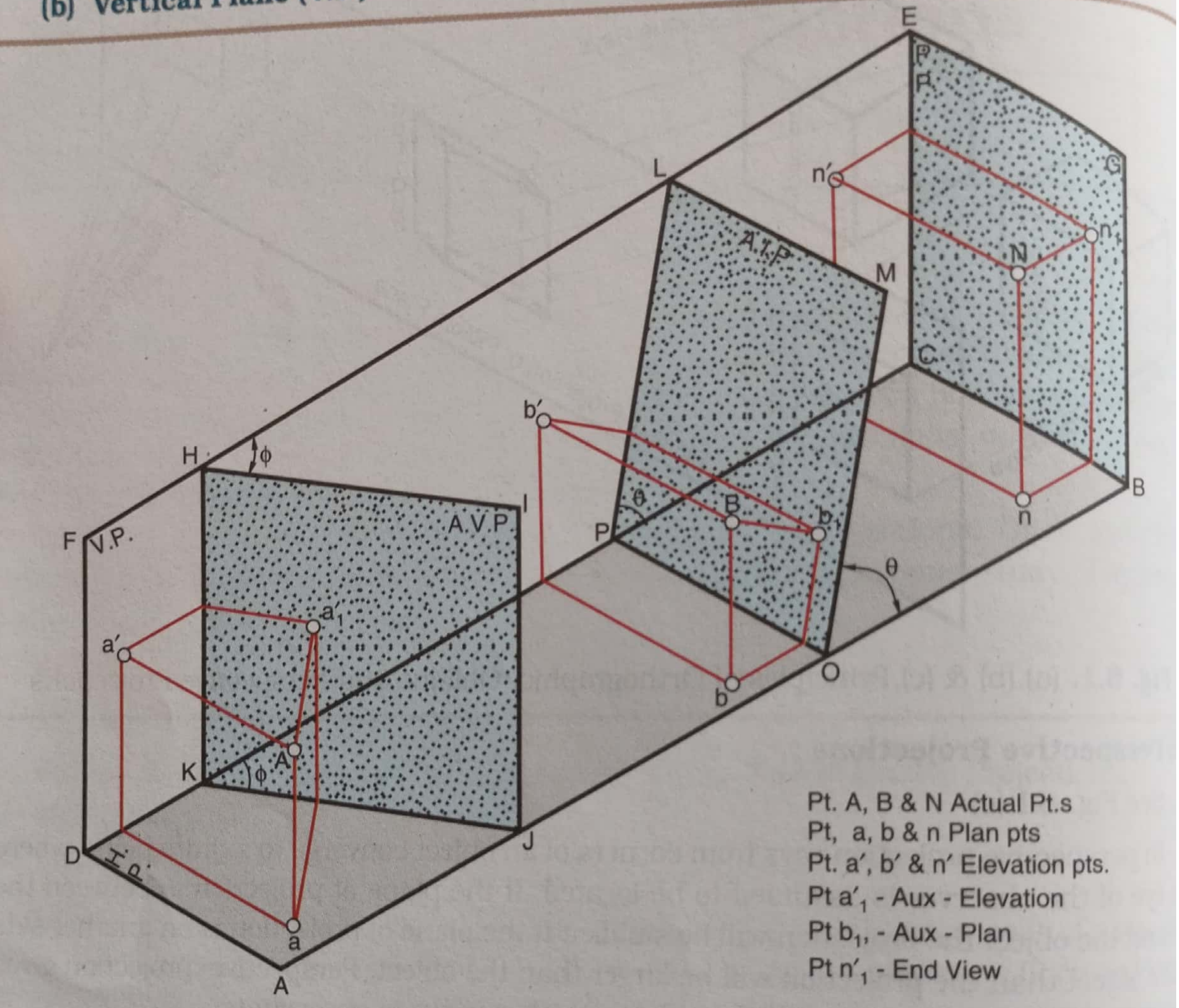


Fig. 6.2. Planes of Projections and Corresponding Orthographic Projections.

Any one plane conveniently selected out of the planes perpendicular to H.P. is known vertical plane. Orthographic projection on V.P. is known as elevation. If an object is view from the front then it is known as front elevation or front view and if it is viewed from re then it is known as rear elevation or rear view. Rear view is rarely used. (a') and (b') Fig. 6.2 are elevations of points (A) and (B) respectively.

(c) Profile Plane or Auxiliary Vertical Plane (P.P. or A.V.P.)

A plane perpendicular to H.P. and V.P. both is known as profile plane. Orthograph projection on P.P. is known as end view, side view or end elevation. If the object is view from left then the view is known as left hand end view and similarly if it is viewed from rig then it is known as right hand end view.

(ii) Auxiliary Planes :

There are two types of auxiliary planes. They are perpendicular to one of the principal planes and inclined to other principal plane. Projections on auxiliary planes are known as auxiliary views. Types of auxiliary planes are : (a) Auxiliary Vertical Plane (A.V.P.) (HIJK in Fig. 6.2) and (b) Auxiliary Inclined Plane (A.I.P.) (LMOP in Fig. 6.2).

(a) Auxiliary Vertical Plane (A.V.P.)

A plane perpendicular to H.P. and inclined to V.P. by an angle Ø is known as auxiliary vertical plane (A.V.P.) and projection on it (a_1') in Fig 6.2 is known as auxiliary elevation.

(b) Auxiliary Inclined Plane (A.I.P.)

A plane perpendicular to V.P. and inclined to H.P. by an angle θ is known as auxiliary inclined plane (A.I.P.) and projection (b_1) on it in Fig. 6.2 is known as auxiliary plan.

(iii) Axonometric Planes :

Planes inclined to all the three principal planes are known as Axonometric Planes. There are three types of Axonometric Planes.

(a) Isometric Plane.

(b) Diametric Plane.

(c) Trimetric Plane.

Projections on axonometric planes are known as axonometric projections. Plane QRG in Fig. 6.3 is an axonometric plane. C_2 is known as axonometric projection of point C.

(a) Isometric Plane :

It is an axonometric plane which is equally inclined to all the three principal planes. Projection on it is known as Isometric Projection. It is an important three dimensional projection. We shall study this projection in detail in chapter on Isometric.

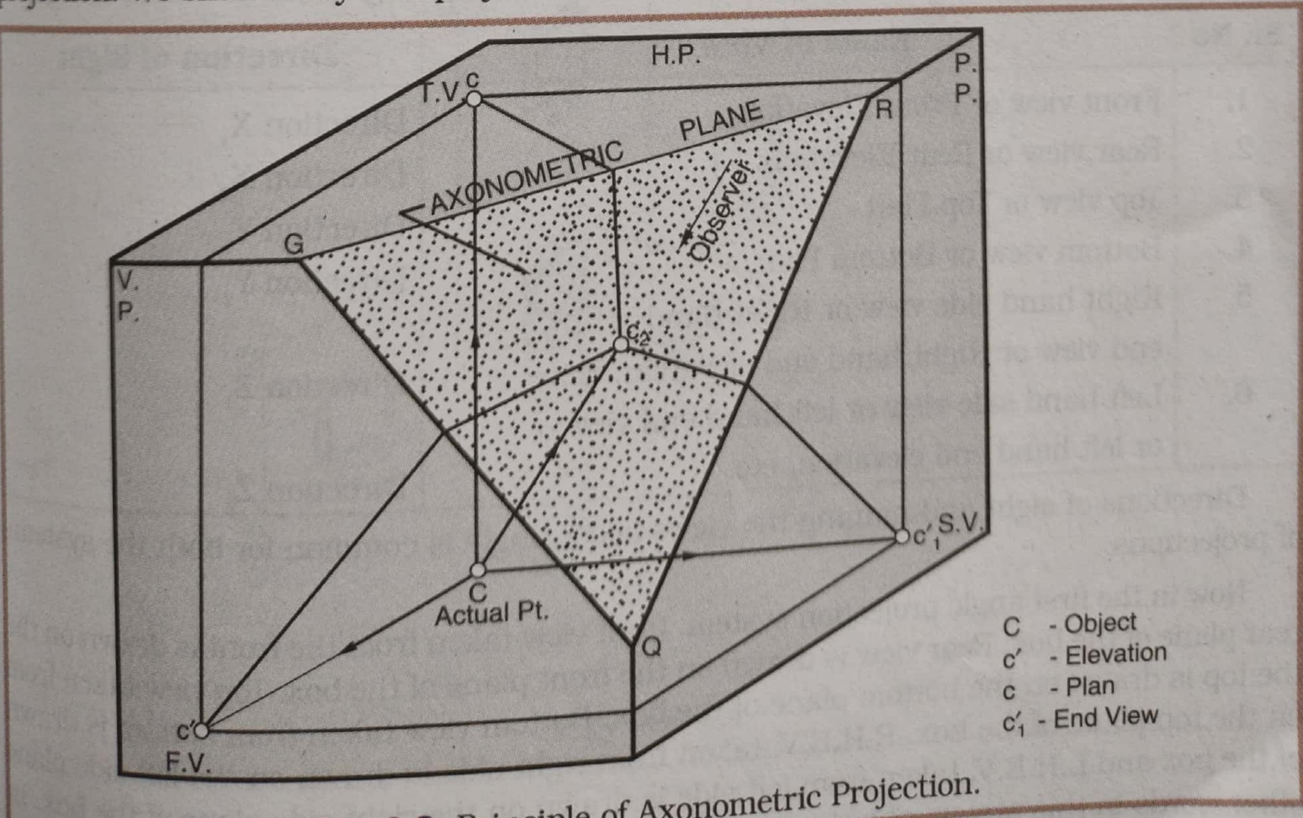


Fig. 6.3. Principle of Axonometric Projection.

(b) Diametric Plane :

It is an axonometric plane which is equally inclined to two of the three principal planes. Projection on it is known as diametric projection. It is little different than isometric projection.

(c) Trimetric Plane :

It is an axonometric plane which has different inclinations with all the three principal planes. Projection on it is known as trimetric projection. It is little more different than isometric projection.

4. SYSTEMS OF ORTHOGRAPHIC PROJECTIONS

There are two systems of orthographic projections.

(i) First angle projection system.

(ii) Third angle projection system.

At present first angle projection system is recommended for practice in engineering field as per I.S. Before few years third angle projection system was used in India. So we shall study both the systems of projections.

Although principle involved is the same, treatment appears to be different for problems of (a) Orthographic Projections of objects and (b) Problems of Solid Geometry for the same system of projections. So we shall study the same system for both the type of problems.

(a) First Angle Projection System for Orthographic Projections of Objects :

See Fig. 6.4 (i) (ii) (iii).

In Fig. 6.4 (i) object, whose orthographic projections are required to be drawn, is kept inside a glass box and is viewed from all six directions perpendicular to the 6 planes of the box and six views are achieved.

These six views of the object are named as under according to the direction of sight.

Sr. No	Name of View	Direction of Sight
1.	Front view or Front Elevation.	Direction X_1
2.	Rear view or Rear Elevation.	Direction X_2
3.	Top view or Top Plan.	Direction Y_1
4.	Bottom view or Bottom Plan.	Direction Y_2
5.	Right hand side view or Right hand end view or Right hand end elevation, etc.	Direction Z_1
6.	Left hand side view or left hand end view or left hand end elevation, etc.	Direction Z_2

Directions of sight and naming the views accordingly is common for both the systems of projections.

Now in the first angle projection system, front view taken from the front is drawn on the rear plane of the box. Rear view is drawn on the front plane of the box. Top view taken from the top is drawn on the bottom plane of the box. Bottom view taken from bottom is drawn on the top plane of the box. R.H.E.V. taken from right side is drawn on the left side plane of the box and L.H.E.V. taken from left side is drawn on the right side plane of the box. In other words in this system object remains in between picture plane and observer.

After drawing six views on six planes of the box, all planes of the box are opened about the plane which contains the front view. Front view is considered as the main and important view. F.V. in this system is on rear plane of the box and so all planes are opened around edges of rear plane. See Fig. 6.4 (ii).

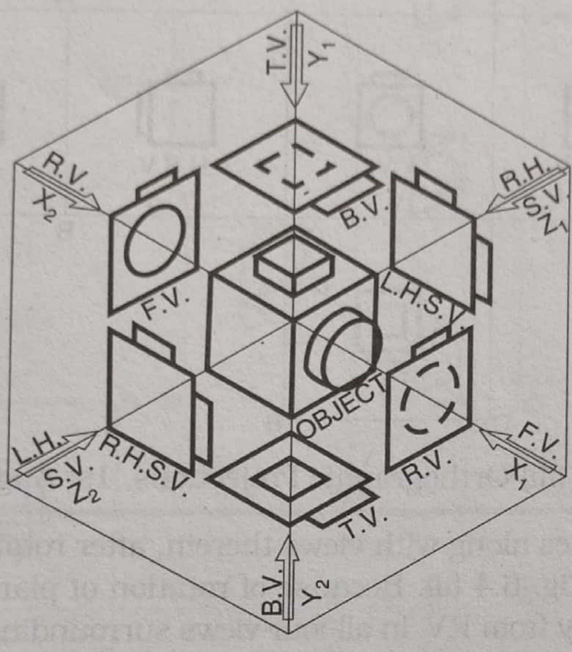


Fig. 6.4. (i)

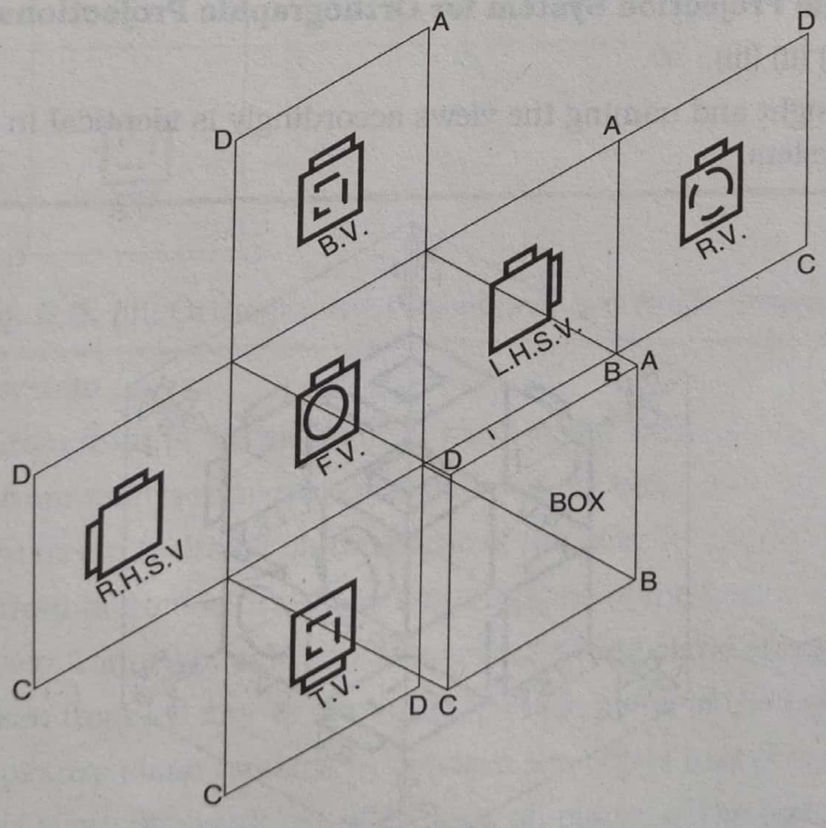


Fig. 6.4. (ii)

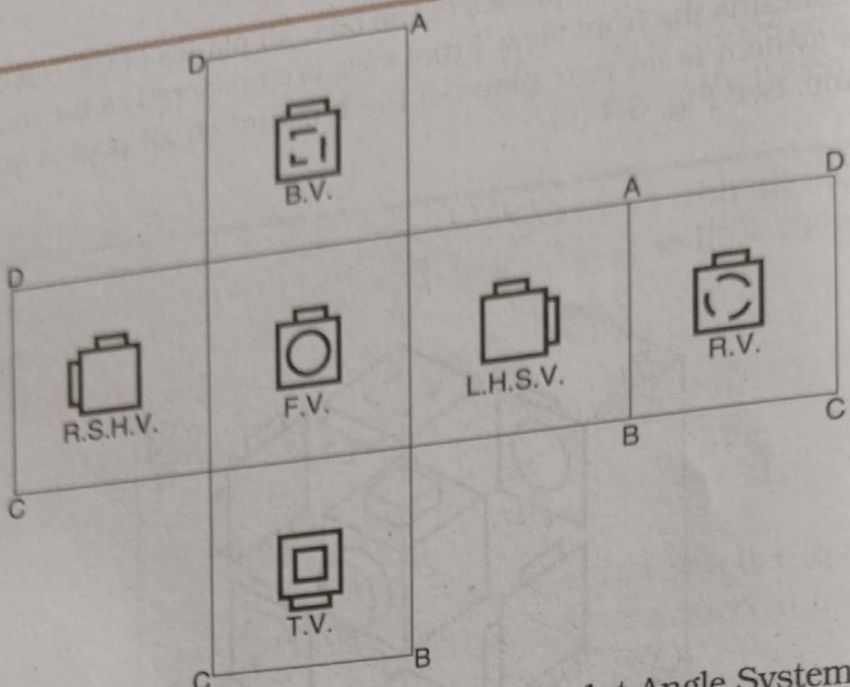


Fig. 6.4. (iii) Orthographic Projections. 1st Angle System

Position of all six planes along with views therein, after rotation are shown in line with plane containing F.V. in Fig. 6.4 (ii). Because of rotation of planes front part or portion of the object will be seen away from F.V. in all four views surrounding F.V. See Fig. 6.4 (iii) also.

Students are requested to co-relate Fig. 6.4 (i), Fig. 6.4 (ii) and Fig. 6.4 (iii) and try to visualize the rotation of planes and its effect on views.

(ii) (a) Third Angle Projection System for Orthographic Projections of Objects

See Fig. 6.5 (i) (ii) (iii).

Directions of sight and naming the views accordingly is identical in first angle system and third angle system.

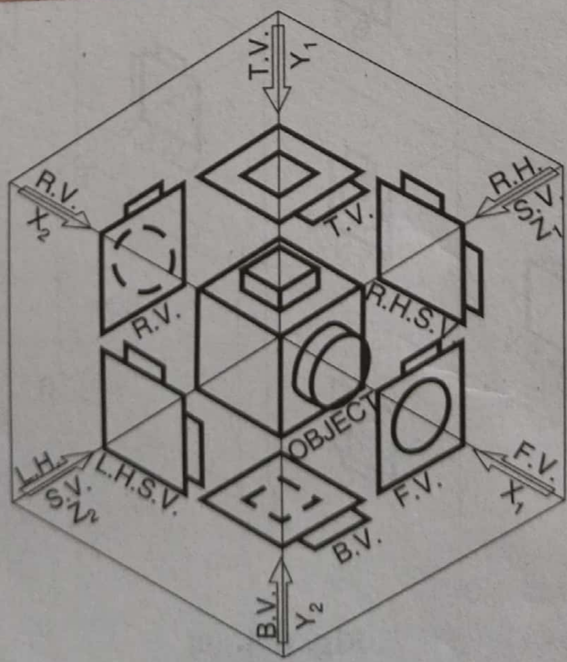


Fig. 6.5. (i)

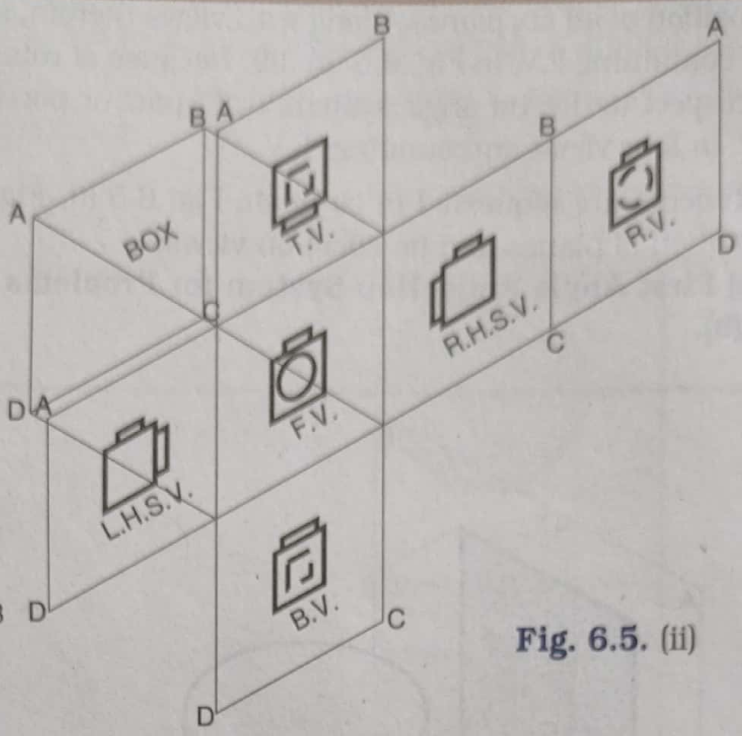


Fig. 6.5. (ii)

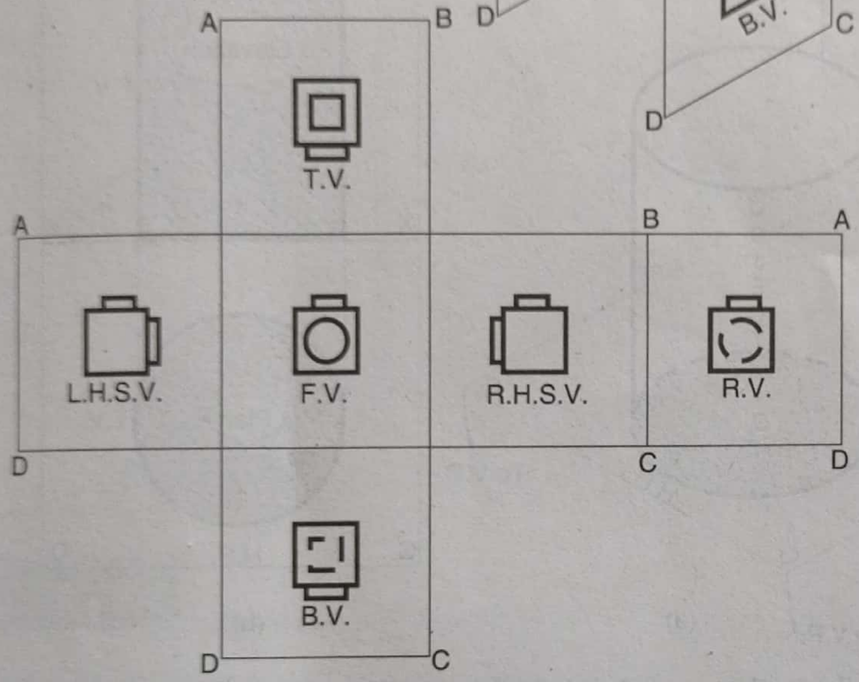


Fig. 6.5. (iii) Orthographic Projections. 3rd Angle System.

In third angle system :

1. F.V. taken from front is drawn on front plane of the box.
2. R.V. taken from rear is drawn on rear plane of the box.
3. T.V. taken from top is drawn on top plane of the box.
4. B.V. taken from bottom is drawn on bottom plane of the box.
5. R.H.S.V. taken from right side is drawn on right side plane of the box and
6. L.H.S.V. taken from left side is drawn on left side plane of the box.

In other words picture plane remains in between the object and observer.

After drawing six views on six planes of the box, all planes of the box are opened about the plane which contains front view. F.V. in this system is on front plane of the box and so all planes are opened around edges of front plane. See Fig. 6.5 (ii).

Position of all six planes, along with views therein, after rotation are shown in line wi plane containing F.V. in Fig. 6.5 (ii) (iii). Because of rotation of planes in different directio with respect to the 1st angle system, front part or portion of the object will be seen near to F.V. in four views surrounding F.V.

Students are requested to co-relate Fig. 6.5 (i), Fig. 6.5 (ii) & (iii) and try to visualiz the rotation of planes and its effect on views.

(i) (b) First Angle Projection System for Problems of Solid Geometry : See Fig. 6 (a) & (b).

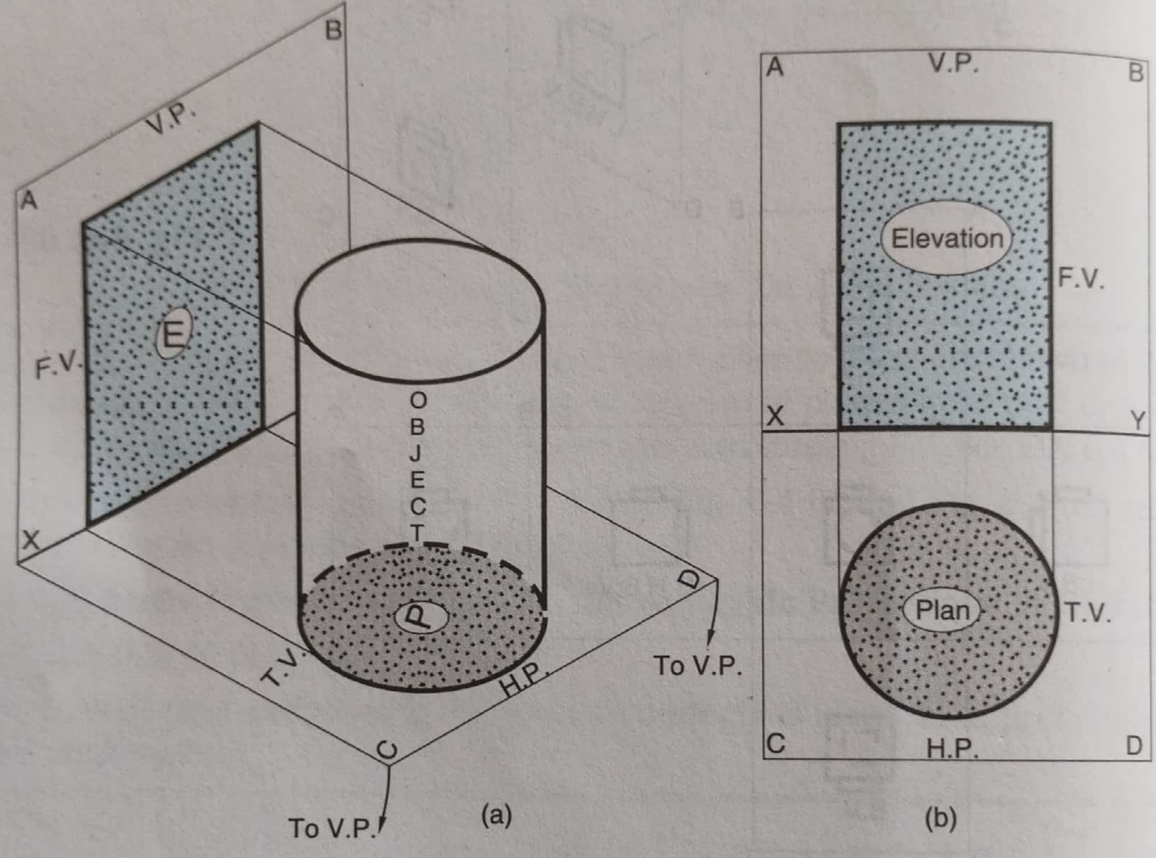


Fig. 6.6. (a) & (b) Problem of Solid Geometry. [1st Angle System]

In this system solid remains in first quadrant and generally resting on H.P. Elevati and plan of the solid are drawn on V.P.(Upper) and H.P. (Front) respectively. Afterwar H.P. (F) plane is given rotation of 90° as shown in Fig. 6.6 (a) to bring it in line with V.P.

In Fig. 6.6 (b) position of two planes V.P. and H.P. after rotation are shown along wi projections, elevation and plan drawn therein respectively, XY line is generated by intersecti of H.P. and V.P.

In short in this system plan is drawn below xy line and elevation is drawn above xy lin Ground line G.L. will be absent.

(ii) (b) Third Angle Projection System for Problems of Solid Geometry :

In this system solid remains in third quadrant and generally resting on ground. Elevatio and plan of the solid are drawn on V.P. (lower) and H.P. (rear) respectively. Afterwards H. (R) plane is given rotation of 90°, as shown in Fig. 6.7 (a) to bring it in line with V.P.(L).

In Fig. 6.7 (b) position of two planes V.P.(L) and H.P.(R) after rotation are shown alon with projections elevation and plan drawn therein respectively. XY line is generated by intersection of H.P. and V.P. and G.L. line is generated by intersection of ground and V.P.

In short in this system elevation is drawn between parallel G.L. line and xy line and plan is drawn above xy line as shown in Fig. 6.7 (b).

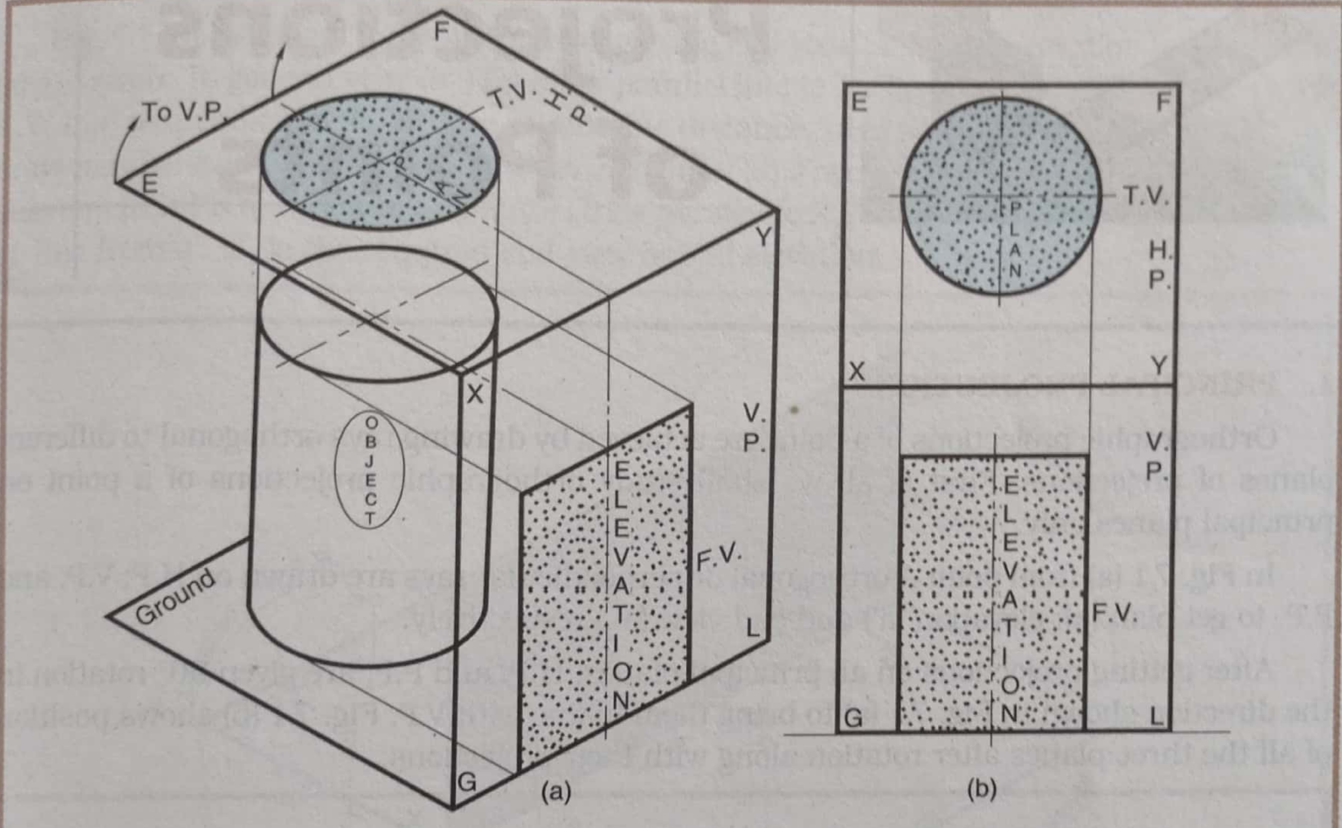
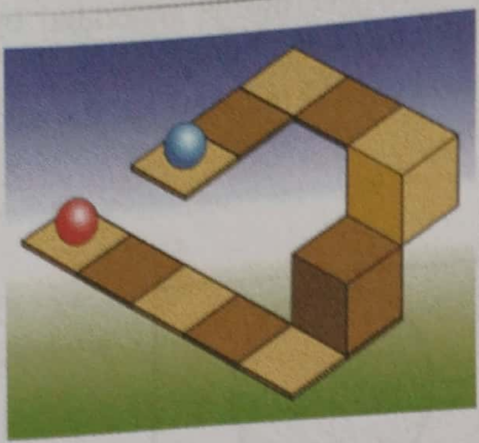


Fig. 6.7. (a) & (b) Problem of Solid Geometry [3rd Angle System]

EXERCISE

1. Name different types of projections you know. Explain the difference between them.
2. What is orthographic projection ? How it is named ?
3. Enlist clearly two systems of projections. Show clearly by sketch each one.
4. Explain clearly two systems of projections. How will you find out that particular projections given are in 1st angle system or in 3rd angle system ?
5. Explain with sketch how two systems of projections are applied in the solution of problems of solid geometry.
6. How the different projections are named ?
7. Explain box method used for orthographic projection.
8. Why orthographic projections are extensively used even though they are not three dimensional ?
9. Name three dimensional projections you know.



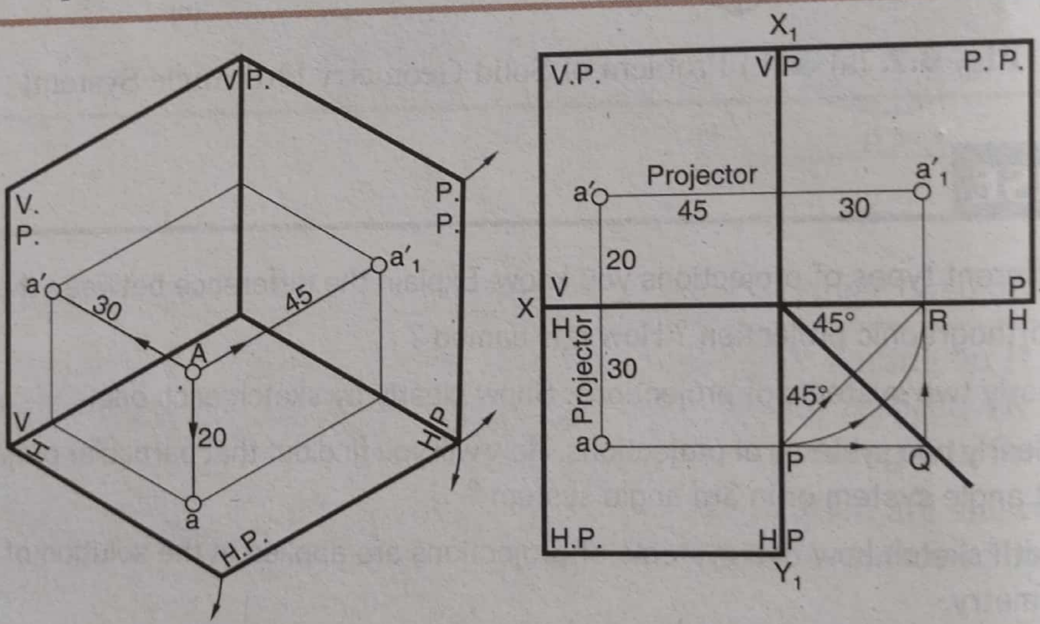
Orthographic Projections of Points

1. PRINCIPAL PROJECTIONS

Orthographic projections of a point are achieved by drawing rays orthogonal to differen[t] planes of projections. First of all we shall study orthographic projections of a point on principal planes only.

In Fig. 7.1 (a) from point A orthogonal or perpendicular rays are drawn on H.P; V.P. and P.P. to get plan (a); elevation (a') and end view (a'_1) respectively.

After getting projections on all principal planes, H.P. and P.P. are given 90° rotation in the direction shown in Fig. 7.1 (a) to bring them in line with V.P. Fig. 7.1 (b) shows position of all the three planes after rotation along with their projections.



(a) (b)

Fig. 7.1. (a) & (b) Principal Projections of A Point

It will be seen from Fig. 7.1 (a) and (b) that distance of point A from H.P.(20) is seen in elevation and end view from XY line or $\left[\frac{V}{H}\right] \left[\frac{P}{H}\right]$ line. Distance of the point A from V.P. (30 is seen in plan from XY line or $\left[\frac{V}{H}\right]\left[\frac{P}{H}\right]$ line. Distance of the point A from P.P. (45) is seen in elevation and plan from X_1Y_1 line or $\left[\frac{V}{P}\right]\left[\frac{H}{P}\right]$ line.

Fig. 7.1 (b) indicates three methods of getting end view (a'_1) when elevation (a') and plan (a) are given. To get end view (a'_1) (1) draw parallel line to XY line from elevation a'. (2) Draw X_1Y_1 line perpendicular to XY line at suitable distance, here it is 45 mm (3) From plan (a) draw parallel line to XY line upto P on X_1Y_1 line. (4) Transfer P to R by 45° line PR or by quarter circle PR or by square PQR. (5) Draw parallel to X_1Y_1 from R to get a'_1 by intersection on line from a'. a'_1 is the required end view or end elevation.

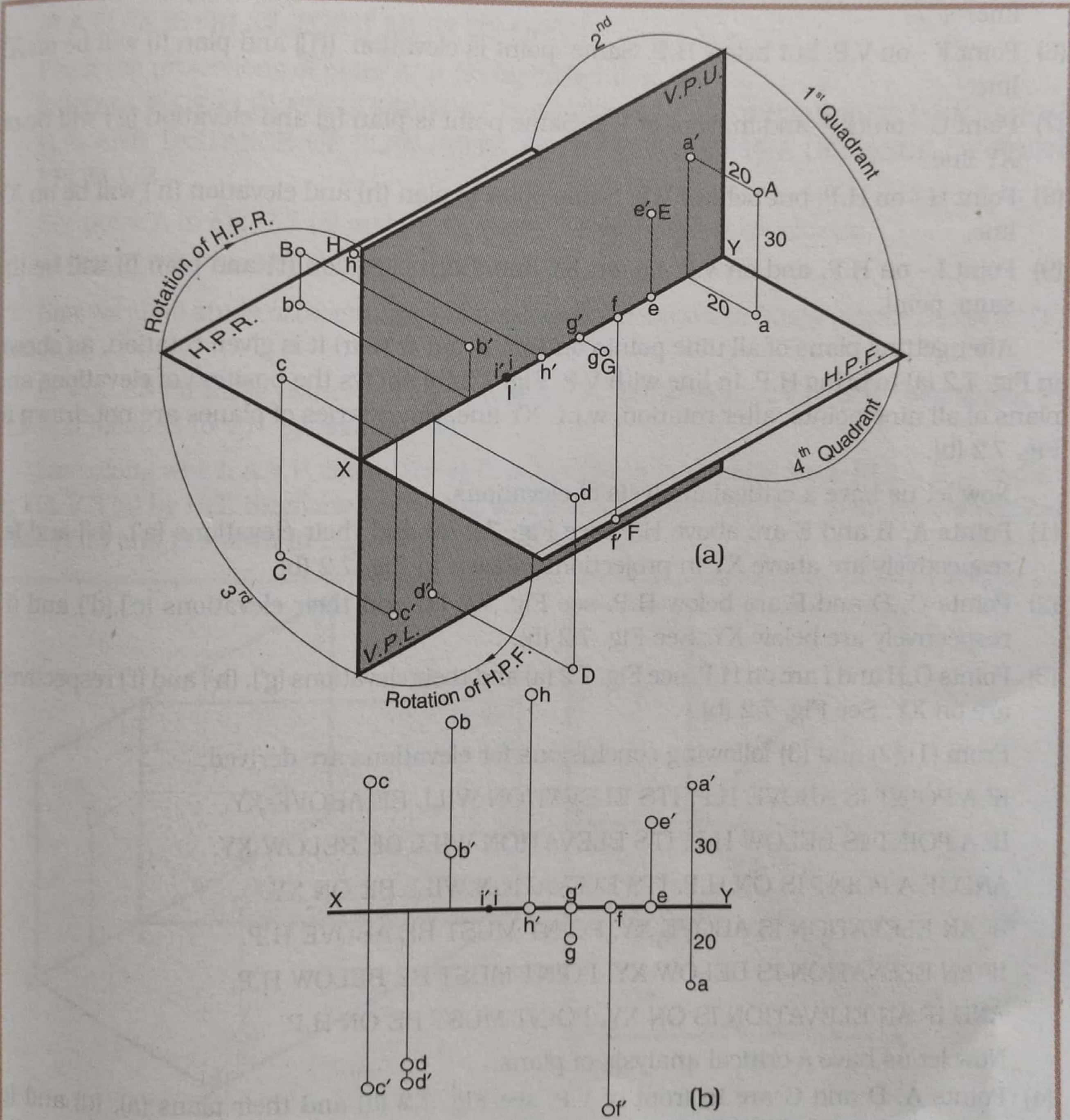


Fig. 7.2. (a) & (b) Nine Types of Points

If elevation and end view are given, plan can be drawn by following reverse method to above.

There are nine different types of points in space w.r.t H.P. and V.P. as shown in Fig. 7.2

(1) Point A - 30 mm above H.P. and 20 mm in front of V.P. It is in 1st quadrant. (a') is elevation and (a) is plan.

(2) Point B - above H.P. and behind V.P. It is in the 2nd quadrant. (b') is elevation and (b) is plan.

(3) Point C - below H.P. and behind V.P. It is in the 3rd quadrant. (c') is elevation and (c) is plan.

(4) Point D - below H.P. and in front of V.P. It is in the 4th quadrant. (d') is elevation and (d) is plan.

(5) Point E - On V.P. and above H.P. Same point is elevation (e') and plan (e) will be on XY line.

(6) Point F - on V.P. but below H.P. Same point is elevation (f') and plan (f) will be on XY line.

(7) Point G - on H.P. and in front of V.P. Same point is plan (g) and elevation (g') will be on XY line.

(8) Point H - on H.P. but behind V.P. Same point is plan (h) and elevation (h') will be on XY line.

(9) Point I - on H.P. and on V.P. i.e. on XY line. Both elevation (i') and plan (i) will be the same point.

After getting plans of all nine points on H.P. (front & rear) it is given rotation, as shown in Fig. 7.2 (a) to bring H.P. in line with V.P. Fig. 7.2 (b) shows the position of elevations and plans of all nine points, after rotation, w.r.t. XY line. Boundaries of planes are not drawn in Fig. 7.2 (b).

Now let us have a critical analysis of elevations.

(1) Points A, B and E are above H.P. see Fig. 7.2 (a) and their elevations (a'), (b') and (e') respectively are above XY in projections, shown in Fig. 7.2 (b).

(2) Points C, D and F are below H.P. see Fig. 7.2 (a) and their elevations (c'),(d') and (f') respectively are below XY. See Fig. 7.2 (b).

(3) Points G,H and I are on H.P. see Fig. 7.2 (a) and their elevations (g'), (h') and (i') respectively are on XY. See Fig. 7.2 (b).

From (1),(2) and (3) following conclusions for elevations are derived;

IF A POINT IS ABOVE H.P. ITS ELEVATION WILL BE ABOVE XY.

IF A POINT IS BELOW H.P. ITS ELEVATION WILL BE BELOW XY.

AND IF A POINT IS ON H.P. ITS ELEVATION WILL BE ON XY.

IF AN ELEVATION IS ABOVE XY, POINT MUST BE ABOVE H.P.

IF AN ELEVATION IS BELOW XY, POINT MUST BE BELOW H.P.

AND IF AN ELEVATION IS ON XY, POINT MUST BE ON H.P.

Now let us have a critical analysis of plans.

(4) Points A, D and G are in front of V.P. see Fig. 7.2 (a) and their plans (a), (d) and (g) respectively are below XY. See Fig. 7.2 (b).

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(5) Points B, C and H are behind V.P. see Fig. 7.2 (a) and their plans (b), (c) and (h) respectively are above XY. See Fig. 7.2 (b).

(6) Points E, F and I are on V.P. see Fig. 7.2 (a) and their plans (e), (f) and (i) respectively are on XY. See Fig. 7.2 (b).

From (4), (5) and (6) following conclusions for plans are derived;

IF A POINT IS IN FRONT OF V.P. ITS PLAN WILL BE BELOW XY.

IF A POINT IS BEHIND V.P. ITS PLAN WILL BE ABOVE XY, AND

IF A POINT IS ON V.P. ITS PLAN WILL BE ON XY.

IF A PLAN IS BELOW XY, POINT MUST BE IN FRONT OF V.P.

IF A PLAN IS ABOVE XY, POINT MUST BE BEHIND V.P. AND

IF A PLAN IS ON XY, POINT MUST BE ON V.P.

From the projections of point A, it is concluded that -

DISTANCE OF ELEVATION FROM XY IS EQUAL TO THE DISTANCE OF POINT FROM H.P. AND DISTANCE OF PLAN FROM XY IS EQUAL TO THE DISTANCE OF POINT FROM V.P.

See point A in Fig. 7.2 (a) and (b) for understanding above conclusion.

2. AUXILIARY PROJECTIONS ON A.V.P.

Now we shall study orthographic projections of a point on auxiliary planes i.e. on A.V.P. and A.I.P.

In Fig. 7.3 (a) three planes H.P; V.P; and A.V.P. are shown. A.V.P. is perpendicular to H.P and inclined to V.P. by an angle Ø.

Line along which A.V.P. meets the H.P. is known as horizontal trace (H.T.). It is shown in Fig. 7.3 (a) by H.T. Similarly line along which A.V.P. meets the V.P. is known as vertical trace (V.T.) and is shown in Fig. 7.3 (a) by V.T.

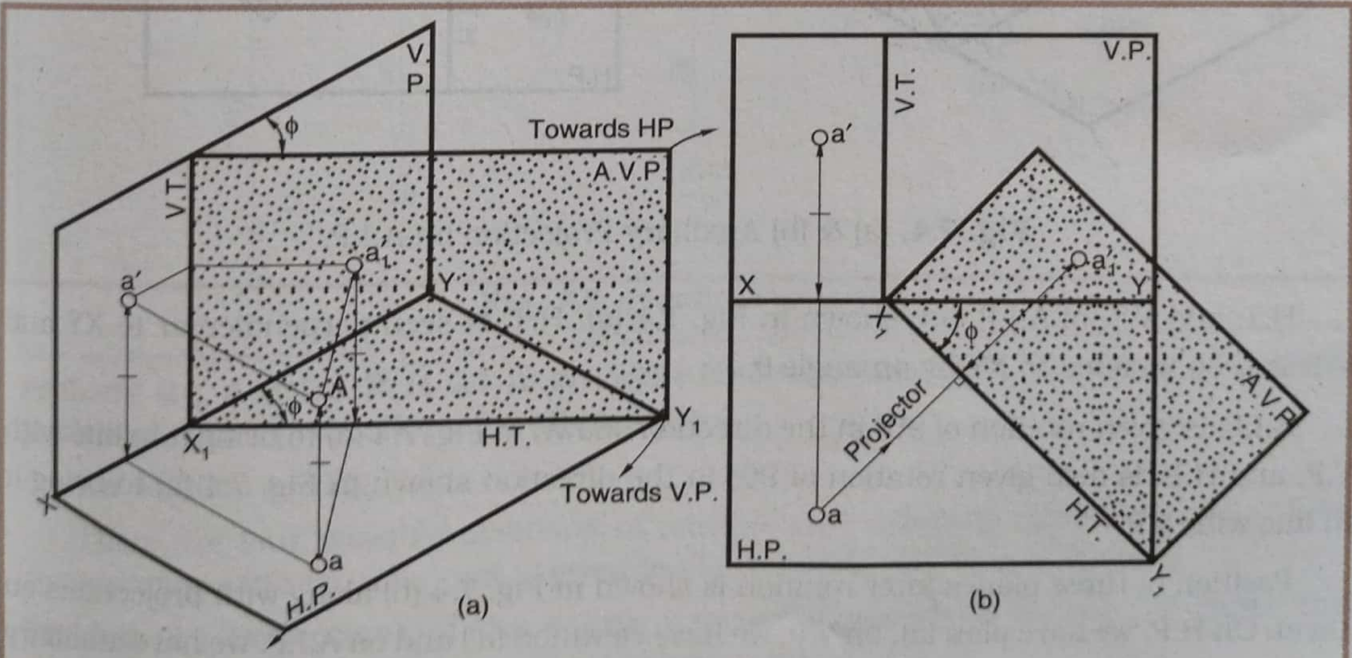


Fig. 7.3. (a) & (b) Auxiliary Projection on A.V.P.

V.T. is seen perpendicular to XY and H.T. is seen inclined to XY by an angle Ø.

H.P. is given rotation of 90° in the direction shown in Fig. 7.3 (a) to bring it in line with V.P. and A.V.P. is also given rotation of 90° in the direction shown in Fig. 7.3 (a) to bring it in line with H.P. and V.P. both.

Position of three planes after rotation is shown in Fig. 7.3 (b) along with projections on them. On H.P. we have plan (a), on V.P. we have elevation a' and on A.V.P. we have auxiliary elevation a'_1.

It is clear from Fig. 7.3 (a) and (b) that to get auxiliary elevation, plan is required to be projected on new X_1Y_1 line, which is nothing but H.T. of A.V.P. drawn at an angle Ø with XY. On projector, from plan (a), on X_1Y_1 take distance of elevation from XY and mark from X_1Y_1 to get a'_1, as shown in Fig. 7.3 (b).

3. AUXILIARY PROJECTIONS ON A.I.P.

Similarly, in Fig. 7.4 (a) three planes H.P; V.P. and A.I.P. are shown, A.I.P. is perpendicular to V.P. and inclined to H.P. by an angle θ.

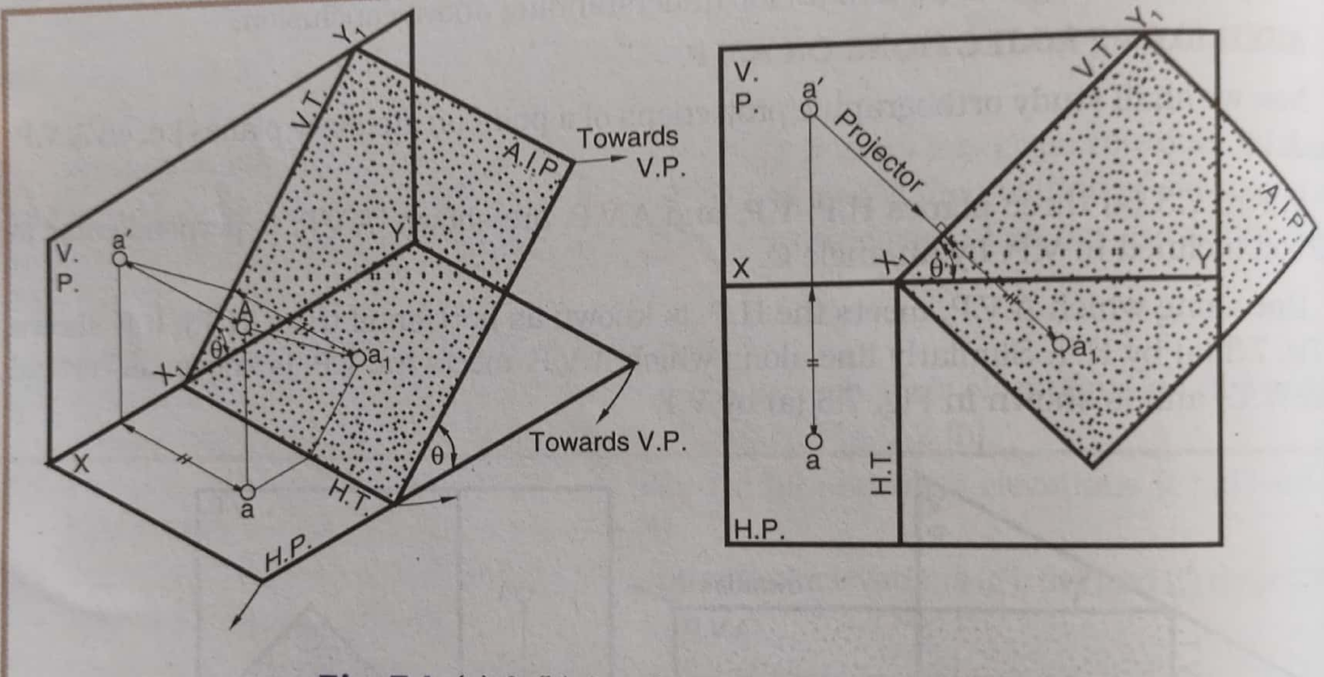


Fig. 7.4. (a) & (b) Auxiliary Projection on A.I.P.

H.T. and V.T. of A.I.P. are shown in Fig. 7.4 (a). H.T. is seen perpendicular to XY and V.T. is seen inclined to XY by an angle θ.

A.I.P. is given rotation of 90° in the direction shown in Fig. 7.4 (a) to bring it in line with V.P. and H.P. is also given rotation of 90° in the direction shown in Fig. 7.4 (a) to bring it in line with V.P.

Position of three planes after rotation is shown in Fig. 7.4 (b) along with projections on them. On H.P. we have plan (a), on V.P. we have elevation (a') and on A.I.P. we have auxiliary plan a_1.

It is clear from Fig. 7.4 (a) and (b) that to get auxiliary plan a_1, elevation a' is required to be projected on new X_1Y_1 line, which is nothing but V.T. of A.I.P. drawn at an angle θ with XY. On projector from elevation (a') on X_1Y_1 take distance of plan from XY and mark from X_1Y_1 to get a_1, as shown in Fig. 7.4 (b).

Boundaries of planes are shown just for understanding but in actual practice boundaries are not drawn.

4. PRACTICE PROBLEMS

Students are requested to keep conclusions ready on hand while solving problems.

Problem 1 : Draw the projections of the following points on the same x - y line.

(i) Point A in V.P. 30 mm below H.P.

(ii) Point B in H.P. 20 mm in front of V.P.

(iii) Point C 20 mm above H.P. and 20 mm behind V.P.

(iv) Point D 25 mm below H.P. and 40 mm behind V.P.

(v) Point E on H.P. and on V.P.

(vi) Point F 40 mm above H.P. and 10 mm in front of V.P.

(vii) Point G on V.P. 35 mm above H.P.

For solution see Fig. 7.5.

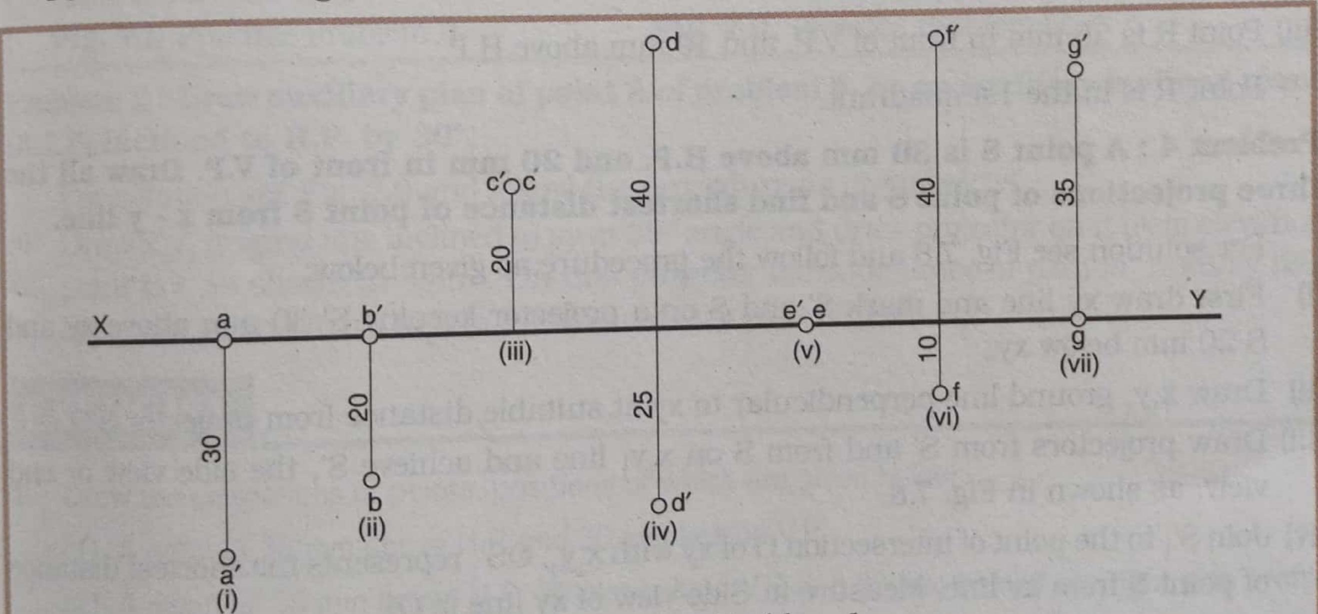


Fig. 7.5. Practice Problem 1.

Problem 2 : A point R is 25 m.m. from both the reference planes in all possible positions. Draw projections for all positions.

For solution see Fig. 7.6.

There are four possible positions of point R in 4 different quadrants and solution corresponding to each quadrant is given in Fig. 7.6 as R_1, R_2, R_3 and R_4.

Problem 3 : Projections of the points P,Q and R are given in Fig 7.7. State their positions w.r.t H.P. and V.P. and state their quadrants.

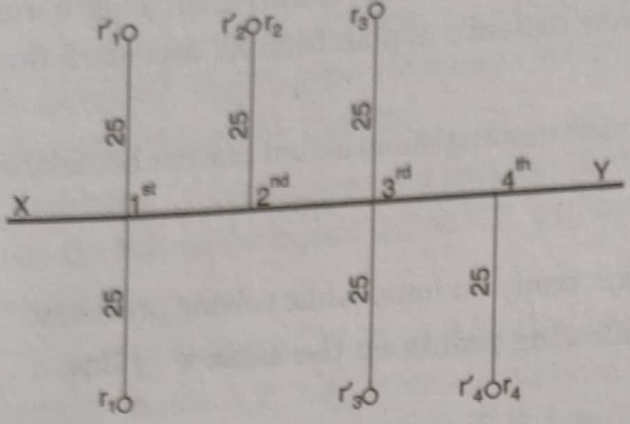


Fig. 7.6. Practice Problem 2

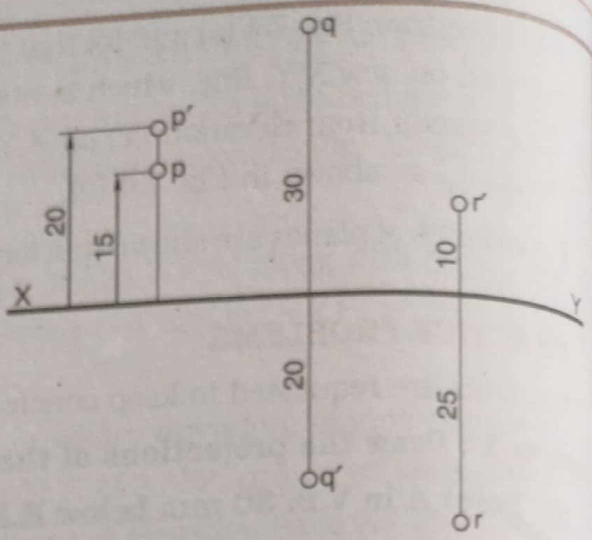


Fig. 7.7. Data of Practice Problem 3

Answer

(i) Point P is 20 m.m. above H.P. and 15 mm behind V.P.

Point P is in the 2nd quadrant.

(ii) Point Q is 30 mm behind V.P. and 20 mm below H.P.

Point Q is in the 3rd quadrant.

(iii) Point R is 25 mm in front of V.P. and 10 mm above H.P.

Point R is in the 1st quadrant.

Problem 4 : A point S is 30 mm above H.P. and 20 mm in front of V.P. Draw all th three projections of point S and find shortest distance of point S from x - y line.

For solution see Fig. 7.8 and follow the procedure as given below:

(i) First draw xy line and mark S' and S on a projector keeping S' 30 mm above xy an S 20 mm below xy.

(ii) Draw x_1y_1 ground line perpendicular to xy at suitable distance from projector S'S.

(iii) Draw projectors from S' and from S on x_1y_1 line and achieve S'_1 the side view or en view, as shown in Fig. 7.8.

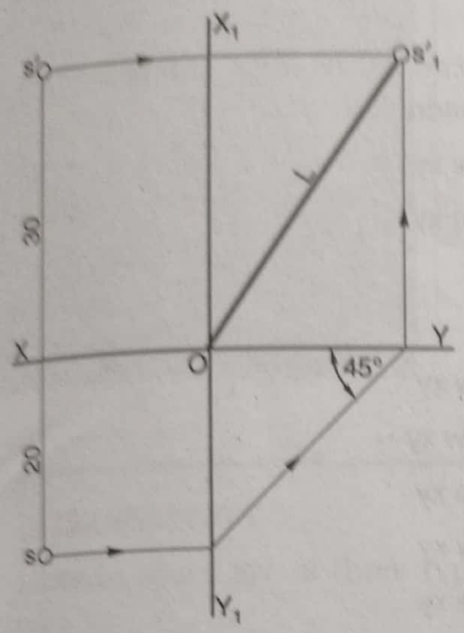
(iv) Join S'_1 to the point of intersection O of xy with x_1y_1. OS'_1 represents the shortest distanc of point S from xy line. Measure it. Side view of xy line is O.

Problem 5 : Point A is 20 mm behind V.P. and 25 mm below H.P. Draw its projection and project its auxiliary elevation on A.V.P. which makes 60° with V.P.

For solution see Fig. 7.9 and follow the procedure as given below :

(i) First draw plan (a) and elevation (a') of point A.

(ii) Draw x_1y_1 ground line inclined to xy at 60° angle and draw projector on it from plan point (a) as shown in Fig. On this projector mark distance of (a') from xy (25) beyond x_1y_1 and mark (a'_1). Then (a'_1) is an auxiliary elevation of point A.



Answer : L = 36 mm

Fig. 7.8 Practice Problem 4

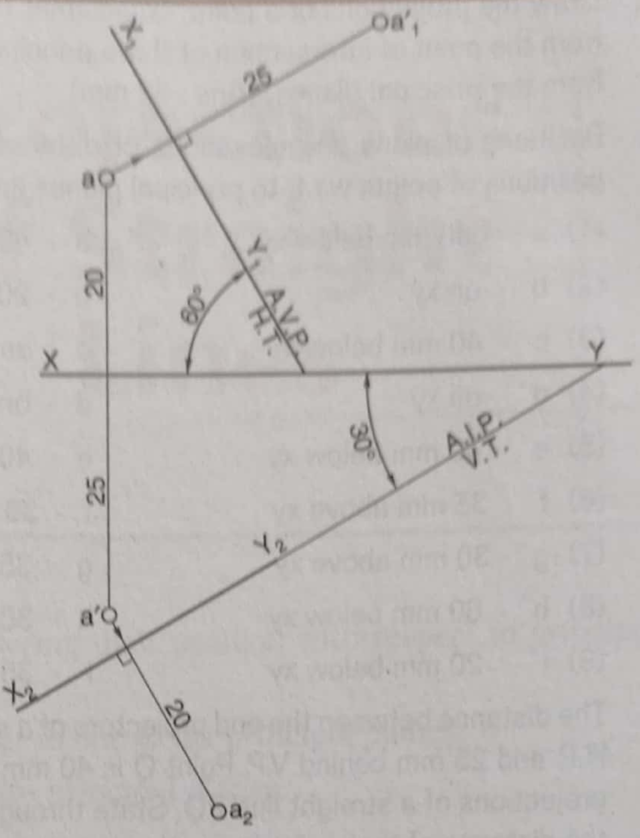


Fig. 7.9. Practice Problems 5 & 6

Problem 6 : Draw auxiliary plan of point A of problem 5, on an auxiliary inclined plane (A.I.P) inclined to H.P. by 30°.

For solution see Fig. 7.9 and follow the procedure as given below:

(i) Draw x_2y_2 ground line inclined to xy at 30° angle and draw projector on it from elevation point (a'), as shown in figure. On this projector mark distance of plan (a) from xy (20) beyond x_2y_2 and mark (a_2). (a_2) is an auxiliary plan of point A.

EXERCISE

1. Draw the projections of points, positions of which are given below :
   (1) A point 'A' 30 mm above H.P. and 30 mm behind V.P.
   (2) A point 'B' 35 mm below H.P; 25 mm behind V.P. and 20 mm behind the right side profile plane (P.P).
   (3) A point 'C' on H.P.; on V.P. and on P.P.
   (4) A point 'D' 40 mm below H.P. and 40 mm in front of V.P.
   (5) A point 'E' on H.P. and 40 mm in front of V.P.
   (6) A point 'F' on V.P; 25 mm above H.P. and 40 mm in front of right side P.P.
   (7) A point 'G' is 50 mm from H.P.; V.P. Draw its projections in all possible positions.
2. Draw the projections of a point 'P' 20 mm above H.P. and in 1st quadrant if its shortest distance from xy line is 40 mm Find the distance of point P from V.P. [Ans : 34.5 m.m.]

3. Draw the projections of a point, equidistant from three principal planes, the shortest distance from the point of intersection of three principal planes is 90 mm Find the distance of the point from the principal planes. [Ans : 52 mm]

4. Positions of plans and elevations of different points w.r.t. xy line is given below. State the positions of points w.r.t. to principal planes and state quadrants.

(1) a′ - 30 mm below xy	a - 40 mm above xy
(2) b′ - on xy	b - 20 mm above xy
(3) c′ - 40 mm below xy	c - on xy
(4) d′ - on xy	d - on xy
(5) e′ - 40 mm below xy	e - 40 mm below xy
(6) f′ - 35 mm above xy	f - 30 m.m. below xy
(7) g′ - 30 mm above xy	g - 35 mm above xy
(8) h′ - 60 mm below xy	h - 30 mm below xy
(9) i′ - 20 mm below xy	i - 35 mm above xy

5. The distance between the end projectors of a straight line PQ is 70 mm. Point P is 30 mm above H.P. and 25 mm behind V.P. Point Q is 40 mm above H.P. and 20 mm in front of V.P. Draw the projections of a straight line PQ. State through which plane the line will pass and what will be the distance of that point from other principal plane.

[Ans : V.P.(U); 36 mm above H.P.]

CHAPTER - 8

Projections of Straight Lines

1. INTRODUCTION

Lines in space are of three types considering their position with respect to principal planes :

1. Line parallel to two and perpendicular to one of the principal planes.
2. Line parallel to one and inclined to two principal planes.
3. Line inclined to all the three principal planes.

To study the projections of straight lines one must study five important theories dealt with in this chapter and understand the conclusions/discussions, etc. given at the end of theories. Theory No. 3 and Theory No. 5 are most important theories. Most of the problems on straight lines are based on the above two theories.

Projections of straight lines have nothing to do with 1st angle or 3rd angle system of projection. Projections of straight lines are drawn according to the positions of the end points of the straight lines w.r.t. H.P. and V.P. Therefore, three dimensional drawings are given in each theory with straight line drawn in 1st quadrant. As it is easy to understand the drawing of the first quadrant, three dimensional drawings are drawn with lines taken in 1st quadrant. Conclusion/discussion given at the end of each theory is also true for a line in any quadrant.

2. IMPORTANT THEORY - 1

Line parallel to two P.P.s and perpendicular to one principal plane :-

See Fig. 8.1 (a) and (b).

In Fig. 8.1 (a) we have lines AB, CD and EF perpendicular to H.P; V.P. and P.P. respectively. It is clear from Fig. 8.1(a) and (b) (i) that when line (AB) is perpendicular to H.P., plan (ab) is a point view and two other views i.e. elevation (a'b') and end view (a_1' b_1') are having true length and parallel to ground line generated by V.P. and P.P. i.e. X_1Y_1 or $\left[\frac{V}{P}\right]$ line.

Similarly, when line (CD), as shown in Fig. 8.1 (a), (b) (ii), is perpendicular to V.P., elevation (c'd') is a point view and two other views i.e. plan (cd) and end view (c_1' d_1') are having true length and parallel to ground line generated by H.P. and P.P. i.e. X_1Y_1/XY or $\begin{bmatrix} H \\ \hline P \end{bmatrix}$ line.

Similarly, when line (EF), as shown in Fig. 8.1 (a) (b) (iii), is perpendicular to P.P., e… view (e_1' f_1') is a point view and other two views i.e. elevation (e'f') and plan (ef) are havin… true length and parallel to ground line generated by V.P. and H.P. i.e. XY line or $\begin{bmatrix} V \\ \hline H \end{bmatrix}$ lin…

Conclusion

(1) WHEN A LINE IS PERPENDICULAR TO ONE OF THE PRINCIPAL PLANES IT I… PARALLEL TO TWO OTHER PRINCIPAL PLANES.

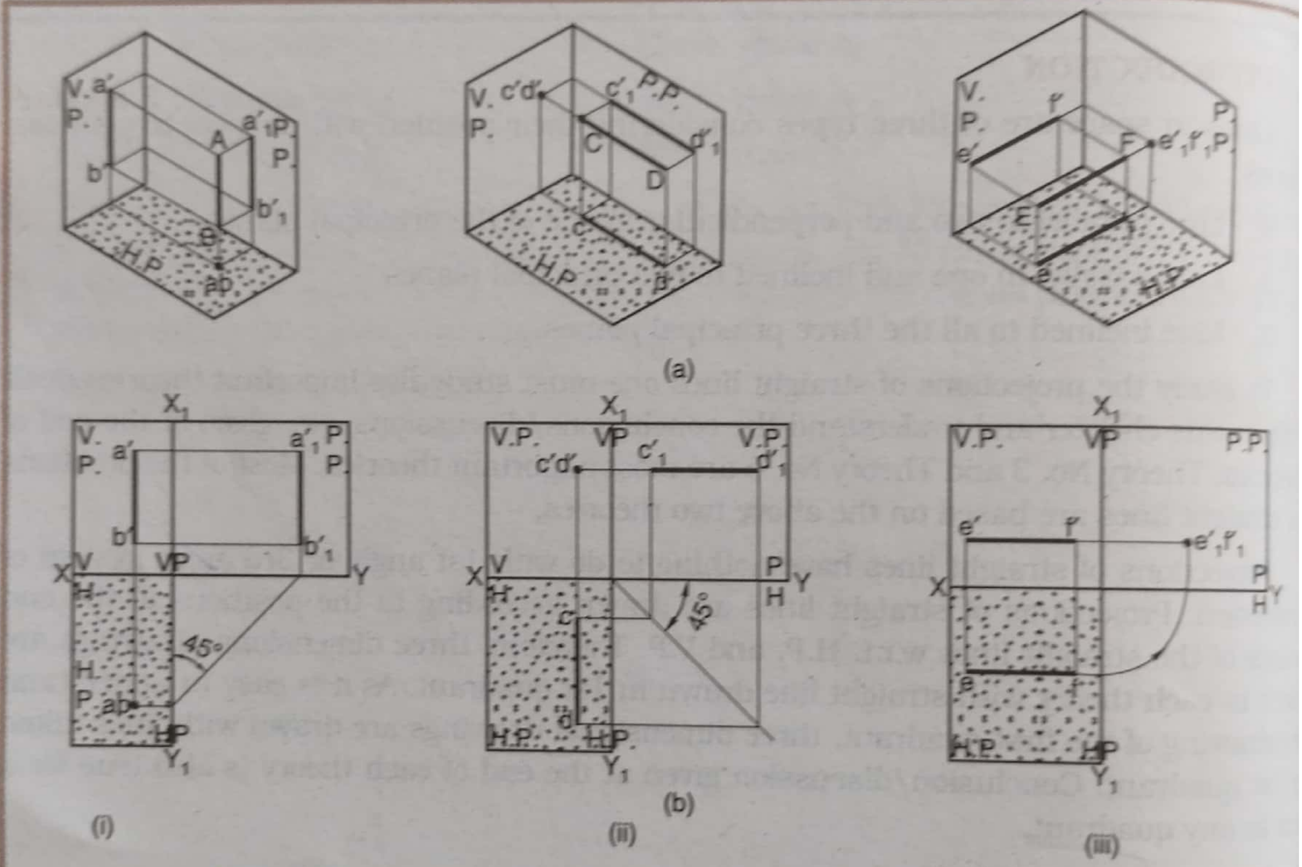


Fig. 8.1. Line Parallel to Two P. Ps and Perpendicular to one.

(2) PROJECTION OF A LINE ON A PLANE TO WHICH IT IS PERPENDICULAR IS A POIN… VIEW AND PROJECTIONS ON OTHER TWO PRINCIPAL PLANES ARE OF TRUE LENGT… AND PARALLEL TO RESPECTIVE GROUND LINE. ALL CONDITIONS GO TOGETHE…

(3) ABOVE TWO CONDITIONS GO TOGETHER.

Having done the study of theory of straight line perpendicular to one of the principa… planes and having gone through the conclusions and discussions we shall now solve fe… problems based on this theory.

Problem 1 : A line PJ, 50 mm long, is perpendicular to H.P. and it is below H.P. Poin… P is on H.P. and 30 mm behind V.P. Draw the projections of the line PJ.

For solution see Fig. 8.2 and follow the procedure as given below :

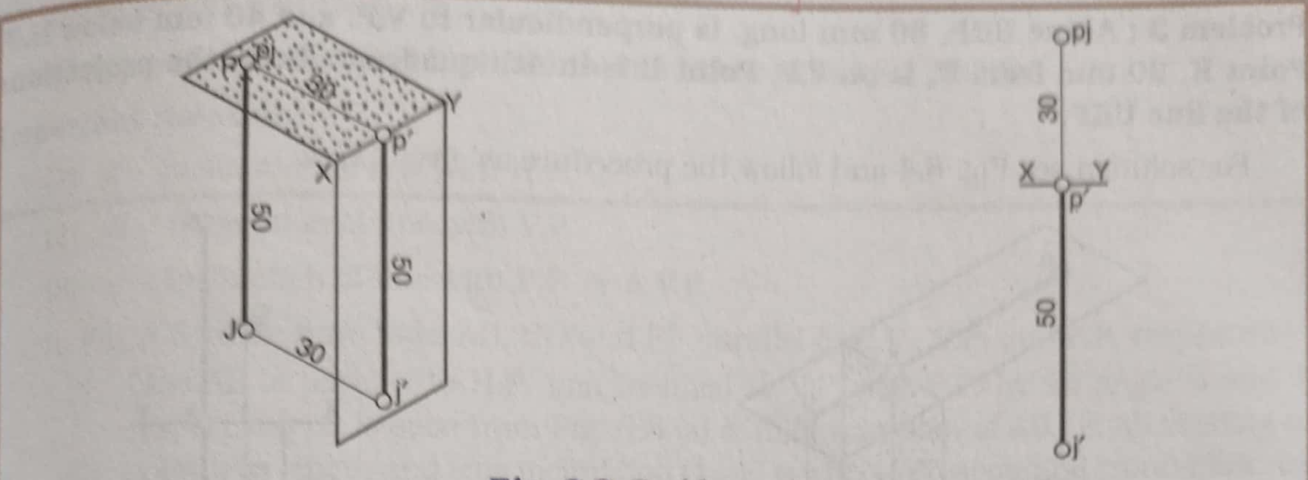


Fig. 8.2. Problem - 1.

Three dimensional drawing is given for understanding the location of the straight line w.r.t. planes. It is expected that it will also develop three dimensional thinking of the students.

(i) First of all mark the position of p and p'. As the point P is on H.P., p' will be on xy line and as it is 30 mm behind V.P., its plan p will be 30 mm above xy line.

(ii) As the line is perpendicular to H.P., its plan will be point view and it's elevation will be true length (50 mm) and perpendicular to xy line. So draw elevation p'j' of 50 mm length perpendicular to xy and below it and mark j at p only.

Problem 2 : A line MJ, 35 mm long, is perpendicular to the profile plane. The end M is 20 mm below H.P., 30 mm behind V.P. and 10 mm to the left of P.P. Draw all the three principal projections of the straight line MJ.

For solution see Fig. 8.3 and follow the procedure as given below:

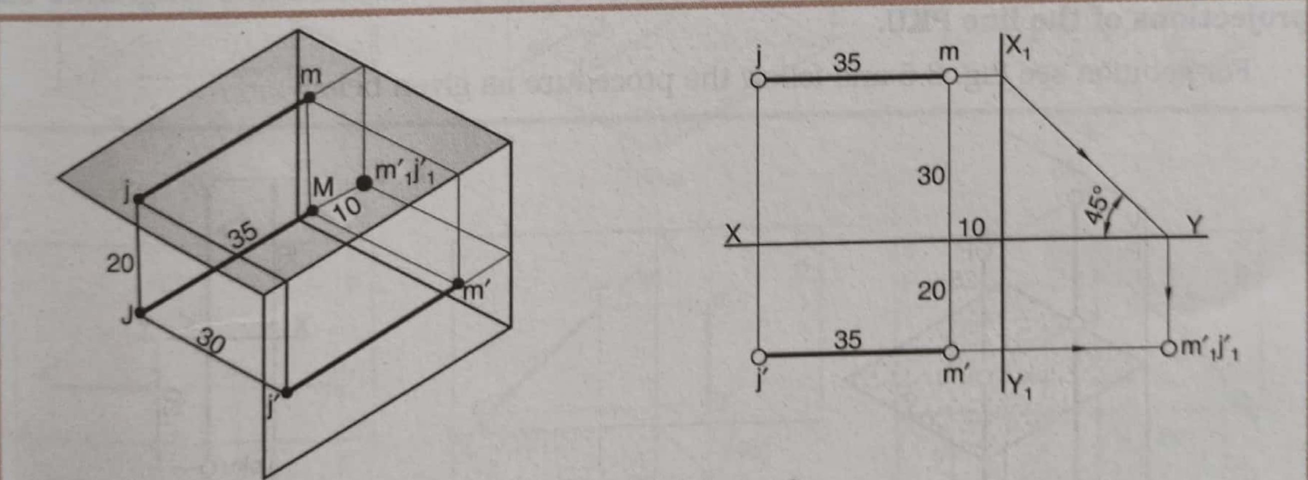


Fig. 8.3. Problem - 2.

(i) First draw xy and x_1y_1 lines and mark positions of projections m, m¢ and m¢$_1$ of point M as per the given data. Follow the procedure of drawing end view, as studied earlier.

(ii) As the line is perpendicular to P.P., its end view will be point view and its elevation and plan will be true length (35) and parallel to xy line or perpendicular to x_1y_1 line. So draw jm and j'm' parallel to xy and of 35 mm length, as shown in Fig.8.3.

(iii) Point m¢$_1$ and j¢$_1$ will be the same point since end view is going to be point view.

Problem 3 : A line UKP, 60 mm long, is perpendicular to V.P. and 40 mm below H.P. Point K, 20 mm from U, is on V.P. Point U is in 4th quadrant. Draw the projections of the line UKP.

For solution see Fig. 8.4 and follow the procedure as given below :

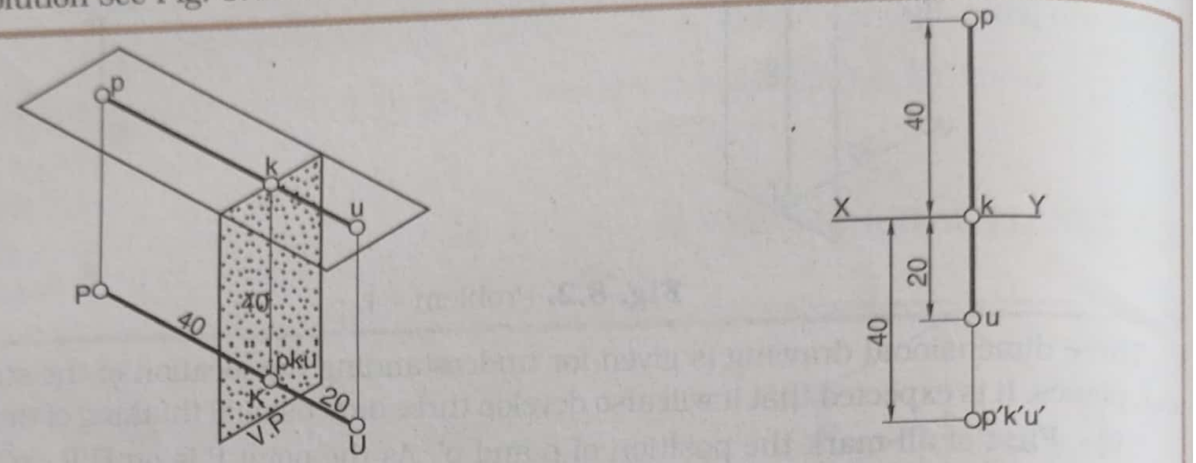


Fig. 8.4. Problem - 3.

(i) Elevation will be point view and plan will be perpendicular to xy line and will be of true length (60). So mark elevation p'k'u' 40 mm below xy line, since the line is given 40 mm below H.P.

(ii) As the point K is on V.P. mark plan point k on xy line. Mark u 20 mm below xy line and point p 40 mm above xy line on a perpendicular line to xy through k.

Problem 4 : A line PKU, 75 mm long, is perpendicular to H.P. Point P 20 mm away from V.P., is in the first quadrant, and point K is on H.P. PK is 25 mm long. Draw the projections of the line PKU.

For solution see Fig. 8.5 and follow the procedure as given below :

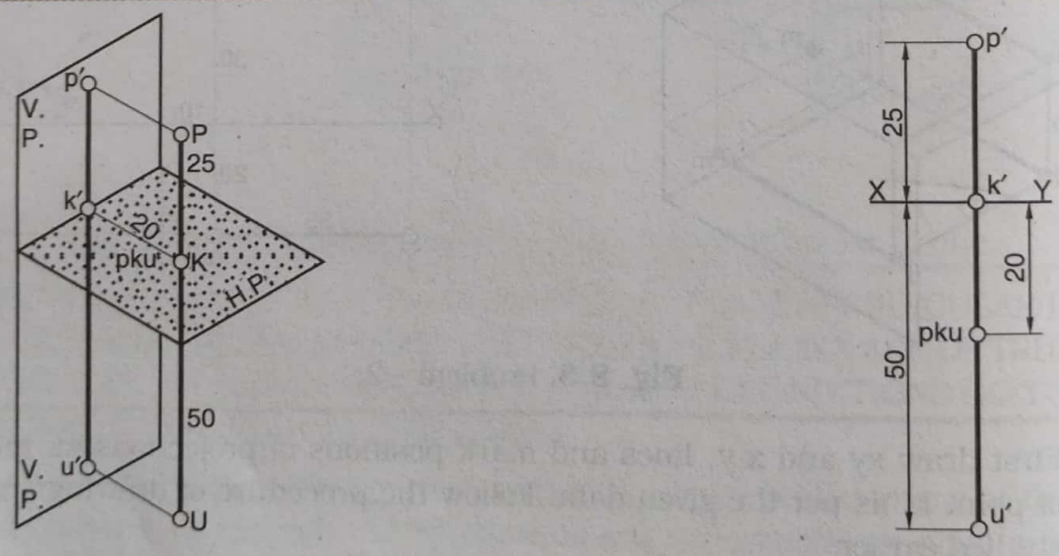


Fig. 8.5. Problem - 4.

Here plan will be point view and elevation will be true length (75) and perpendicular to xy line since the line is perpendicular to H.P. Follow the procedure similar to Problem 3.

3. IMPORTANT THEORY - 2

Line parallel to one and inclined to two other principal planes.

Important Notations :-

(1) θ - Inclination of line with H.P.

(2) Ø - Inclination of line with V.P.

(3) ψ - Inclination of line with P.P. or A.V.P.

In Fig. 8.6 (a) we have lines AB, CD and EF parallel to H.P., V.P. and P.P. respectively.

(i) Line AB is parallel to H.P. and inclined to V.P. and P.P. by an angle Ø and ψ respectively. It is seen from Fig. 8.6 (a) & (b)(i) that plan of AB i.e. ab is going to show true length and true inclination Ø and ψ with corresponding ground line. Its elevation a'b' and end view $a'_1b'_1$ will be less than true length and will be parallel to corresponding ground line Further it is clear from the figure that (Ø+ψ) = 90°.

(ii) Similarly, line CD is parallel to V.P. and inclined to H.P. and P.P. by an angle θ and ψ respectively. It is seen from Fig. 8.6 (a) and (b) (ii) that elevation of CD i.e. c'd' is going to show true length and true inclination θ and ψ with corresponding ground lines. Its plan cd and end view $c¢_1$ $d¢_1$ will be less than true length and will be parallel to corresponding ground line. Here (θ+ψ) = 90°.

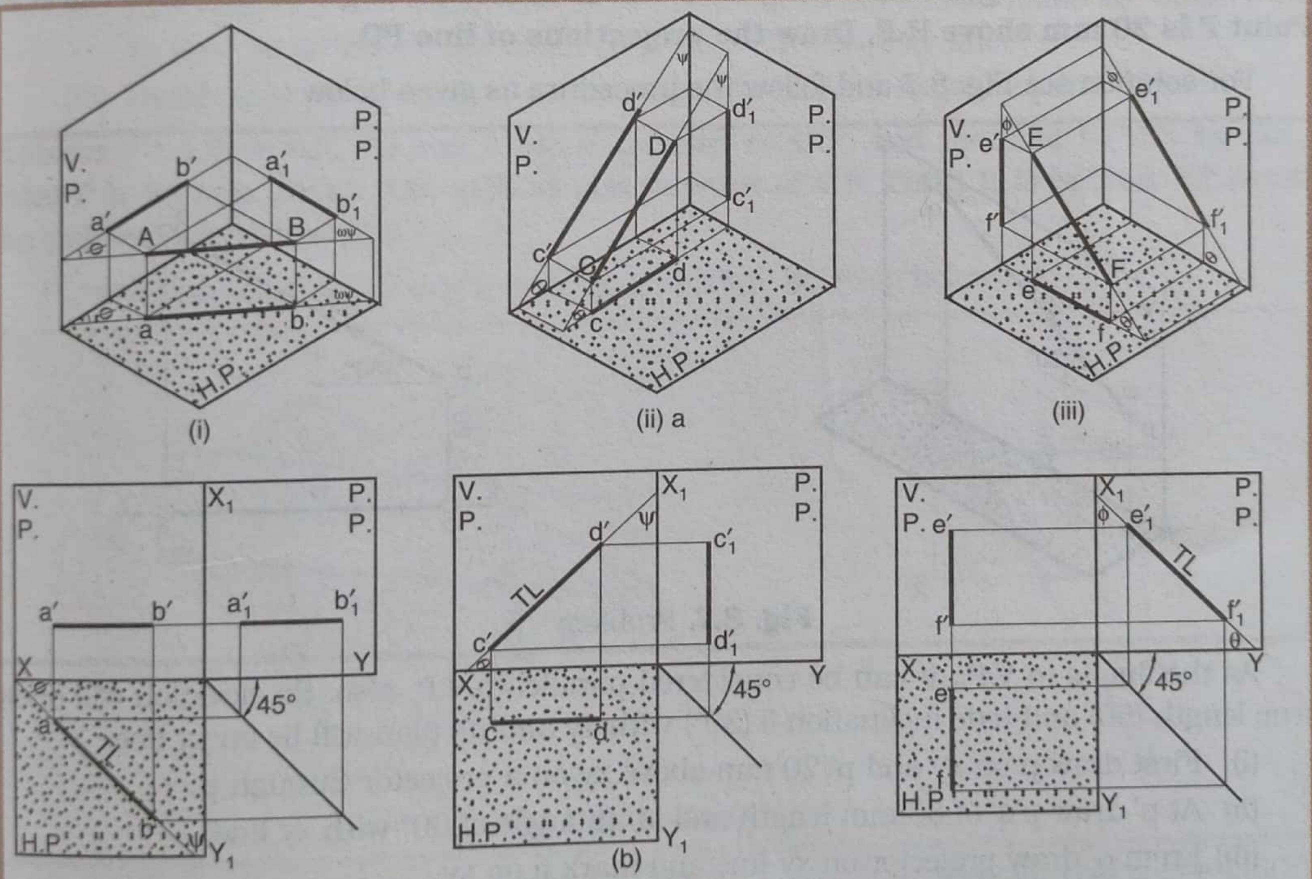


Fig. 8.6. Line Parallel to One and Inclined to Two P.Ps.

(iii) Similarly, line EF is parallel to P.P. and inclined to H.P. and V.P. by an angle θ and Ø respectively. It is seen from Fig. 8.6 (a) and (b) (iii) that end view $e'_1f'_1$ is going to show true length and true inclination θ and Ø with corresponding ground line. Its plan ef and elevation e'f' will be less than true length and will be parallel to corresponding ground line. Here (θ+Ø) = 90°. Further it is seen that, elevation and plan will be on a single projector.

CONCLUSION :

(1) WHEN A LINE IS PARALLEL TO ONE AND INCLINED TO TWO OTHER PRINCIPAL PLANES, ITS PROJECTION ON PLANE TO WHICH IT IS PARALLEL WILL SHOW TRUE LENGTH AND WILL ALSO SHOW TRUE INCLINATIONS WITH OTHER TWO PLANES WITH CORRESPONDING GROUND LINES.

(2) PROJECTIONS ON PLANES TO WHICH IT IS INCLINED AND NOT PARALLEL ARE SHORTER THAN THE TRUE LENGTH AND THEY WILL BE PARALLEL TO CORRESPONDING GROUND LINES.

(3) OUT OF 3 ANGLES θ, Ø and ψ, ONE WILL BE ZERO AND SUMMATION OF TWO OTHER WILL BE 90°.

(4) ALL THINGS GO TOGETHER AND CANNOT BE SEPARATED.

Having studied second theory of a straight line parallel to one of the principal planes and inclined to two other, we shall solve few problems based on it. Students are requested to go through conclusion/discussion etc. based on theory No.2 before solving problems based on it.

Problem 5 : A line PQ, 50 mm long, is in V.P. It makes an angle of 30° with the H.P. Point P is 20 mm above H.P. Draw the projections of line PQ.

For solution see Fig. 8.7 and follow the procedure as given below :

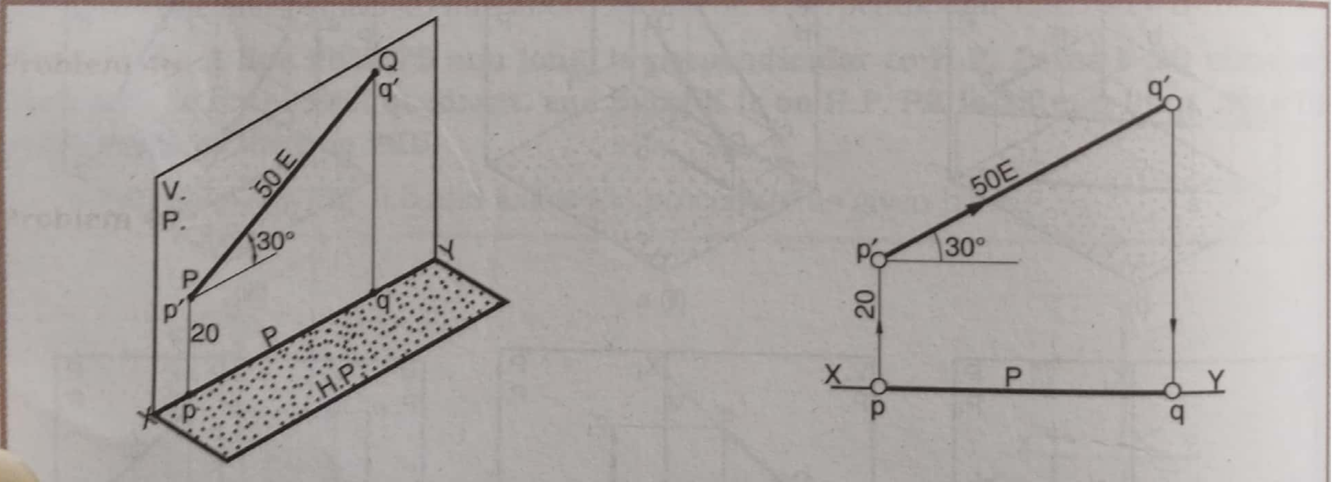


Fig. 8.7. Problem - 5.

As the line is in V.P., it can be considered parallel to V.P. also. So elevation will show true length (50) and true inclination θ (30°) with xy and its plan will lie on xy line.

(i) First draw p on xy and p' 20 mm above xy on a projector through p.

(ii) At p' draw p'q' of 50 mm length and at an angle of 30° with xy line.

(iii) From q' draw projector on xy line and mark q on xy.

Problem 6 : A line VK, 75 mm long, is parallel to V.P. and inclined to H.P. by an angle 45°. Point V is 30 mm below H.P. and 20 mm in front of V.P. Point K is in first quadrant. Draw the projections of the straight line VK.

For solution see Fig. 8.8 and follow the procedure as given below :

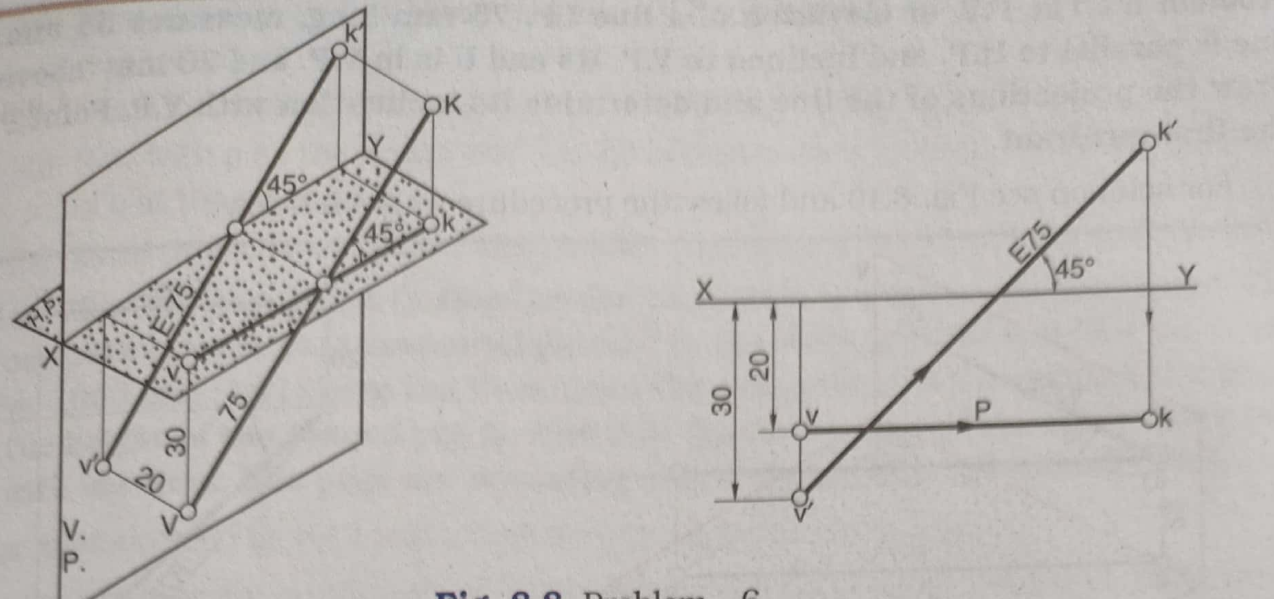


Fig. 8.8. Problem - 6.

Here, as the line is parallel to V.P. its elevation v'k' must show T.L. 75 mm and must make 45° with xy and its plan vk must be parallel to xy line.

(i) First plot v and v' on a projector 20 mm and 30 mm below xy respectively.

(ii) At v' draw v'k' of 75 mm length on upperside and it should make 45° angle with xy line. Since K is in 1st quadrant k' must be above xy line.

(iii) Draw projector through k' and parallel to xy from v to get k by intersection.

Problem 7 : A line KP, 70 mm long, is parallel to H.P. and inclined to V.P. by 60°. Point P is 20 mm above H.P. and 30 mm in front of V.P. Point K is behind V.P.Draw the projections of line KP.

For solution see Fig. 8.9 and follow the procedure as given below :

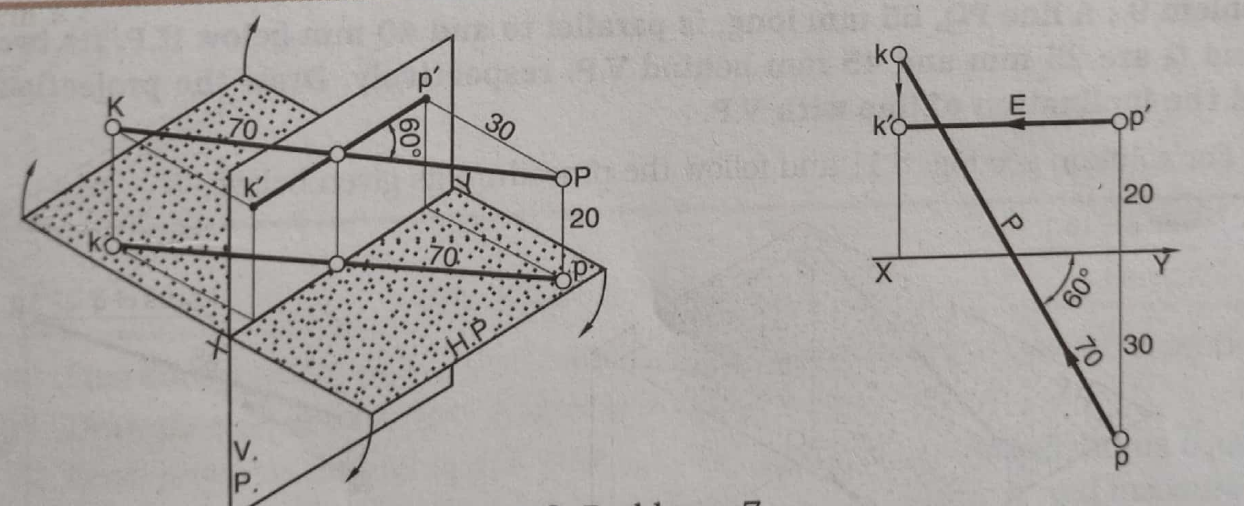


Fig. 8.9. Problem - 7.

As the line is parallel to H.P., plan kp will show T.L.(70) and will make true angle Ø(60°) with xy line and its elevation k'p' will be shorter and parallel to xy line.

(i) First plot p and p'.

(ii) From p draw pk line of T.L. 70 mm length at an angle of 60° with xy. Plan k should be above xy, since K is behind V.P.

(iii) Draw projector through k and parallel to xy from p' to get k' by intersection.

Problem 8 : The F.V. or elevation of a line UP, 75 mm long, measures 55 mm. The line is parallel to H.P. and inclined to V.P. Its end U is in V.P. and 20 mm above H.P. Draw the projections of the line and determine its inclination with V.P. Point P is in the first quadrant.

For solution see Fig. 8.10 and follow the procedure as given below :

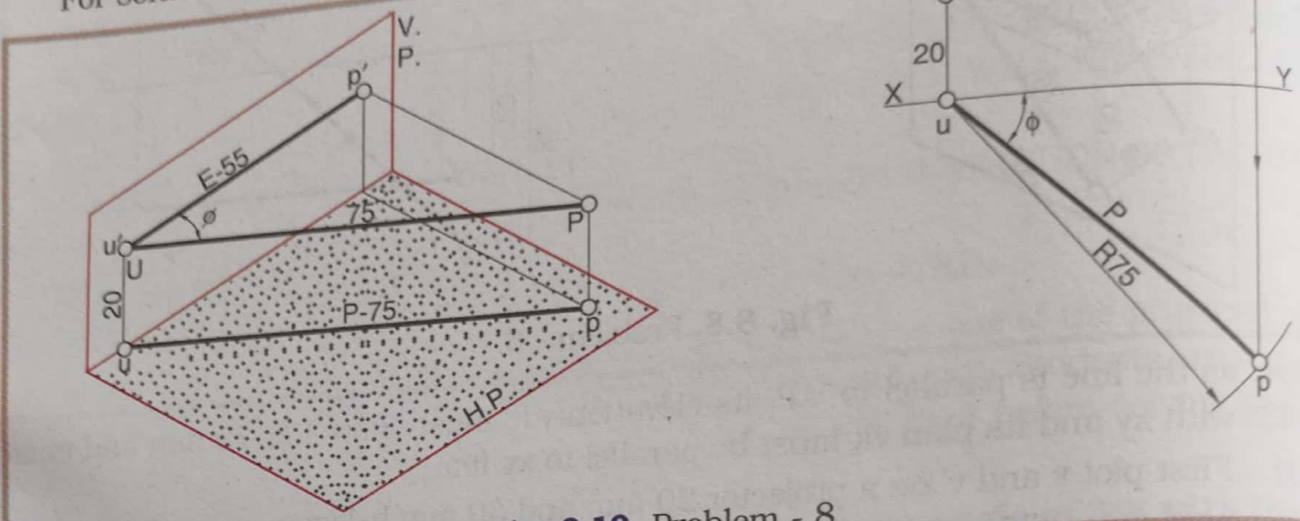


Fig. 8.10. Problem - 8.

As the line is parallel to H.P., elevation (55) will be parallel to xy and plan will be of T.L. (75), and will show true angle Ø with xy line.

(i) First draw u and u'. From u' draw line u'p' of 55 mm length and parallel to xy

(ii) Draw projector through p' and draw arc with u as the centre and radius equal T.L. (75) to get p, as shown in figure. Join up and measure angle of up with i.e. Ø .

(iii) Measured Ø = 43° (Ans).

Problem 9 : A line PQ, 65 mm long, is parallel to and 40 mm below H.P. Its two ends P and Q are 25 mm and 45 mm behind V.P. respectively. Draw the projections and find the inclination of line with V.P.

For solution see Fig. 8.11 and follow the procedure as given below:

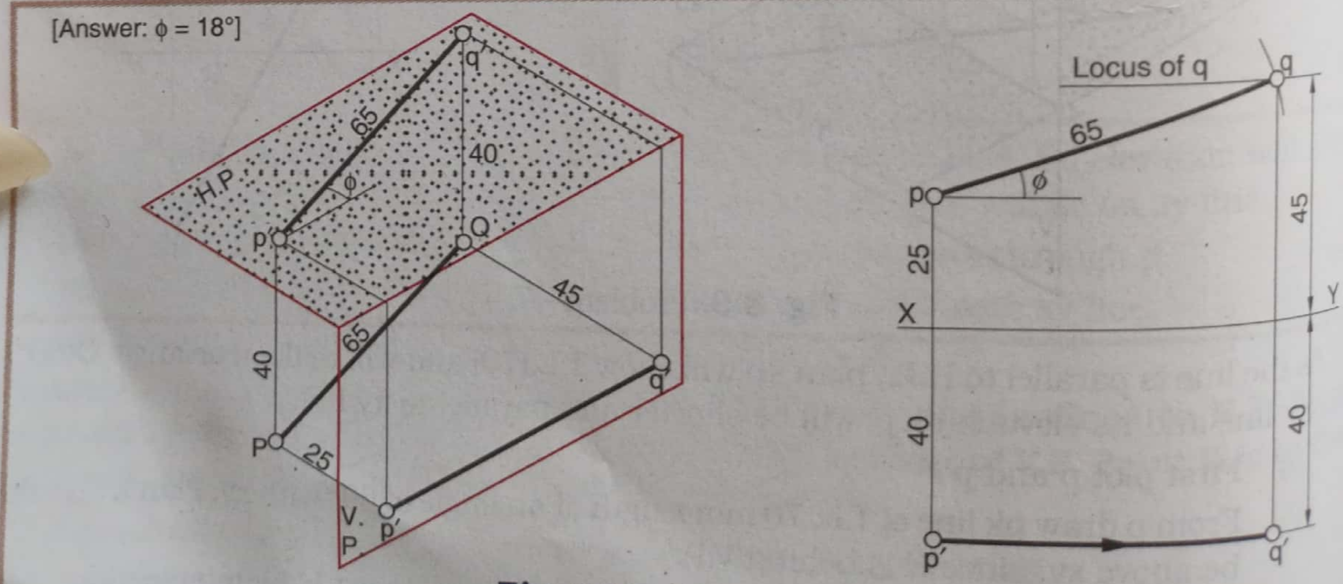


Fig. 8.11. Problem - 9.

(i) First draw p and p', on a projector, 25 mm above xy and 40 mm below xy respectively.

(ii) Draw locus of q, 45 mm above xy, since Q is 45 mm behind V.P.

(iii) Now with p as the centre and T.L. 65 mm as radius draw an arc to cut the locus of q at the point q.

(iv) Draw projector through q and parallel to xy from p' to get q' by intersection.

Problem 10 : Two pegs P and Q, fixed on the same wall, are 3000 mm (3m) apart. The distance between the pegs measured parallel to the floor is 2400 mm (2.4 m). If the peg P is 1000 mm (1m) above the floor, draw the projections of line joining two pegs. Find the height of the second peg Q. Also find the inclination of the line joining two pegs with the floor. The pegs are protuding out of the wall by 150 mm (0.15 m).

For solution see Fig. 8.12 and follow the procedure as given below :

As the two pegs are protuding out by the same length (150 mm) from a wall, line joining two pegs will be parallel to wall. Consider the wall as V.P. and floor as H.P. So line will be parallel to V.P. and inclined to H.P. Visualise this from the three dimensional drawing given in Fig. 8.12 Distance (2400) between two pegs measured parallel to the floor is going to be plan length.

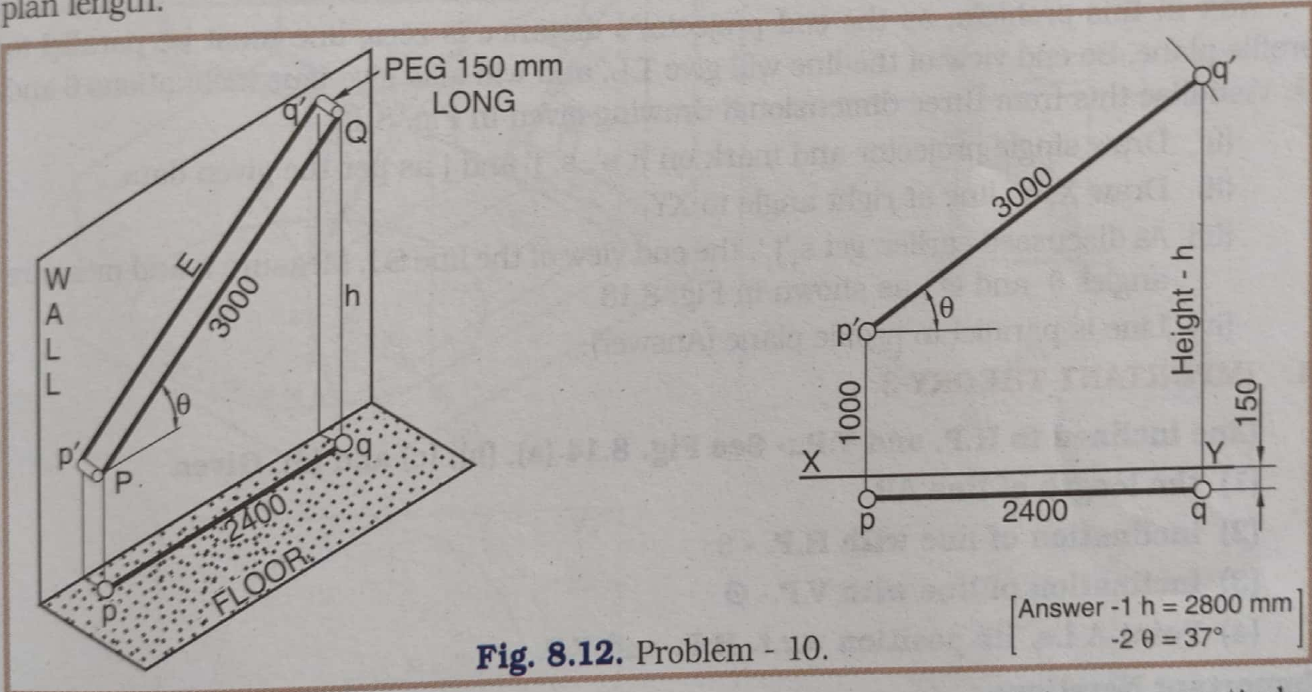


Fig. 8.12. Problem - 10.

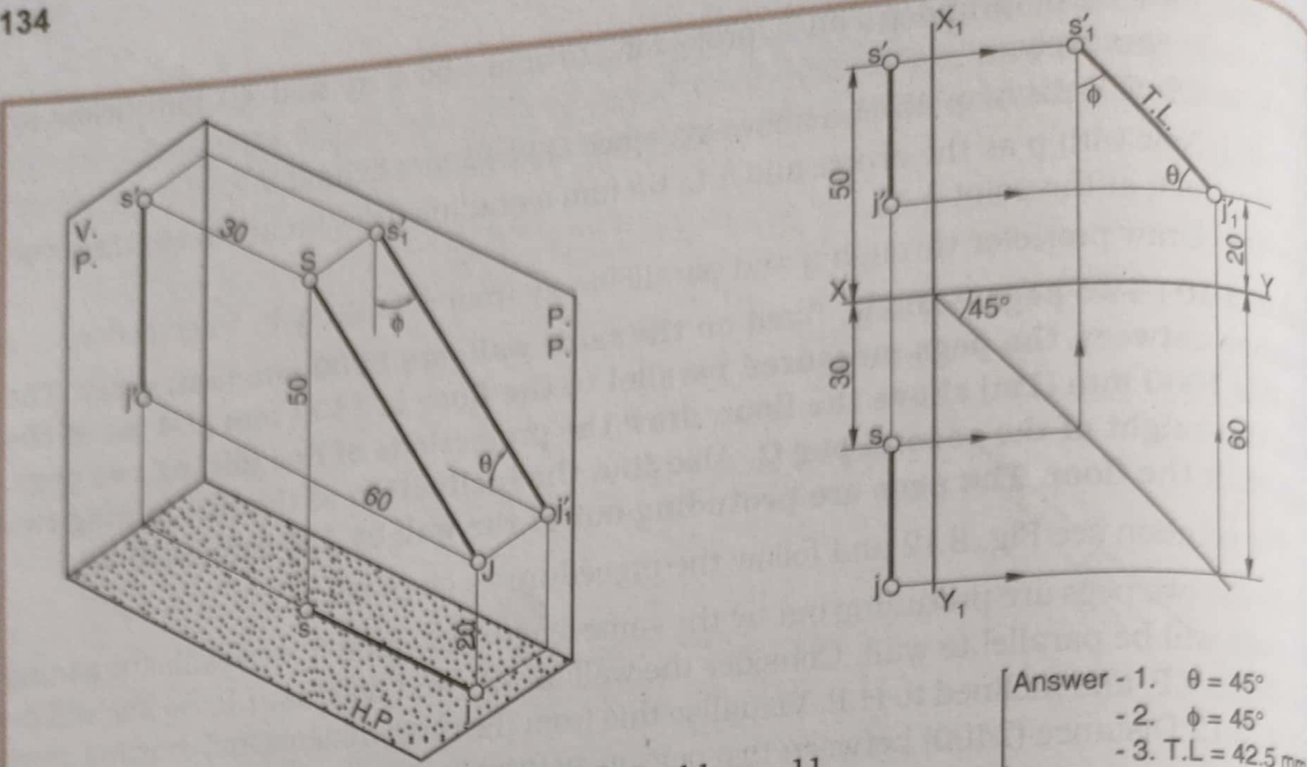
(i) First draw p' and p, on a projector, 1000 mm above and 150 mm below xy respectively.

(ii) Draw plan pq of 2400 mm length and parallel to xy.

(iii) Draw projector through q and draw arc with p' as the centre and radius equal to T.L. 3000 mm (3m) to get by intersection the point q'. Join p'q' and measure the angle θ and height h of point q' from xy line. θ = 37° is the inclination of the line with the floor and h = 2800 mm (2.8 m) height of the peg Q above the floor.

Problem 11 : The distance between the end projectors of a line SJ is zero. Point S is 50 mm above H.P. and 30 mm in front of V.P. Point J is 20 mm above H.P. and 60 mm in front of V.P. Draw the projections and find the angle of the line with H.P. and V.P. and also find the true length of the line. State the position of the line w.r.t. P.P.

For solution see Fig. 8.13 and follow the procedure as given below :



Answer - 1. θ = 45°
- 2. ϕ = 45°
- 3. T.L = 42.5 mm

Fig. 8.13. Problem - 11.

Now in this problem, as the end projector's distance is zero, line must be parallel to profile plane. So end view of the line will give T.L. and will also give true inclinations θ and Ø. Visualise this from three dimensional drawing given in Fig. 8.13.

(i) Draw single projector and mark on it s', s, j' and j as per the given data.

(ii) Draw X_1 Y_1 line at right angle to XY.

(iii) As discussed earlier, get $s_1'j_1'$, the end view of the line SJ. Measure it and measure angles θ and Ø, as shown in Fig. 8.13.

(iv) Line is parallel to profile plane (Answer).

4. IMPORTANT THEORY-3

Line inclined to H.P. and V.P.:- See Fig. 8.14 (a), (b), (c) and (d). Given

(1) the length of line AB

(2) inclination of line with H.P. - θ

(3) inclination of line with V.P.- Ø

(4) Point A i.e. its position w.r.t. H.P. and V.P.

Important Notations :

(1) Actual line - AB

(2) Plan of line - ab

(3) Elevation of line - a'b'

(4) Line AB_1 parallel to V.P. - position of the line AB during its rotation about the point A keeping inclination θ with H.P. the same.

(5) Line AB_2 parallel to H.P. - position of the line AB during its rotation about the point A keeping inclination with V.P. the same.

(6) Inclination of elevation with XY line is α and inclination of plan with XY line is β.

Our line is AB, see Fig. 8.14 (a). Line AB is rotated about the point A, keeping the inclination of the line AB with H.P. constant and equal to θ.

During rotation the point B will rotate around the periphery of a horizontal circle and the line AB will generate a cone. Plan of this horizontal circle is also a circle of same size with plan(a) of point A as the centre. This circle is a circle on which plan of actual point B is going to lie and so we call it as the locus of b. Elevation of horizontal circle is a straight line parallel to XY. The elevation of the actual point B is going to lie on this straight line and so we will call it as the locus of b'.

Now during this rotation, the line AB will have its position as AB_1 when it will become parallel to V.P. See Fig. 8.14 (a).

As AB_1 is parallel to V.P., as per conclusion of line parallel to V.P. (See Fig. 8.14 (b) also),

(1) elevation $a'b'_1$ will be true length;

(2) elevation $a'b'_1$ will make an angle θ with XY line ; and

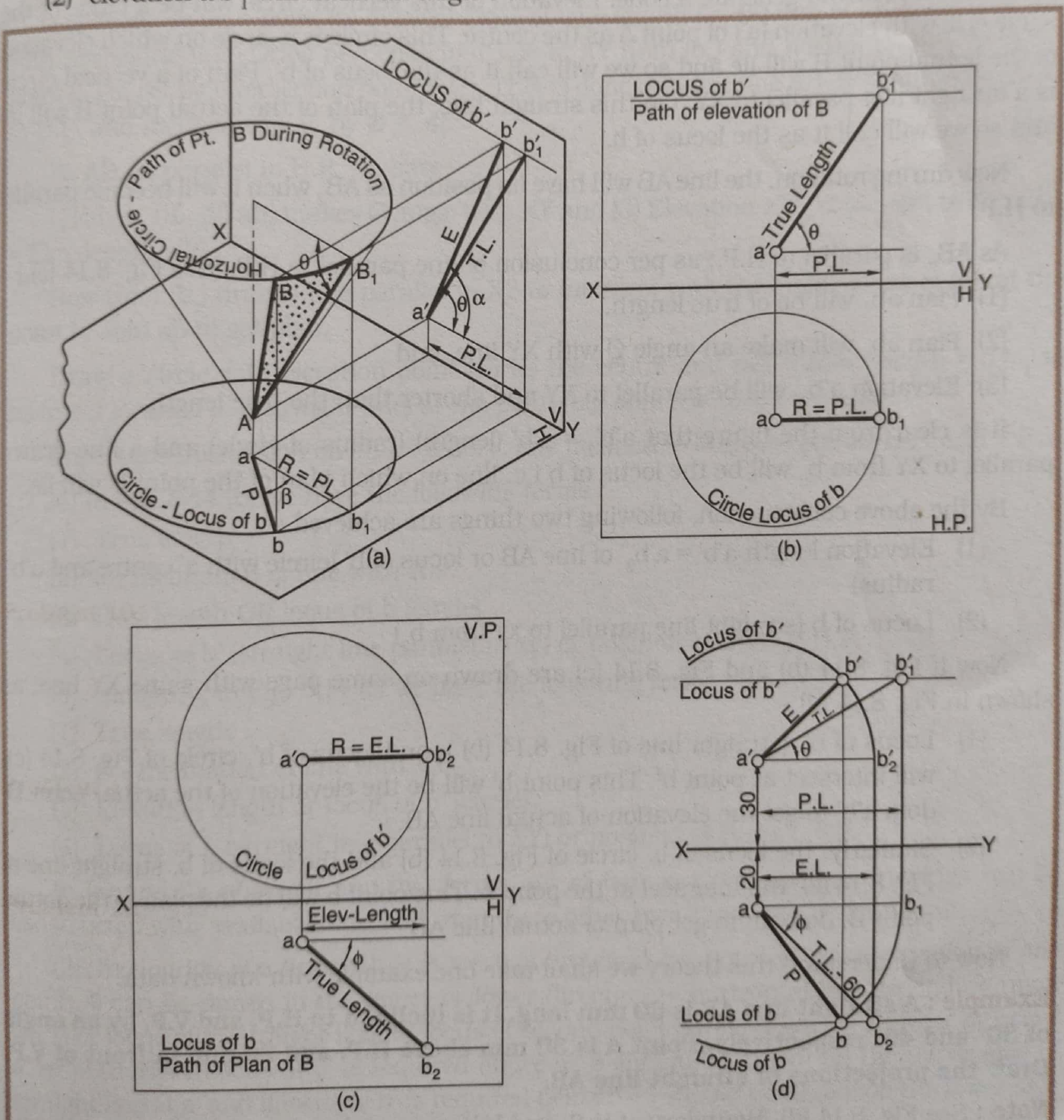


Fig. 8.14. Theory - Line Inclined to All P. Ps.

(3) plan ab_1 will be parallel to XY and shorter than the true length.

It is clear from the figure that ab_1 = ab (length) (radius of circle) and a line drawn parall to XY from b'_1 is going to be locus of b' i.e. line on which elevation of point B is going to li See Fig. 8.14 (b) also. By this construction, the following two things are achieved. See Fig. 8.14 (b) also.

By this construction, the following two things are achieved.

(1) Plan length ab = ab_1 of line AB or locus of b (circle with a centre and ab_1 radius)

(2) Locus of b'. (straight line parallel to XY from b'_1)

Similarly, a line AB is rotated about the point A keeping the inclination of line A with V.P. constant and equal to Ø. As drawing is not given, imagine a similar drawing t Fig. 8.14 (a) with the path of B as vertical circle.

During rotation, the point B will rotate around the periphery of vertical circle (parallel t V.P.) and line AB will generate a cone. Elevation of this vertical circle will be a circle of th same size with elevation (a') of point A as the centre. This circle is a circle on which elevatio of the actual point B will lie and so we will call it as the locus of b'. Plan of a vertical circ is a straight line parallel to XY. On this straight line, the plan of the actual point B will l and so we will call it as the locus of b.

Now during rotation, the line AB will have its position as AB_2 when it will become parall to H.P.

As AB_2 is parallel to H.P.; as per conclusion of line parallel to H.P. [See Fig. 8.14 (c).]

(1) Plan ab_2 will be of true length.

(2) Plan ab_2 will make an angle Ø with XY line, and

(3) Elevation $a'b_2'$ will be parallel to XY and shorter than the true length.

It is clear from the figure that $a'b'_2$ = a'b' (length) (radius of circle) and a line draw parallel to XY from b_2 will be the locus of b i.e. line on which plan of the point B will lie.

By the above construction, following two things are achieved :

(1) Elevation length a'b' = $a'b_2'$ of line AB or locus of b' (circle with a' centre and a'b radius)

(2) Locus of b (straight line parallel to xy from b_2)

Now if Fig. 8.14 (b) and Fig. 8.14 (c) are drawn on same page with same XY line, a shown in Fig. 8.14 (d) -

(1) Locus of b', (straight line of Fig. 8.14 (b)), and locus of b', circle of Fig. 8.14 (c will intersect at point b'. This point b' will be the elevation of the actual point B Join a'b' to get the elevation of actual line AB.

(2) Similarly, the locus of b, circle of Fig. 8.14 (b) and the locus of b, straight line o Fig. 8.14 (c), will intersect at the point b. This point b will be the plan of the actua point B. Join ab to get plan of actual line AB.

Now to understand this theory we shall take one example with known data.

Example : A straight line AB is 60 mm long. It is inclined to H.P. and V.P. by an angl of 30° and 45° respectively. Point A is 30 mm above H.P. and 20 mm in front of V.P Draw the projections of straight line AB.

Note : See Fig. 8.14 (d). Boundary of H.P. and V.P. are not drawn and circles are partiall drawn instead of full.

Solution : First draw XY line and draw (a') and (a) 30 mm above and 20 mm below XY line respectively. At point (a') draw $a'b'_1$ line of true length i.e. 60 mm at an angle $\theta = 30°$ with horizontal line. Draw perpendicular line to XY from (b'_1) and horizontal line from point (a) to get by intersection point (b_1). This is done because line AB_1 is assumed parallel to V.P. and inclined to H.P. by $\theta = 30°$.

As AB_1 is parallel to V.P. we have -

(1) $a'b'_1$ = T.L, (2) $a'b'_1$ makes θ angle with XY and (3) plan ab_1, is parallel to XY, ab_1 = Plan length ab.

Now from (b_1') draw a line parallel to XY which is the locus of (b') (straight line) and draw a circle with plan point (a) as the centre and plan length ab_1 as the radius which is locus of b (circle). Full circle is not required and hence not drawn.

Similarly, at plan point (a) draw ab_2 line of true length i.e. 60 mm at an angle Ø = 45° with horizontal line. Draw perpendicular line to XY from (b_2) and horizontal line from the point (a') to get by intersection point (b¢$_2$). This is done because line AB_2 is assumed parallel to H.P. and inclined to V.P. by Ø = 45°.

As AB_2 is parallel to H.P, we have -

(1) ab_2 = T.L, (2) ab_2 makes Ø angle with XY and (3) Elevation $a'b'_2$ is parallel to XY. $a'b'_2$ = Elev-length a'b'.

Now from (b_2) draw a line parallel to XY to intersect with the circle, locus of (b), at the point b. Join ab to get plan.

Draw a circle with elevation point (a') as the centre and rad = Elev. length = $a'b'_2$ to intersect locus of (b') (straight line) at the point (b'). Join a'b' to get elevation.

Important Discussion on above theory, line inclined to both the planes H.P. and V.P. :-

(1) (a) In Fig. 8.14 (b) we have the following terms :-

(1) True length
(2) θ - Inclination of line with H.P.
(3) Plan length OR locus of b (circle)
(4) Locus of b' (straight line parallel to xy) or position of B w.r.t. H.P.

(b) Similarly, in Fig. 8.14 (c) we have the following terms :-

(1) True length
(2) Ø - Inclination of line with V.P.
(3) Elevation length or locus of b' (circle)
(4) Locus of b (straight line parallel to XY) or position of B w.r.t. V.P.

In (a) and (b) above, if any two terms out of four are given, same drawings can be constructed with available data and from there other two terms can be found out.

(2) Inclination of a line with H.P. i.e. θ is drawn at the point a' (or on elevation of one point). θ can be drawn in the upper or lower direction to horizontal line depending upon whether locus of b' (straight line) or locus of second point (straight line) is above a' or below a' respectively. If locus of b' is required above a', θ should be drawn on the upper side of straight line at a' and if locus of b' is required below a', θ should be drawn on the lower side of horizontal straight line at a'.

Similarly, inclination of line with V.P. i.e. Ø is drawn at the point (a) (or on plan of one point). Ø can be drawn in the upper or lower direction to horizontal line at (a) depending upon whether locus of (b) (straight line) is required above (a) or required below (a) respectively.

(3) Out of many points (e.g. two end points, V.T., H.T., mid point, etc) given of any straight line, only one point can be marked positively and for rest of the points only their loci can be drawn parallel to XY for elevations as well as for plans.

In the begining, simple examples are solved, then some applied problems are solved and at the end most typical problems from university papers are solved.

Problem 12 : The distance between the end projectors of a straight line AB is 60 mm. Point A is 5 mm above H.P. and 30 mm in front of V.P. Point B is 40 mm above H.P. and 50 mm behind V.P. Draw the projections and find the inclination of straight line AB with H.P. and V.P. and the true length of the line. Find also the traces.

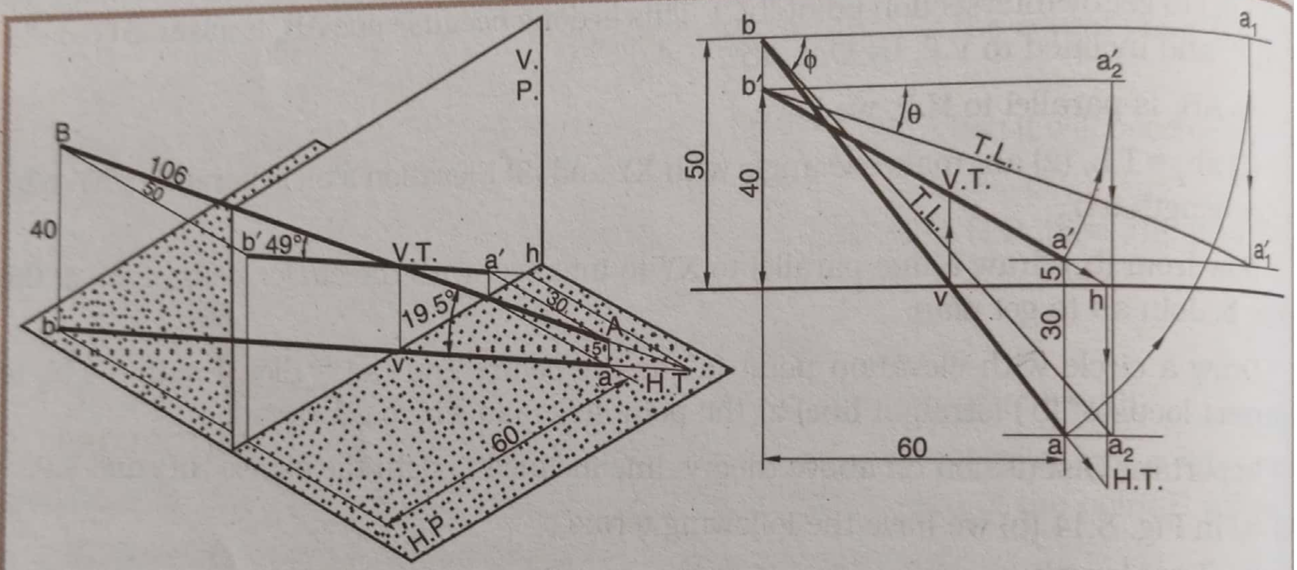


Fig. 8.15. Problem -12.

Answer : (1) $\theta = 19.5°$ (2) $\phi = 49°$
(3) T.L. - 106 mm.

This problem can be solved by two methods. One by theory No.3 and another by theory No.5. Solution is done with the help of theory No.3. See Fig. 8.15 for solution and follow the procedure as given below :

(i) First draw two end projectors 60 mm apart and plot b, b' and a, a' as per given data. Join plan ab and elevation a'b'.

(ii) Rotate the plan ba about b to position ba_1 to make it parallel to xy so that its corresponding elevation $b'a'_1$ gives us true length and true inclination θ, as shown in Fig. 8.17. Measured value of $\theta = 19.5°$ and T.L. = 106 mm.

(iii) Similarly, rotate the elevation b'a' about b' to new position $b'a'_2$ (parallel to xy) so that its corresponding plan ba_2 gives us true length and true inclination Ø, as shown in Fig. 8.17. Measured value of T.L. = 106 mm and measured Ø= 49°.

(iv) To get H.T. extend elevation b'a' upto xy line and mark the point as h. Now draw projector through h and extend plan ba. They will intersect at H.T.

(v) Similarly, mark point v where plan ba intersects with xy line and draw projector through v to intersect with elevation at V.T.

Problem 13: A line VH, 70 mm long, has its end V in V.P. and end H in H.P. Line is inclined to H.P. by 60° and to V.P. by 30°. Draw the projections and find traces of a line VH.

For solution see Fig. 8.16 and follow the procedure as given below:

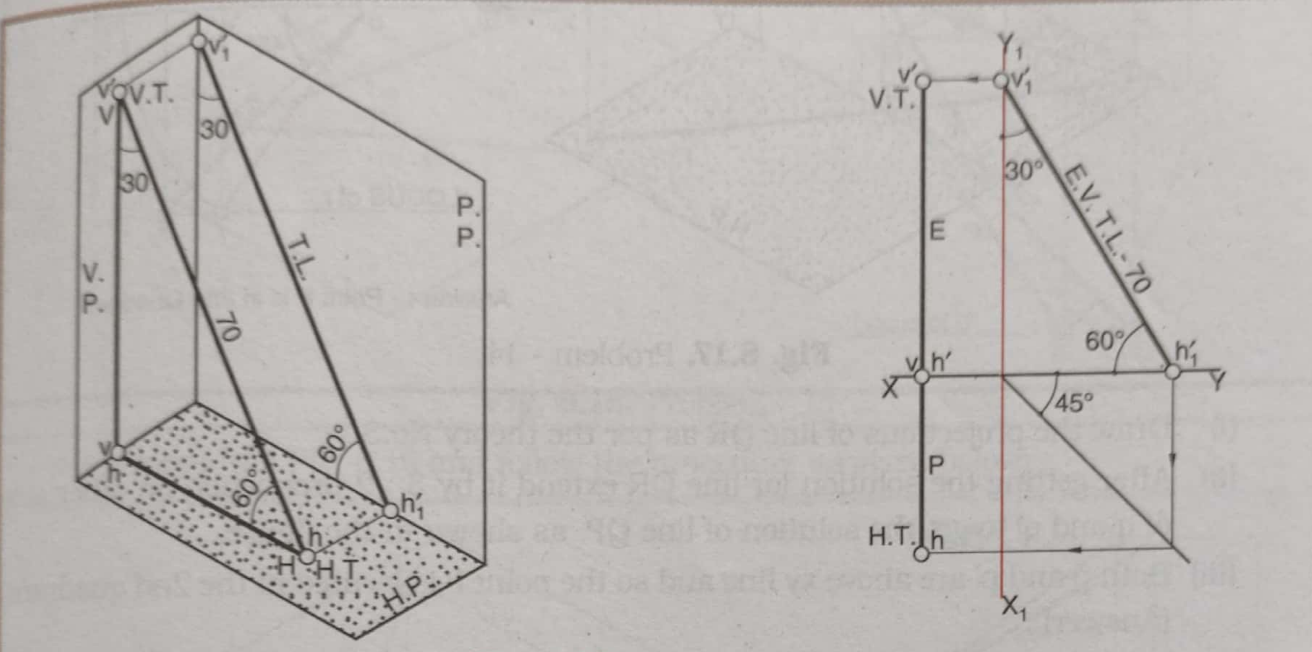


Fig. 8.16. Problem - 13.

Whenever (θ+Ø) = 90°, the line must be parallel to P.P. And when line is parallel to P.P., its end view will be true length (70) and it will show θ with xy line and Ø with x_1y_1 line. Distance between end projectors will be zero. Given data satisfy this.

(i) Draw xy and x_1y_1 lines mutually at right angle.

(ii) First draw end view $v'_1h'_1$ of 70 mm length in such a way that angle of $v_1'h_1'$ with xy line will be 60° and hence with x_1y_1 line 30°.

(iii) Draw v and h' on xy line since the point V is in V.P. and H is in H.P. Find v' and h by projection method, as studied earlier.

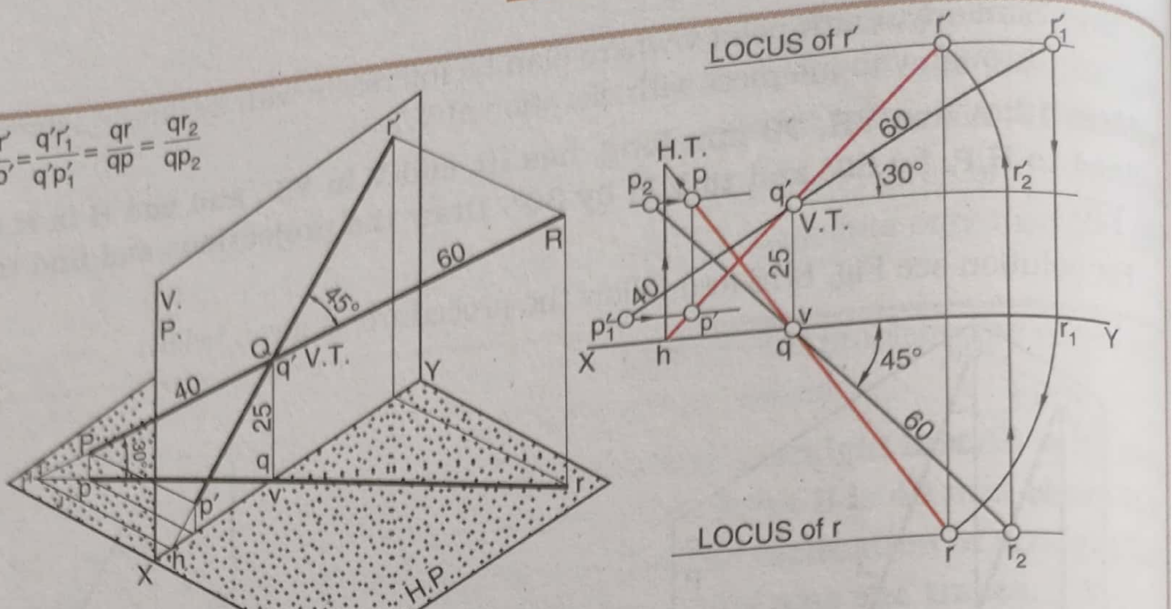
(iv) Point H (h) and point V (v') are H.T. and V.T. respectively.

Problem 14 : A line PQR, 100 mm long, is inclined to H.P. by 30° and V.P. by 45°. PQ : QR : 2 : 3. Point Q is in V.P. and 25 mm above H.P. Draw the projections of the line PQR when point R is in the 1st quadrant. Find the position of point P. Draw also the traces of line PQR.

For solution see Fig. 8.17 and follow the procedure as given below :

As position of point Q is given and as the quadrant of point R is given, do the solution first considering only the line QR. Length of QR will be 60 mm. So now for line QR, the point Q is completely given, T.L. = 60 mm, θ - 30° and Ø = 45°.

$$\frac{3}{2} = \frac{q'r'}{q'p'} = \frac{q'r'_1}{q'p'_1} = \frac{qr}{qp} = \frac{qr_2}{qp_2}$$



Answer : - Point P is in 2nd Quadrant.

Fig. 8.17. Problem - 14.

(i) Draw the projections of line QR as per the theory No.3.

(ii) After getting the solution for line QR extend it by 3 : 2 proportion on other side of q and q' to get the solution of line QP, as shown in the figure.

(iii) Both p and p' are above xy line and so the point P is located in the 2nd quadrant. (Answer)

(iv) Get the traces as usual.

Problem 15 : The top view and the front view, of the line CD, measure 65 mm and 53 mm respectively. The line is inclined to H.P. and to V.P. by 30° and 45° respectively. The end C is on the H.P. and 12 mm in front of V.P. Other end D is in the 1st quadrant. Draw the projections of the line CD and find its true length and draw traces.

For solution see Fig. 8.18 and follow the procedure as given below:

(i) First draw c and c', 12 mm below xy line and on xy line respectively.

(ii) Draw $c'd'_2$ = 53 mm parallel to xy and draw cd_1 = 65 mm parallel to xy. Their corresponding plan cd_2 and elevation c¢d¢$_1$ are achieved by drawing 45° angle line at c and 30° angle line at c' respectively. $c'd'_1$ and cd_2 both will give T.L.

(iii) From d'_1 and d_2 draw parallel lines to xy line to get locus of d' and locus of d respectively.

(iv) Now with c' as the centre and radius equal to 53 mm draw an arc to get d' on locus of d'. Similarly, with c as the centre and radius equal to 65 mm draw an arc to get d on locus of d. Join c'd' and cd.

(v) Find the traces as usual.

Problem 16 : A line EF, 75 mm long, has its end E 20 mm below H.P. and 25 mm behind V.P. The end F is 50 mm below H.P. and 65 mm behind VP. Draw the projections of line EF and find its inclinations with H.P. and V.P.

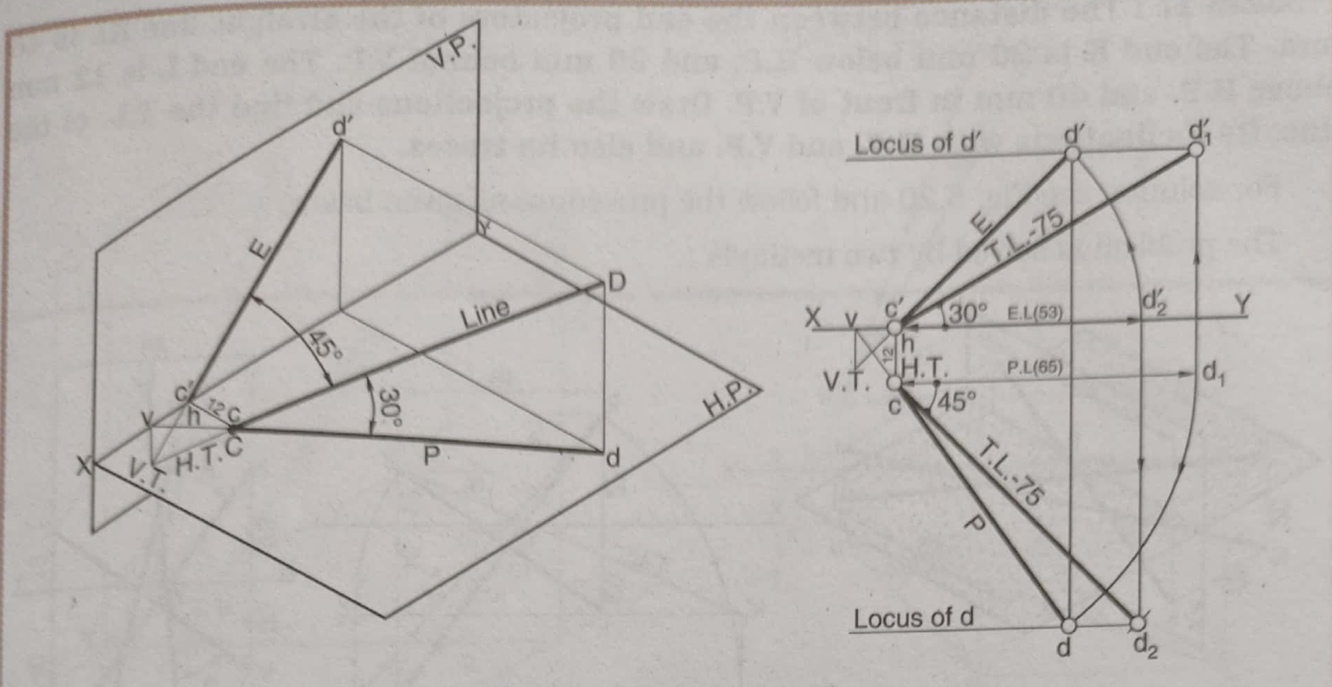
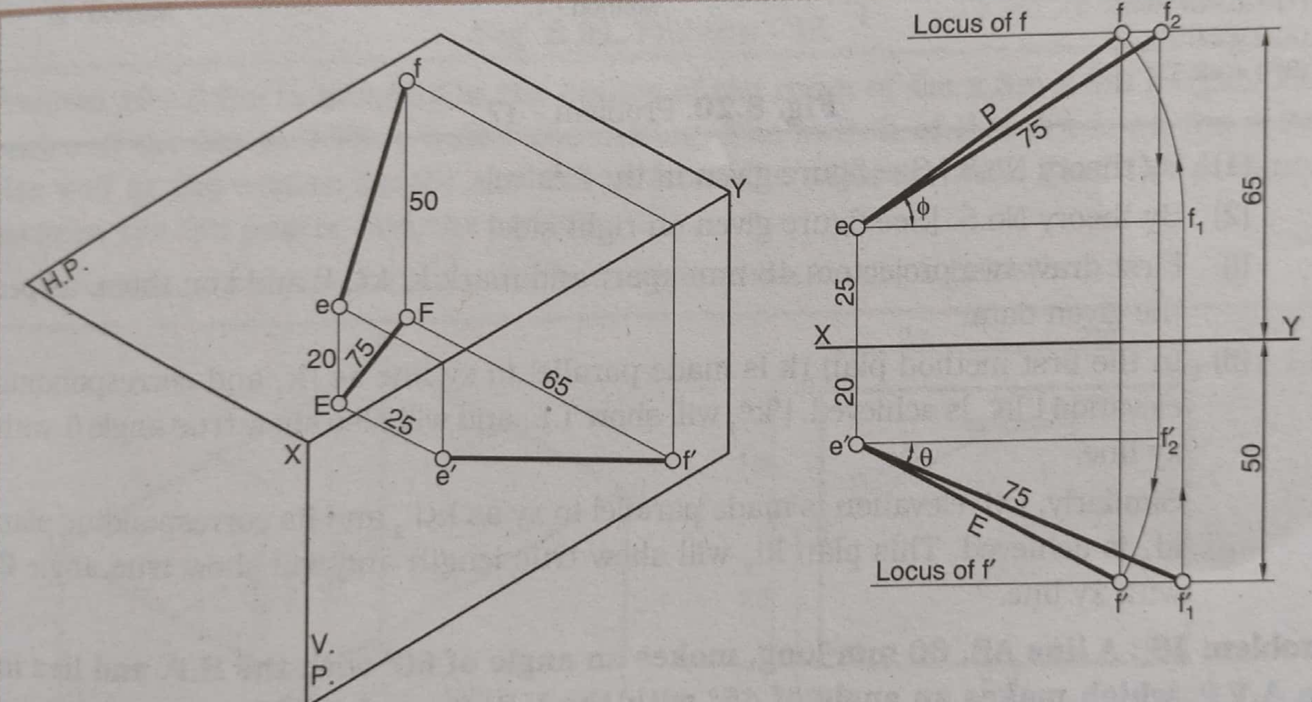


Fig. 8.18. Problem - 15

For solution see Fig. 8.19 and follow the procedure as given below:



[Answer - 1. θ = 23.5°
- 2. ϕ = 32°]

Fig. 8.19. Problem - 16.

(i) First of all mark e and e' and locus of f and locus of f¢, as per given position of the point E and the point F from two reference planes.

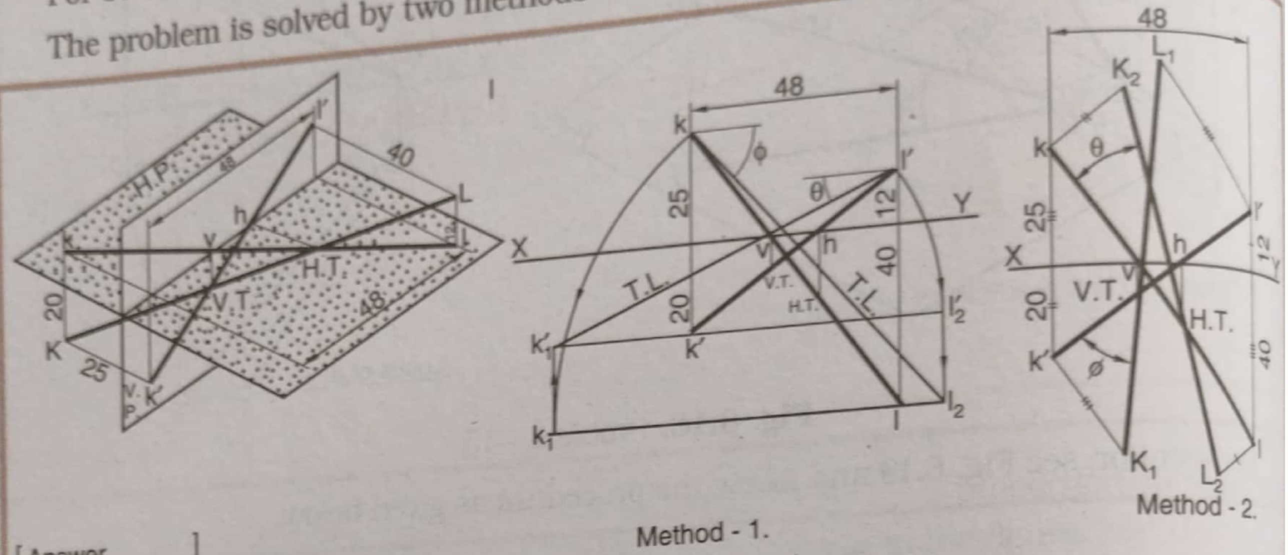
(ii) Now with e as the centre and radius equal to 75 mm (T.L.) draw an arc to cut locus of f at f_2. Join ef_2 and measure angle Ø that ef_2 makes with xy. Similarly, with e' as the centre and radius equal to 75 mm draw an arc to cut locus of f¢ at f¢$_1$. Join e¢f¢$_1$ and measure angle θ that e¢f¢$_1$ makes with xy.

(iii) Get plan ef and elevation e¢f¢ by the procedure, as done earlier by proceeding in the directions of arrows, as shown in Fig. 8.19.

Problem 17 : The distance between the end projectors of the straight line KL is 48 mm. The end K is 20 mm below H.P. and 25 mm behind V.P. The end L is 12 mm above H.P. and 40 mm in front of V.P. Draw the projections and find the T.L. of the line, its inclinations with H.P. and V.P. and also its traces.

For solution see Fig. 8.20 and follow the procedure as given below:

The problem is solved by two methods :



Answer
(1) T.L.- 87 mm
(2) $\theta = 21.5°$
(3) $\phi = 48.5°$

Fig. 8.20. Problem - 17.

(1) By theory No.3 [See figure given in the centre]

(2) By theory No.5. [See figure given on right side]

(i) First draw two projectors 48 mm apart and mark k, k¢, l' and l on them, as per the given data.

M.1 (ii) In the first method plan lk is made parallel to xy line as lk_1 and corresponding elevation $l'k¢_1$ is achieved. $l'k¢_1$ will show T.L. and will also show true angle θ with xy line.

Similarly, k¢l' elevation is made parallel to xy as $k¢l'_2$ and its corresponding plan kl_2 is achieved. This plan kl_2 will show true length and will show true angle ϕ with xy line.

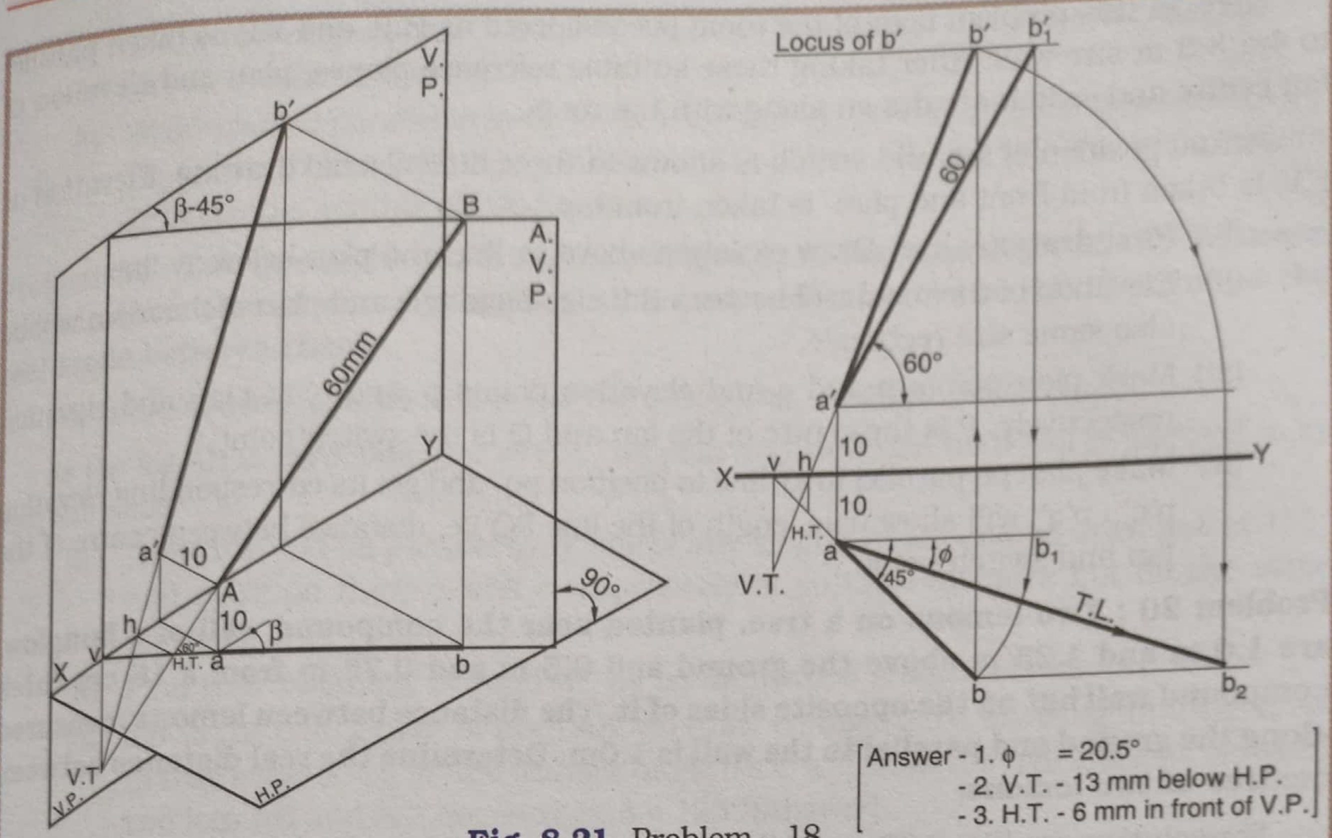
Problem 18 : A line AB, 60 mm long, makes an angle of 60° with the H.P. and lies in an A.V.P. which makes an angle of 45° with the V.P. Its end A is 10 mm away from both the H.P. and V.P. Draw the projections of AB and determine (i) its inclination with V.P. and (ii) its traces. Assume the line in the first quadrant.

For solution see Fig. 8.21 and follow the procedure, as given below :

Hints : As the line lies in an A.V.P. which makes an angle of 45° with the V.P., its plan will make an angle of 45° with xy. See 3-D drawing.

Proceed for solution in the direction of arrows given in the drawing.

Now we shall take few applied problems. In this type of problems generally earth is considered as H.P. or ground and according to situation prevailing in the problem a suitable vertical plane is selected.

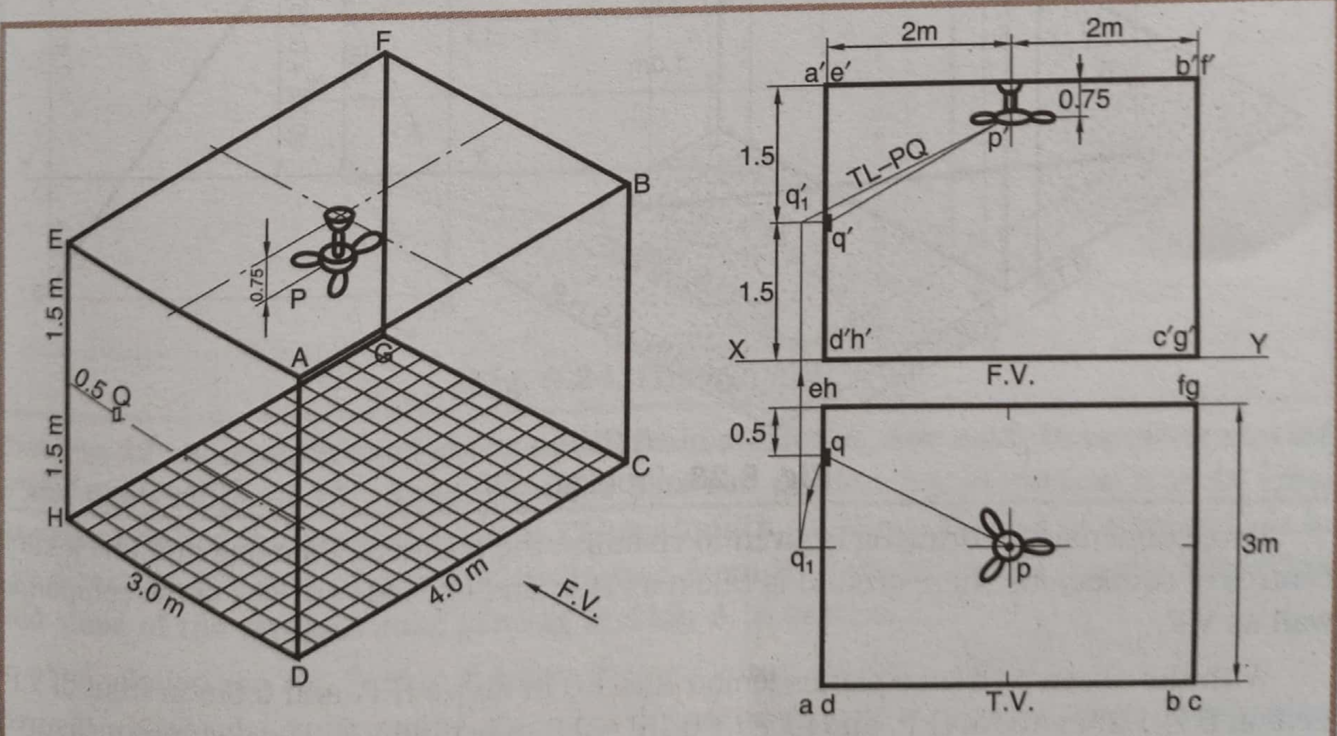


[Answer - 1. φ = 20.5°
- 2. V.T. - 13 mm below H.P.
- 3. H.T. - 6 mm in front of V.P.]

Fig. 8.21. Problem - 18.

Problem 19 : A fan is hanging in the centre of the room of 4m x 3m x 3m height. The centre of the fan is 0.75 m below the ceiling. The switch of this fan is on 3m x 3m size wall at the centre height and 0.5 m from the adjacent wall. Find the distance between the fan centre and the switch.

For solution see Fig. 8.22 and follow the procedure, as given below :



[Answer
(1) T.L. = 2.36 m.]

Fig. 8.22. Problem - 19.

Here, in this problem floor of the room is considered as H.P. and V.P. is taken paralle to 4m × 3 m size wall. After taking these suitable reference planes, plan and elevation of fan centre and switch are drawn along with the room.

Actual position of fan and switch is shown in three dimensional drawing. Elevation or F.V. is taken from front and plan is taken from top.

(i) First draw x-y line. Draw elevation above xy line and plan below xy line.

(ii) Elevation of the room will be 4m x 3m size rectangle and plan of the room will b also same size rectangle.

(iii) Mark plan points p and q and elevation points p' and q' in plan and elevatio respectively. P is the centre of the fan and Q is the switch point.

(iv) Make plan pq parallel to xy line to position pq_1 and get its corresponding elevatio $p'q'_1$. $p'q'_1$ will show true length of the line PQ i.e. distance between centre of th fan and switch point.

Problem 20 : Two lemons on a tree, planted near the compound wall of a bunglow are 1.0 m and 1.25 m above the ground and 0.5 m and 0.75 m from a 15 cm thic compound wall but on the opposite sides of it. The distance between lemons measure along the ground and parallel to the wall is 1.0m. Determine the real distance betwee centres of two lemons.

For solution see Fig. 8.23 and follow the procedure, as given below :

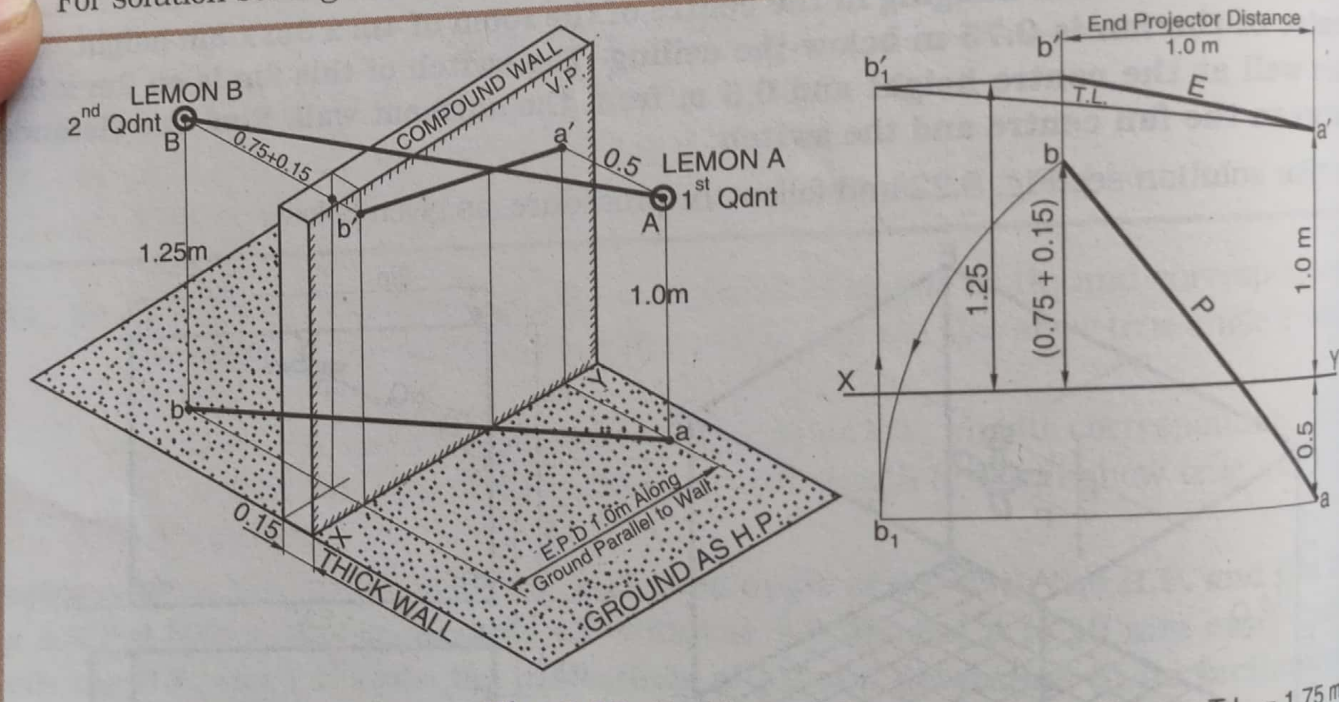


Fig. 8.23. Problem - 20. [Answer T.L. = 1.75 m

Three dimensional drawing is given to visualize the position of lemons and line joinin centres of two lemons. Here ground is taken as H.P. and one side surface of the compour wall as V.P.

With the above reference planes lemon A is 1.0 m above H.P. and 0.5m in front of V.P. Lemon B is 1.25m above H.P. and (0.75 + 0.15) = 0.9 m behind V.P. The distance measure along the ground and parallel to the wall will be nothing but end projectors distance for lin AB.

(i) Draw two projectors 1.0 m apart. Mark (b) and (b') on one projector and (a) and (a') on another projector.

(ii) Make plan ab parallel to xy to position ab_1 and get its corresponding elevation $a'b'_1$, as shown in Fig. 8.30. $a'b'_1$ will show T.L. of line AB and will give real distance between the centres of two lemons.

Problem 21 : Two unequal legs AB and AC, hinged at A, make an angle of 135° between them in their elevation and plan. Leg AB is perpendicular to the P.P. Determine the real angle between them.

For solution see Fig. 8.24 and follow the procedure, as given below :

As the leg AB is perpendicular to P.P., its plan and elevation both will be parallel to xy and will show T.L.

(i) Draw b'a' and ba parallel to xy line of any length. At (a') and (a) draw line at 135° and mark on them c' and c respectively at suitable distance but on the same projector.

(ii) Find true lengths of AC and BC by usual method studied earlier.

(iii) a'b' is the true length of lines AB. So draw triangle a'b'C showing all three lines in true lengths. So in this triangle angle b'a'C will show the real angle (θ) between two legs AB and AC. Measure it. θ = 125° (Answer).

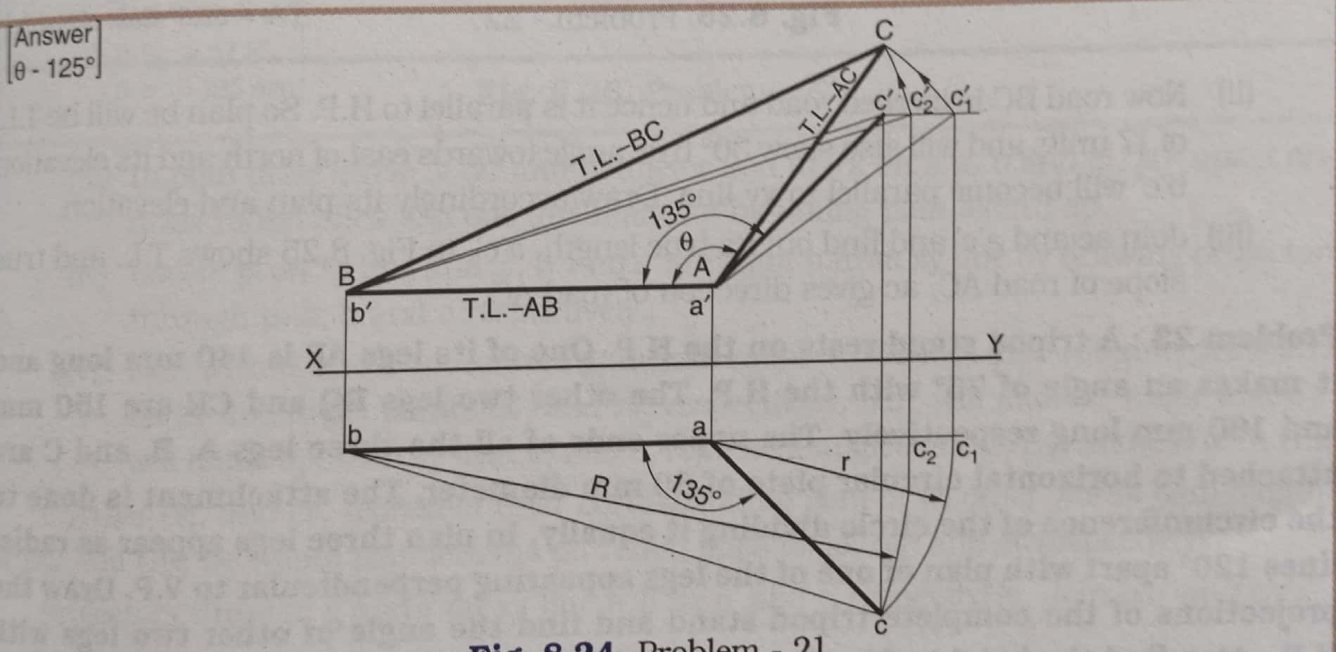


Fig. 8.24. Problem - 21.

Problem 22 : A straight road going uphill from station A, due east, to another station B and has a slope of 15°. Road distance between station A and station B is 14 kms. Another levelled (horizontal) road of 17 kms length, to join station B to station C, is in the direction 30° east of north when looked from B. Determine the length, direction and slope of the straight road joining station A to station C.

For solution see Fig. 8.25 and follow the procedure, as given below :

(i) Here take earth as the H.P. and the vertical plane along east-west direction as V.P. So road AB, as it is going towards east, will become parallel to V.P. and hence a'b' will show T.L. 14 kms and will show true angle 15° with xy line and its plan ab will become parallel to xy line.

So draw a'b' of 14 units at 15° with xy and draw corresponding plan ab parallel to xy line.

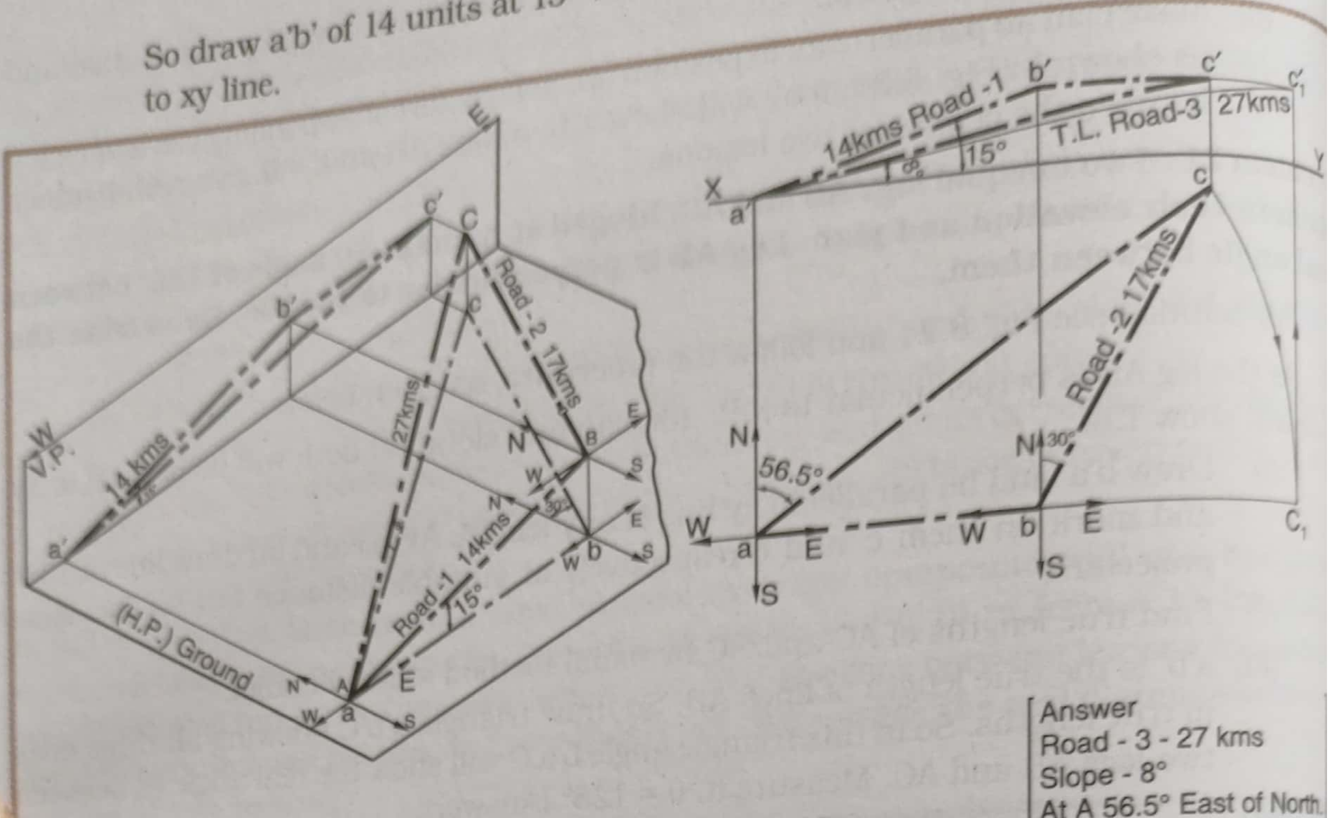


Fig. 8.25. Problem - 22.

(ii) Now road BC is levelled road and hence it is parallel to H.P. So plan bc will be T.L. of 17 units and will also show 30° true angle towards east of north and its elevation b'c' will become parallel to xy line. Draw accordingly its plan and elevation.

(iii) Join ac and a'c' and find out its true length. $a'c'_1$ in Fig. 8.25 shows T.L. and true slope of road AC. ac gives direction of road AC.

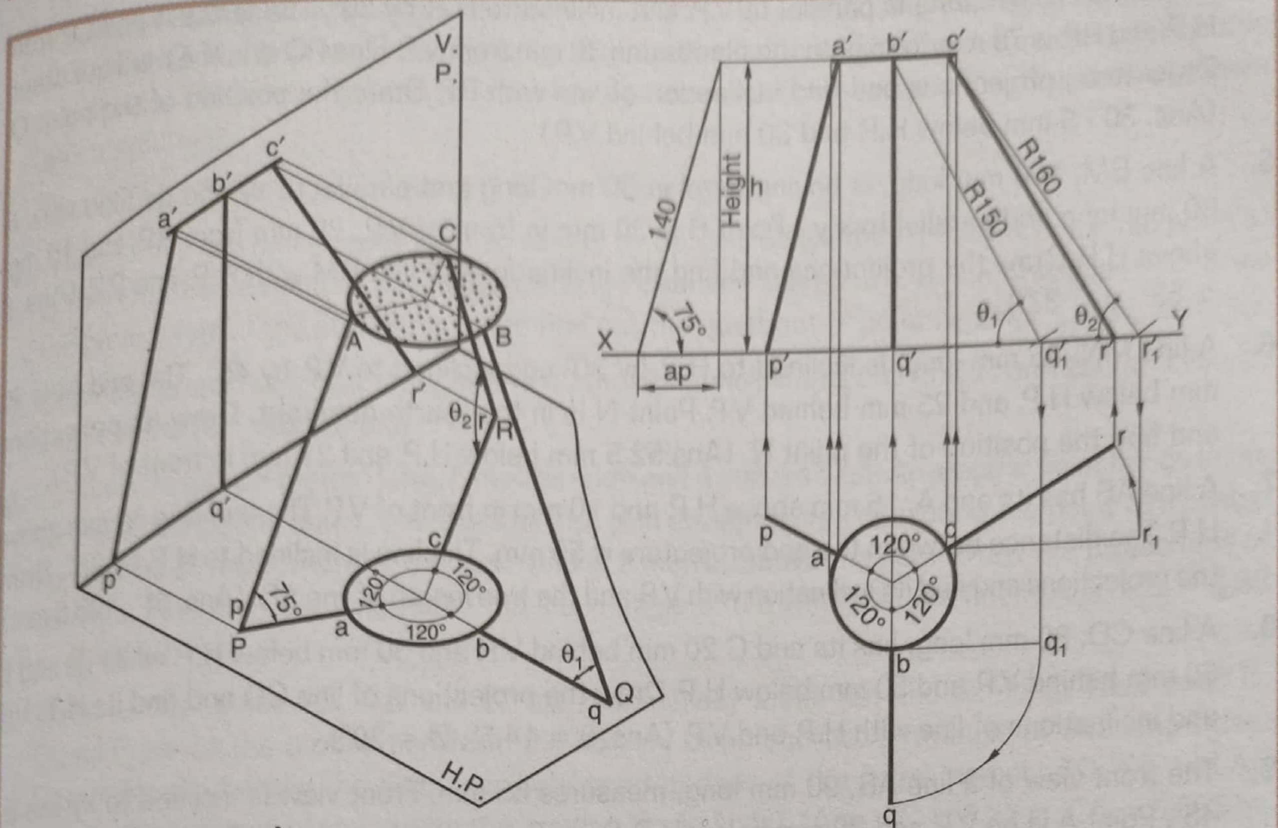
Problem 23 : A tripod stand rests on the H.P. One of its legs AP is 140 mm long and it makes an angle of 75° with the H.P. The other two legs BQ and CR are 150 mm and 160 mm long respectively. The upper ends of all the three legs A, B, and C are attached to horizontal circular plate of 60 mm diameter. The attachment is done to the circumference of the circle dividing it equally. In plan three legs appear as radial lines 120° apart with plan of one of the legs appearing perpendicular to V.P. Draw the projections of the complete tripod stand and find the angle of other two legs with H.P. Also find the height of horizontal circular plate from H.P.

For solution see Fig. 8.26 and follow the procedure, as given below :

Three dimensional drawing is given to visualize the tripod stand and its views on H.P. and V.P. As the stand rests on three legs on H.P. p', q' and r' will be on xy line. Further top surface is horizontal circular plate and so its plan will be circle (abc) and its elevation (a'b'c') will be a straight line parallel to xy line. In plan ap, bq and cr will be radial lines 120° apart out of which bq will be perpendicular to xy line.

(i) Draw xy.

(ii) Draw 140 mm long line at 75° to xy line to get height h and plan length ap, as shown in Fig. 8.33.



[Answer - 1. θ_1 = 64.5°
- 2. θ_2 = 57.5°
- 3. h = 135 mm]

Fig. 8.26. Problem - 23.

(iii) In plan draw circle of 60 mm diameter and mark on it a, b and c 120° apart and draw radial lines. On one line mark ap by taking plan length ap.

(iv) Locate p' on xy line and a', b' and c' at height h from xy line by drawing projectors through p, a, b and c respectively.

(v) Now with b' and c' as centres and radii equal to 150 mm and 160 mm draw arcs on xy line to get points q¢$_1$ and r¢$_1$ respectively. b'q'$_1$ will make θ_1 angle and c'r'$_1$ will make θ_2 , with xy line. θ_1 and θ_2 represent angles of legs BQ and CR with H.P. Further bq$_1$ and cr$_1$ represent their corresponding plan or in other words plan lengths of BQ and CR. So rotate plan lengths on radial lines to get bq and cr respectively. Projectors through q and r on xy line will fix q' and r' respectively. Join b'q' and c'r'. Project also extreme two points of the circle.

EXERCISE

1. A line AB is 75 mm long. It is perpendicular to profile plane. The point A is 30 mm above H.P. and 40 mm in front of V.P. Draw the projections.
2. A line CD, 70 mm long, is parallel to V.P. and P.P. Point C is nearer to H.P., 35 mm below H.P. and is behind V.P. Draw the projections.
3. A line EF, 80 mm long, is having its elevation point view. Point E is 25 mm above H.P. and 30 mm in front of V.P. and the point F is in the second quadrant. Draw its projections and find the ratio of length of line in first quadrant to the length in the second quadrant. (Ans. 3 : 5).

4. A line PQ, 75 mm long is parallel to V.P. and inclined to H.P. by 60°. The farthest point P from H.P. and P.P. is 70 mm from both the planes and 30 mm from V.P. Line PQ is in the third quadrant. Draw three projections and find inclination of line with P.P. State the position of the point Q. (Ans. 30°, 5 mm below H.P. and 30 mm behind V.P.)

5. A line BM, 100 mm long, is having its plan 60 mm long and parallel to xy and its side view is 80 mm long and parallel to x_1y_1. Point B is 30 mm in front of V.P., 80 mm from P.P. and 10 mm above H.P. Draw the projections and find the inclinations of line BM with H.P. and P.P. (Ans. θ = 53°, ψ = 37°).

6. A line MN, 65 mm long, is inclined to H.P. by 30° and inclined to V.P. by 45°. The end M is 20 mm below H.P. and 25 mm behind V.P. Point N is in the fourth quadrant. Draw its projections and find the position of the point N. (Ans.52.5 mm below H.P. and 21 mm in front of V.P.).

7. A line AB has its end A, 15 mm above H.P. and 10 mm in front of V.P. The end B is 60 mm above H.P. The distance between the end projectors is 50 mm. The line is inclined to H.P. by 25°. Draw the projections and find its inclination with V.P. and the true length of line AB. (Ans. 51°; 106.5 mm)

8. A line CD, 80 mm long, has its end C 20 mm behind V.P. and 30 mm below H.P. while its end D 60 mm behind V.P. and 50 mm below H.P. Draw the projections of line CD and find its H.T., V.T. and inclinations of line with H.P. and V.P. (Ans. θ = 14.5°; Ø = 30°).

9. The front view of a line AB, 90 mm long, measures 65 mm. Front view is inclined to xy line by 45°. Point A is on V.P. and 20 mm below H.P. Point B is in third quadrant. Draw the projections and find inclinations of line with V.P. and H.P. Also find traces. (Ans. Ø = 44°, θ = 30.5°).

10. A line ABC, 120 mm long, has point B on xy line. AB:BC:::4:6. Its end A is in the first quadrant and B is in the third quadrant. The line is inclined to H.P. by 45° and to V.P. by 30°. Draw the projections of the line and measure elevation and plan length and state where is H.T. and V.T. (Ans : 104 mm ; 85 mm; H.T. and V.T. will be on xy line).

11. Distance between the end projectors of line CD is 75 mm. Points C and D are 25 mm and 7 mm below H.P. respectively. The line is inclined to V.P. by 30° and its V.T. is 50 mm below H. Draw the projections of CD and find the true length of line and its inclination with H.P. and H. Assume the point C in the third quadrant. (Ans T.L. = 104 mm ; θ = 28.5°)

12. A room is 5 m x 4.5m x 4 m high. Determine by method of projections of straight lines, distance between diagonally (solid) opposite corners of the room. (Ans. 7.83 metre).

13. The distance between end projectors of a straight line PQ is 130 mm. Point P is 40 mm below H.P. and 25 mm in front of V.P. Point Q is 75 mm above H.P. and 30 mm behind V.P. Draw the projections and find out its T.L., θ, Ø, H.T. and V.T. (Ans.T.L. =182 mm, θ = 39°, Ø = 17.5°)

14. Two mangoes A and B on a tree are 0.5 m and 1 m above the ground respectively. P and Q are two compound walls at right angle. Mango A is 0.30 m from the wall P and 0.6m from the wall Q. Mango B is 1.5m from the wall P and 2 m from the wall Q. Draw three projections of the mangoes and find the real distance between their centres. Assume both the mangoes inside the compound.

Also find distances of mangoes A and B from the corner, where two walls and the ground meet (Ans.1.91 m; 0.837 m; 2.69 m).

15. Gurushikhar (P), Achalgadh (Q) and Delwada (R) are three hill stations 4000 m, 3000 m, 2000 above the sea level respectively. They are connected by rope-way with each other. Hill statio

P, Q and R are seen at an angle of elevation of 25°, 20° and 15° respectively from a station in the valley O at sea level. From the point O, P is due south-west, Q is due north and R is due north-west. Find the lengths of the rope for rope-way. Assume ropes tight and so consider them as straight lines.

(Ans. PQ-15.57 km; QR-6.7 km and RQ = 11.5 kms).

16. A line LMN, 120 mm long, is inclined to V.P. by 30° and inclined to H.P. by 45°. Its mid point M is the V.T. of a line and is 20 mm above H.P. Assuming the point L in the first quadrant, draw the projections of the straight line and find out the quadrant of point N. (Ans. Third quadrant).

17. Draw the projections of a straight line 80 mm long inclined at 60° to H.P. and 30° to V.P. with end A in the H.P. and the end B in V.P.

18. A room measures 8 metres long, 5 metres wide and 4 metres high. An electric point hangs in the centre of the ceiling and 1 metre below it. A thin straight wire connecting the point to a switch kept in one of the corners of the room and is 2 metres above the floor. Draw the projections of the wire. Find the true length and the slope angle of the wire with the floor. (Ans. T.L. = 4.82 m, $\theta = 12°$).

19. Three vertical poles AB, CD and EF are respectively 2.5m, 4m and 6m long. Their ends B, D and F are on the ground and form the corners of an equilateral triangle of 5 m long sides. Determine graphically the distances between top ends of the poles, namely AC, CE and AE also the inclinations of these with the ground. Scale 1 : 50. (Ans. AC = 5.22 m, CE = 5.385 m, AE = 6.1m; $\theta_{AC} = 16.7°$, $\theta_{CE} = 21.8°$, $\theta_{AE} = 35°$)

20. Three points A, B and C are connected with each other by rods. A is 4 m above the gorund level, B is on the ground and C is 5 m below the ground level. The angles of depression of A, B and C when seen from a point O on a hillock 10 m above the ground are respectively 10°, 20° and 30°. A is 60° towards east of north, B is due north and C is due south east of O. Find the lengths of the connecting rods. Scale 1 : 100. (Ans. AB = 31.53 m ; BC = 49.64 m ; AC = 38.16 m).

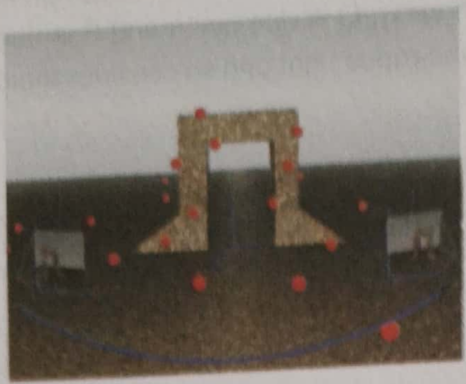
21. Two fan motors hang from the ceiling of a hall, 12 m x 5m x 8m high at a height of 3 m from the floor. The motors are 3 m and 9 m from the end wall, 2 m and 3 m from the front wall. Determine graphically the distance of each motor from a corner of the hall of the floor. (Ans. 4.7 m; 5.2m ; 9.7 m and 9.95 m).

22. A line MN measures 120 mm. Its top and front views measure 80 mm and 96 mm respectively. A point P on the line, dividing it in the ratio of 1 : 2 i.e, MP : PN = 1 : 2, is contained by both the reference planes. Draw the projections of line, and determine its traces and inclinations with the reference planes. (Ans. $\theta = 48°$; Ø = 37°).

23. A chimney of a hostel kitchen is 15 m high and 0.5 m in diameter. This chimney is supported by three guy wires AP, BQ and CR which appear in plan at equal angles to each other. The ends P,Q and R are pegged to the ground at distances 2.5 m, 3.5 m and 4.5 m respectively from the centre of the chimney. The other ends A, B and C of wires are connected to a ring 3 m below the top end of the chimney. Draw the projections and find the lengths of the three guy wires. (Ans. 12.2 m; 12.43m ; and 12.73 m).

24. Two legs of the divider AB and AC are seen at 135° between them in their end elevation and plan. Leg AB is perpendicular to V.P. Draw the projections and find the real angle between two legs of the divider. (Ans. 125°).

CHAPTER - 9



Projections of Planes

1. INTRODUCTION :

A plane surface has only two dimensions. Third dimension thickness of a plane surfa is negligible and assumed to be zero.

Planes are of three types. They are classified according to their three different types position in space with respect to three principal planes, H.P., V.P. and P.P.

(1) Parallel to one and perpendicular to two other principal planes

(2) Perpendicular to one and inclined to two other principal planes

(3) Oblique or inclined to all P.P s.

2. TRACES OF A PLANE :

Just like traces of a line, planes also have traces. A plane, extended if necessary, w meet principal planes along straight lines unless it is parallel to any one of them. Th straight lines are known as traces. Plane will meet, H.P. along horizontal trace (H.T.), V along vertical trace (V.T.) and P.P. along profile trace (P.T.).

3. PLANE PARALLEL TO ONE AND (HENCE) PERPENDICULAR TO TWO OTHER P.

We shall study projections of these types of planes by taking few concrete examples 1st Angle System.

Problem 1 : A circular plate, 30 mm in diameter, is parallel to V.P. Its centre is 20 m above H.P. and 22 mm in front of V.P. Draw its three projections and find its trac

For solution see Fig. 9.1 and follow the procedure as given below. Solution is done 1st angle system.

Looking at the three dimensional drawing of the circular plate in the given positi we can visualize that Elevation will be a circle of the same size. Plan will be straight l parallel to xy line and side view will be straight line parallel to x_1y_1 line. Plan and end vi will have length equal to the diameter.

(i) First plot O' and O, 20 mm above xy and 22 mm below xy respectively.

(ii) With O' as the centre and radius equal to 15 mm draw a circle to get elevatio

(iii) Through O draw line parallel to xy and project extreme points a' and c' of t circle of elevation to get plan.

(iv) Get side view by the usual method.

(v) Since the plane is parallel to V.P. it will not have V.T. Its plan will be H.T. and its side view will be P.T.(Profile Trace).

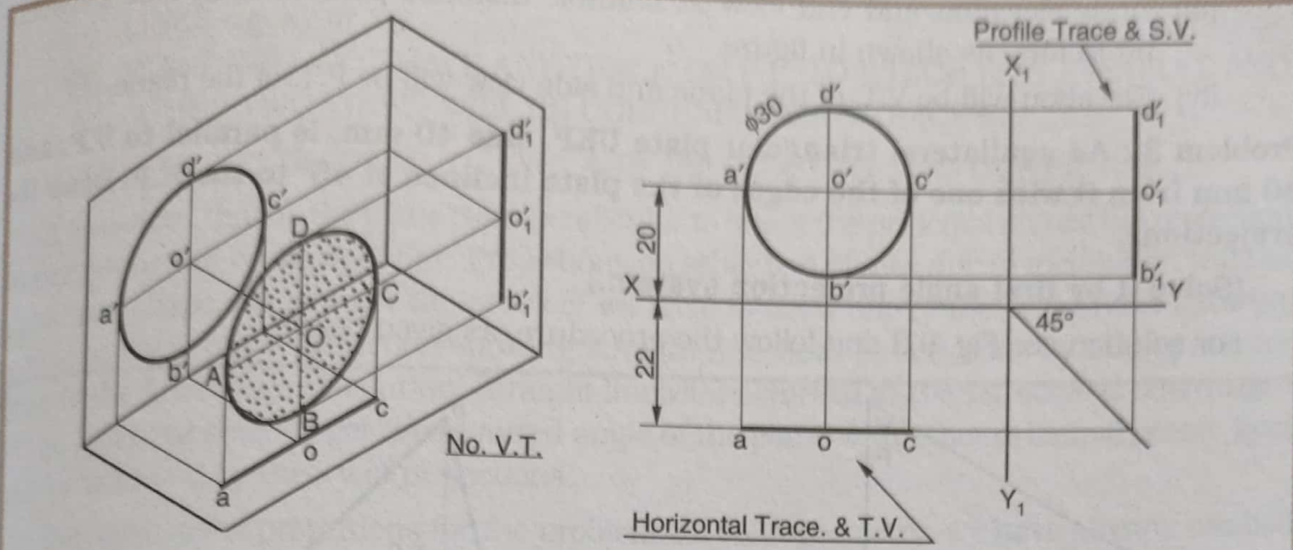


Fig. 9.1. Problem1. Plane Parallel to V.P.

Problem 2 : A square plate ABCD, side 25 mm is parallel to H.P. and is in the first quadrant. One of the edges of square plate is parallel to V.P. Draw its three projections and find out its traces.

For solution see Fig. 9.2 and follow the procedure, as given below

In three dimensional drawing, square plate is shown parallel to H.P. It is seen that plan will show true shape (square of 25 mm side) and its elevation and end view will give straight lines.

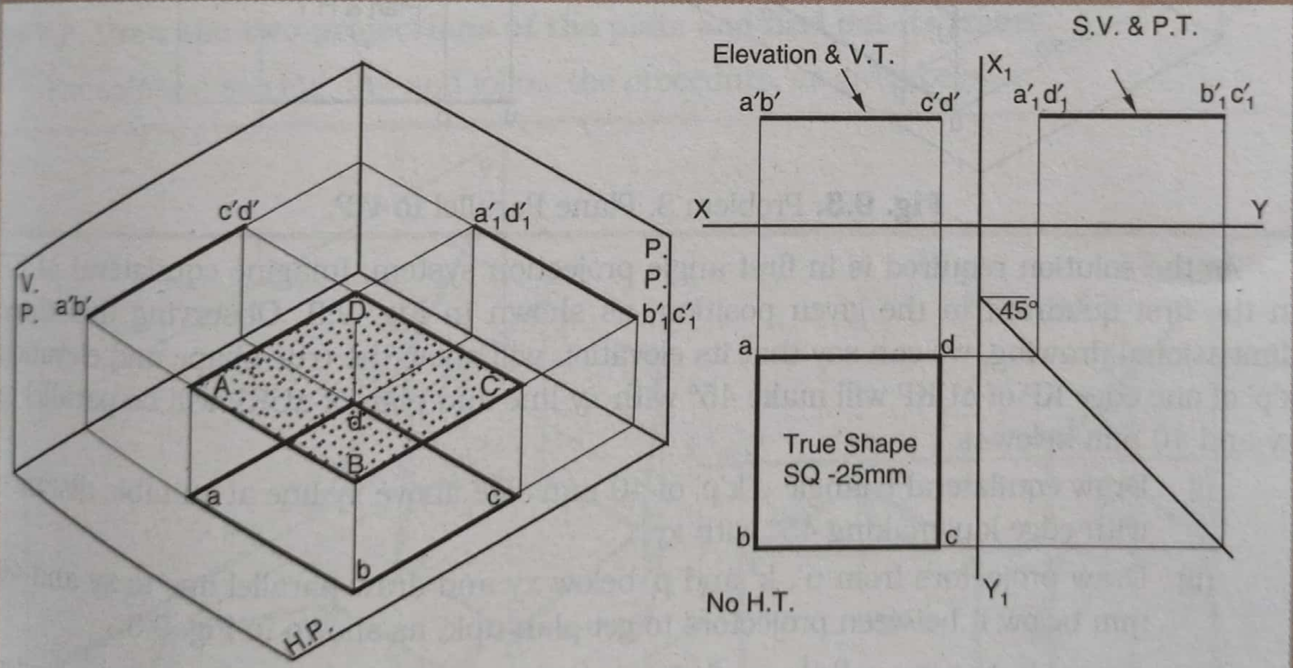


Fig. 9.2. Problem 2 Plane Parallel to H.P.

Further as the plate is in 1st quadrant, plan will be below xy and elevation and end view will be above xy line.

(i) Draw XY and X_1Y_1 lines mutually at right angle.

(ii) Draw plan (square of 25 mm side) below xy with edge parallel to xy at suitable distance and project its corners a,b,c and d for elevation above xy and end view above xy but through x_1y_1 line as per the usual method.

(iii) Draw elevation and end view at suitable distance parallel to xy line between projectors, as shown in figure.

(iv) Elevation will be V.T. of the plane and side view will be P.T. of the plane.

Problem 3 : An equilateral triangular plate UKP, side 40 mm, is parallel to V.P. and 40 mm from it with one of the edges of the plate inclined at 45° to the H.P. Draw its projections.

(Solve it by first angle projection system.)

For solution see Fig. 9.3 and follow the procedure, as given below:

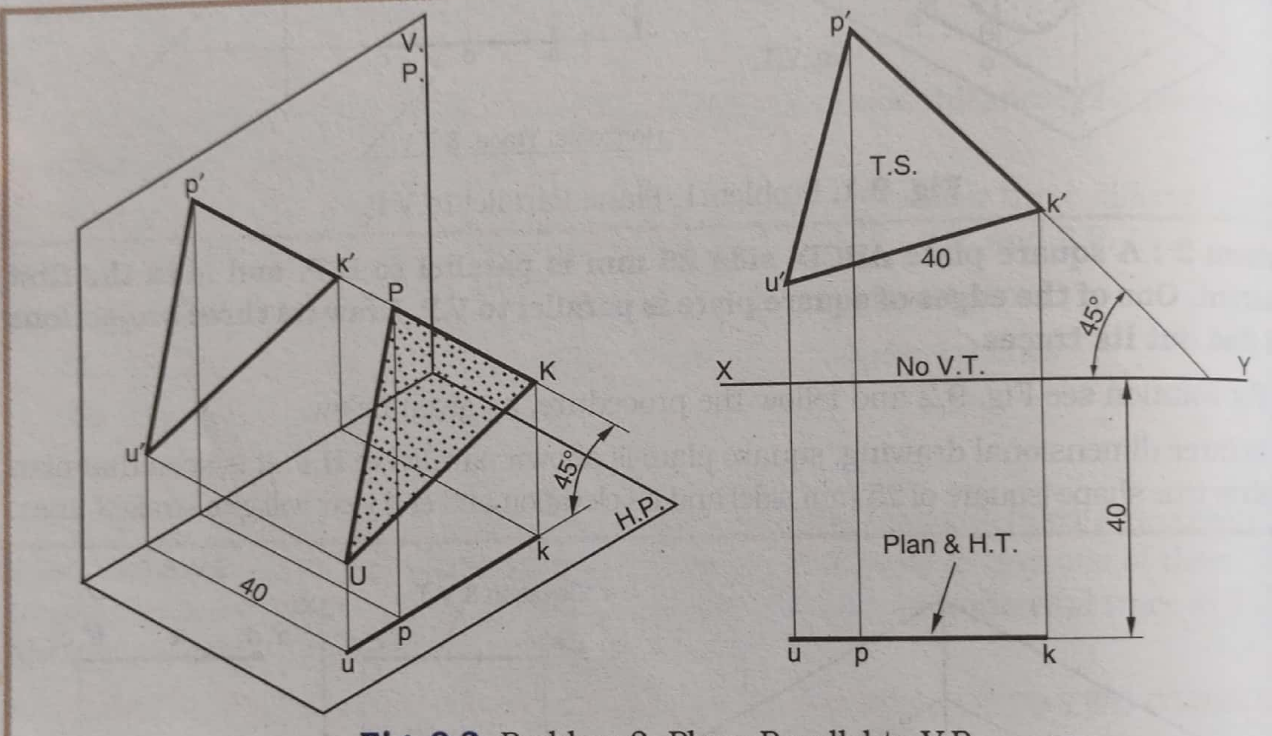


Fig. 9.3. Problem 3. Plane Parallel to V.P.

As the solution required is in first angle projection system, imagine equilateral ΔUKP in the first quadrant in the given position, as shown in Fig. 9.3. Observing this three dimensional drawing, we can say that its elevation will show the true shape and elevation k'p' of one edge KP of ΔUKP will make 45° with xy line and plan of ΔUKP will be parallel to xy and 40 mm below it.

(i) Draw equilateral triangle u'k'p' of 40 mm side above xy line at suitable distance with edge k'p' making 45° with xy.

(ii) Draw projectors from u', k' and p' below xy and draw parallel line to xy and 40 mm below it between projectors to get plan upk, as shown in Fig. 9.3.

Critically analysing the solutions of above three problems, we can conclude as under for plane parallel to one of the P.Ps.

(i) IF A PLANE IS PARALLEL TO ONE OF THE PRINCIPAL PLANES, IT WILL BE PERPENDICULAR TO TWO OTHER P.Ps.

(ii) PROJECTION OF PLANE ON A PRINCIPAL PLANE TO WHICH THE PLANE

PARALLEL WILL SHOW TRUE SHAPE AND TRUE ANGLES OF EDGES WITH OTHER PRINCIPAL PLANES.

(III) PROJECTIONS ON PRINCIPAL PLANES TO WHICH PLANE IS PERPENDICULAR WILL BE STRAIGHT LINES AND PARALLEL TO CORRESPONDING GROUND LINES e.g. xy or x_1y_1.

(iv) PLANE WILL NOT HAVE ANY TRACE ON P.P. TO WHICH IT IS PARALLEL AND OTHER TWO VIEWS WILL BE CORRESPONDING TRACES.

4. PLANE PERPENDICULAR TO ONE AND INCLINED TO TWO OTHER P.Ps.

It is obvious that as the plane is perpendicular to one of the principal planes the projection on that plane will be straight line. Projections on other two planes due to inclination will not show true shape. So to start the problem we have to draw one of the projections showing true shape and other projection showing straight line view. This is done by taking a stage prior to the final stage of solution. Straight line view achieved in the 1st stage is rearranged in the 2nd final stage to get the required angle of the plane with other principal plane. Rest can be achieved by theory of projections.

Two systems of projections for the problems of solid geometry, we have already studied earlier in the chapter of projections, planes of projections and system of projections. Students are requested to go through that before going for solution of problems.

Students are also requested to study the geometry of equilateral triangle, square, regular pentagon, regular hexagon, circle, ellipse, rhombus, etc. and they must do practice of drawing them quickly by the use of board, tee and set-squares.

Now we shall study five problems of this category and arrive at certain conclusions.

Problem 4 : A regular hexagonal plate ABCDEF, 25 mm side, is resting on H.P. on one of the sides/edges with surface of the plate making 45° with H.P. and perpendicular to V.P. Draw the two projections of the plate and find out its traces.

For solution see Fig. 9.4 and follow the procedure, as given below :

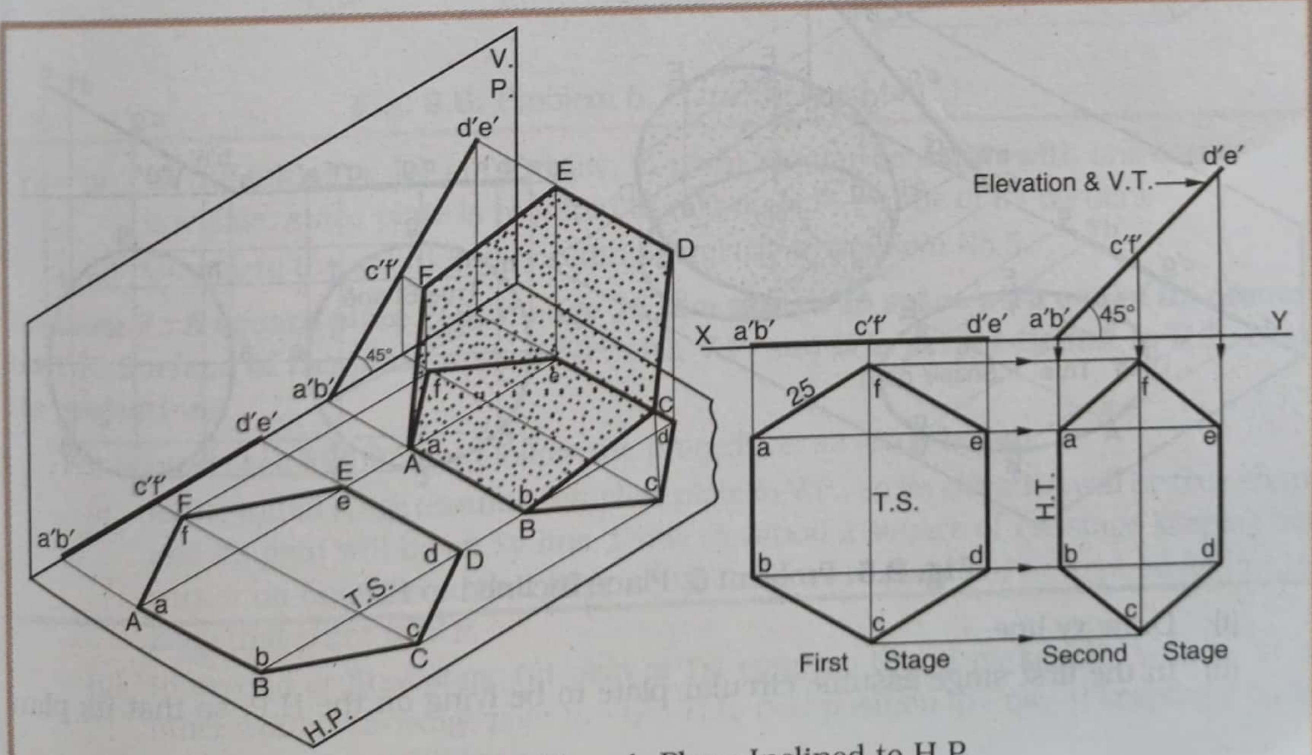


Fig. 9.4. Problem 4. Plane Inclined to H.P.

Whenever it is given that object, plane, solid, etc. rests on H.P. means the object, plane or solid is in the first quadrant, just as it is shown in Fig. 9.4 Whenever it rests on ground instead of H.P., it is in third quadrant.

In first angle system only xy line is drawn. Plan is drawn below xy and elevation is drawn above xy.

In third angle system two parallel ground lines are drawn (1) xy line (2) G.L. line below xy. Elevation is drawn between G.L. line and xy line and plan is drawn above xy line.

(i) First draw regular hexagon a b c d e f, which is the plan of regular hexagonal plate A B C D E F kept on H.P. with one edge out of six perpendicular to xy line since plate is required to rest on H.P. on one of its edges. By keeping one edge a b perpendicular to xy line elevation of that edge a' b' gives us point view (a'b') is kept on xy, to keep two end points (A and B) of edge, i.e. edge, on H.P.

Get on xy line elevation [a¢b¢ – c¢f¢ – d¢e¢] by projecting this true shaped plan.

(ii) Rearrange this straight line elevation to new position (a'b' – c¢f¢ – d¢e¢) keeping a'b' on xy line and straight line elevation making 45° with xy line.

Project all points vertically downward and project all plan points of 1st stage horizontally to get by intersection plan points a, b, c, d, e and f, as shown in Fig.9.4 Join them to get plan.

(iii) Elevation and plan of 2nd stage is the answer. Projections of stage one is merely construction.

(iv) H.T. and V.T. are already shown in the figure.

Problem 5 : A circular plate, 50 mm diameter, is resting on H.P. on one of the points of its periphery with surface of the plate perpendicular to V.P. and inclined to H.P. by 30° . Draw the two projections of the circular plate.

For solution see Fig. 9.5 and follow the procedure, as given below:

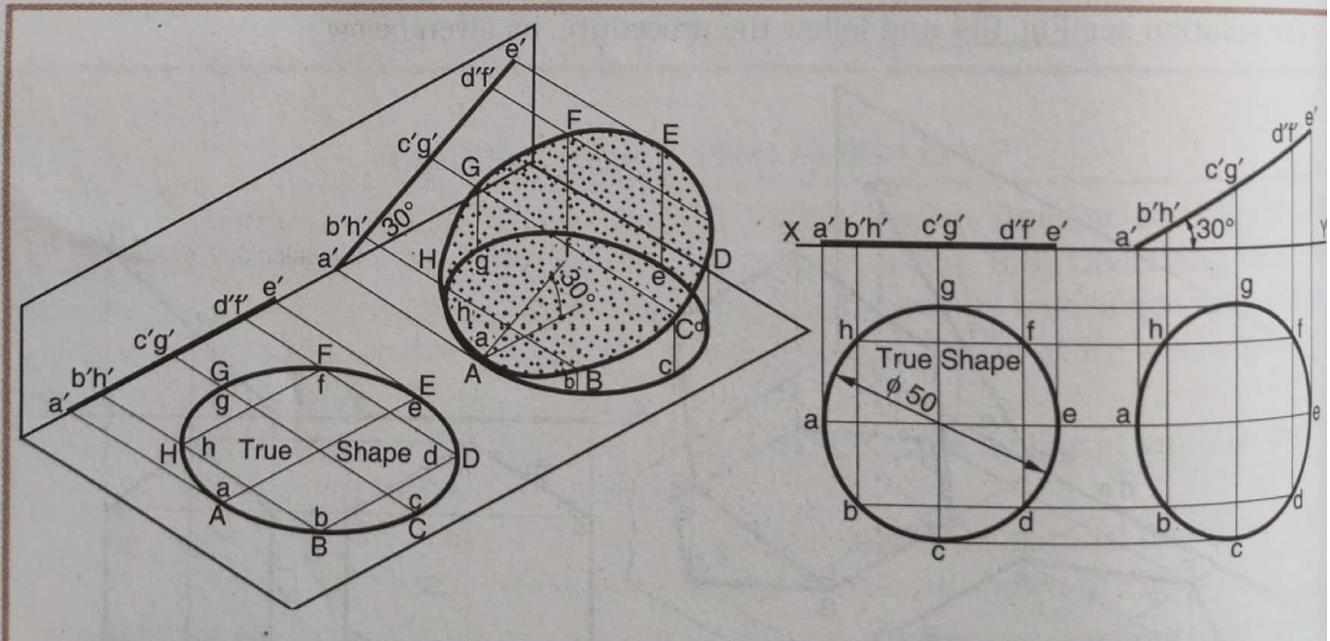


Fig. 9.5. Problem 5. Plane Inclined to H.P.

(i) Draw xy line.

(ii) In the first stage assume circular plate to be lying on the H.P. so that its pla will be a circle of the same size and its elevation will be straight line on xy line. First draw top view or plan (circle) and project elevation on xy line. Take help of 8 points equally spaced on its periphery.

(iii) Tilt circular plate about the point A by 30° or in other words rearrange straight line view of elevation to new position (a' - b'h' - c'g' - d'f' - e') so that it makes 30° with the xy line and point a' remains on xy line.

(iv) Draw projectors from elevation and horizontal lines from plan of previous stage to get by intersection new plan points a,b,c,.... h. Join them by smooth curve to get plan.

Problem 6 : A regular pentagonal plate ABCDE, 25 mm edge/side size, is resting on H.P. on one of its corners with surface of the plate perpendicular to V.P. and inclined to H.P. by 60°. Draw its two projections.

For solution see Fig. 9.6 and follow the procedure, as given below:

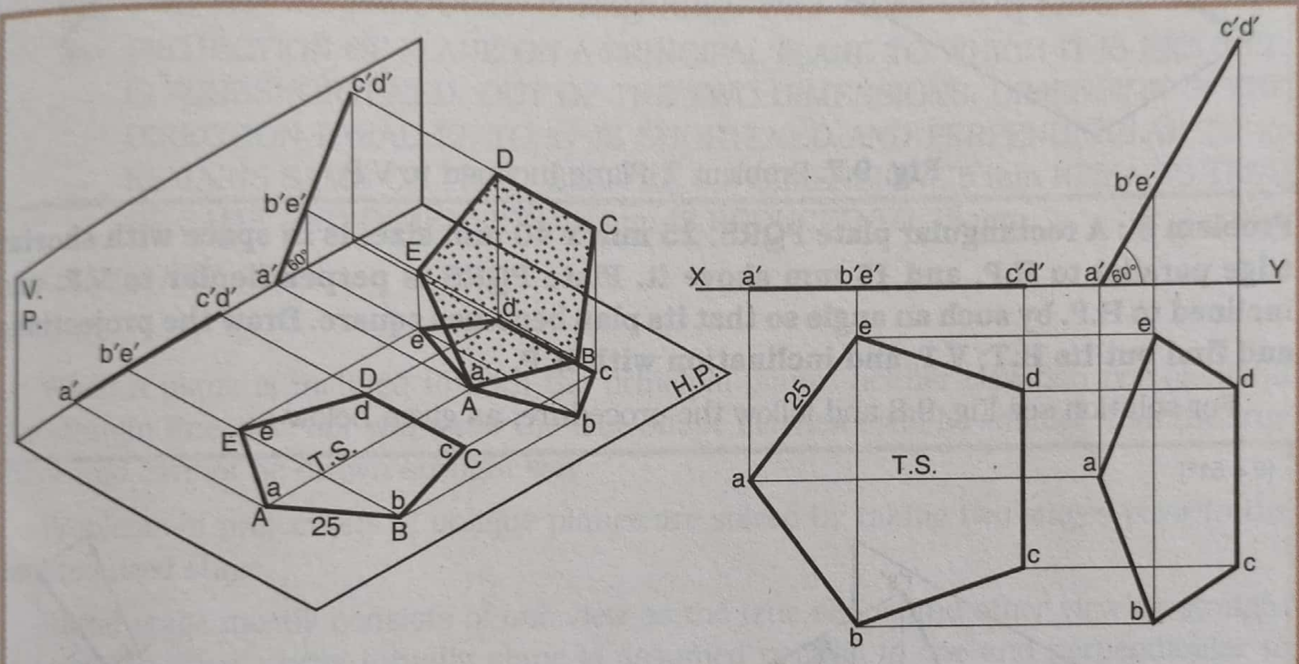


Fig. 9.6. Problem 6. Plane Inclined to H.P.

(i) In the initial or 1st stage draw, in plan, regular pentagon with one corner a on one side, since plate is required to rest on H.P. on one of its corners.

(ii) Complete the solution proceeding similarly to problem No.5.

Problem 7 : A square plate PQRS, edge 25 mm size, is in space with one of its corners on V.P. Surface of the plate makes 50° with V.P. and it is perpendicular to H.P. Draw its projections.

For solution see Fig. 9.7 and follow the procedure, as given below:

(i) In the initial stage assume complete plate in V.P., so its elevation will be true shape and its plan will be on xy line. Draw elevation a square of 1st stage keeping one corner on one side so that later on that corner in plan can be kept on xy line, to keep that point in V.P.

(ii) In second or final stage tilt plan of 1st stage so that it makes 50° with xy or in other words rearrange plan (p - qs - r) to new position (p - qs - r) keeping p on xy and line making 50° with xy.

(iii) Complete the solution as usual.

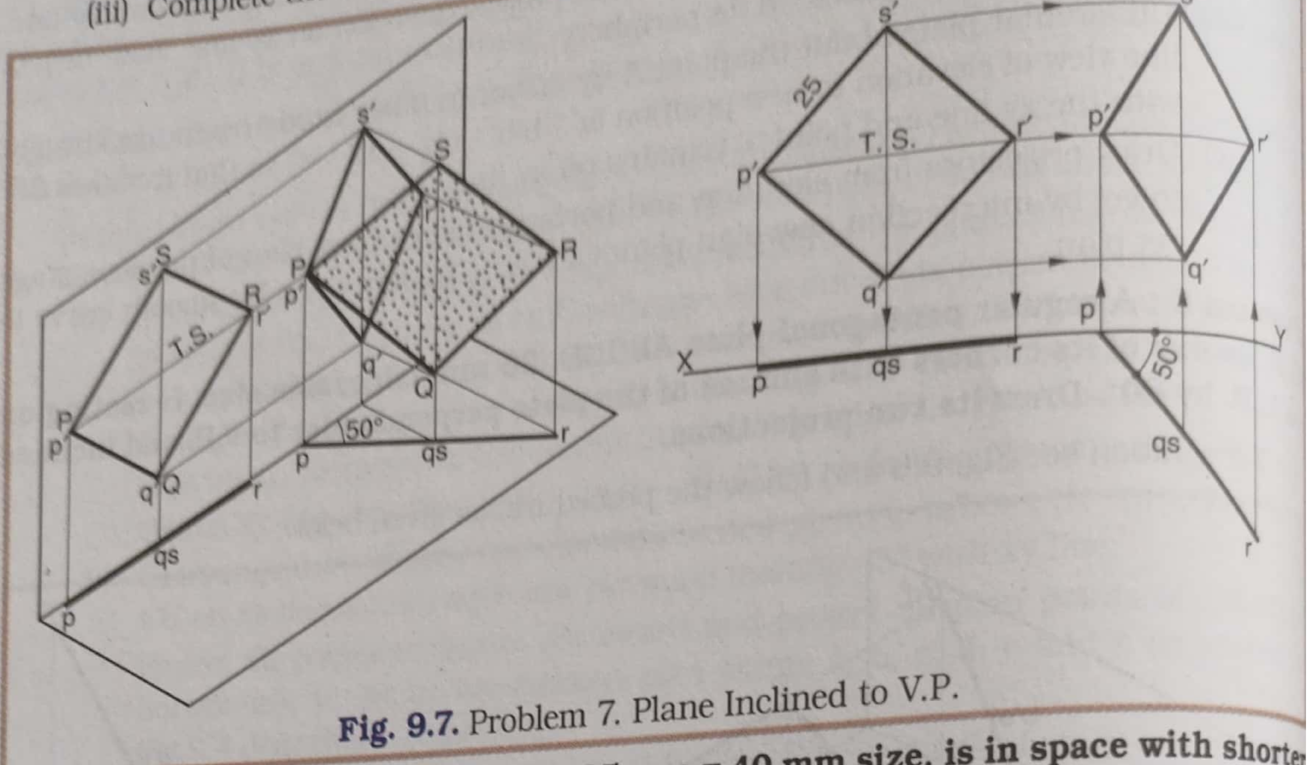


Fig. 9.7. Problem 7. Plane Inclined to V.P.

Problem 8 : A rectangular plate PQRS, 25 mm x 40 mm size, is in space with shorter edge parallel to H.P. and 15 mm above it. Plate PQRS is perpendicular to V.P. and inclined to H.P. by such an angle so that its plan becomes square. Draw the projections and find out its H.T; V.T. and inclination with H.P.

For solution see Fig. 9.8 and follow the procedure, as given below :

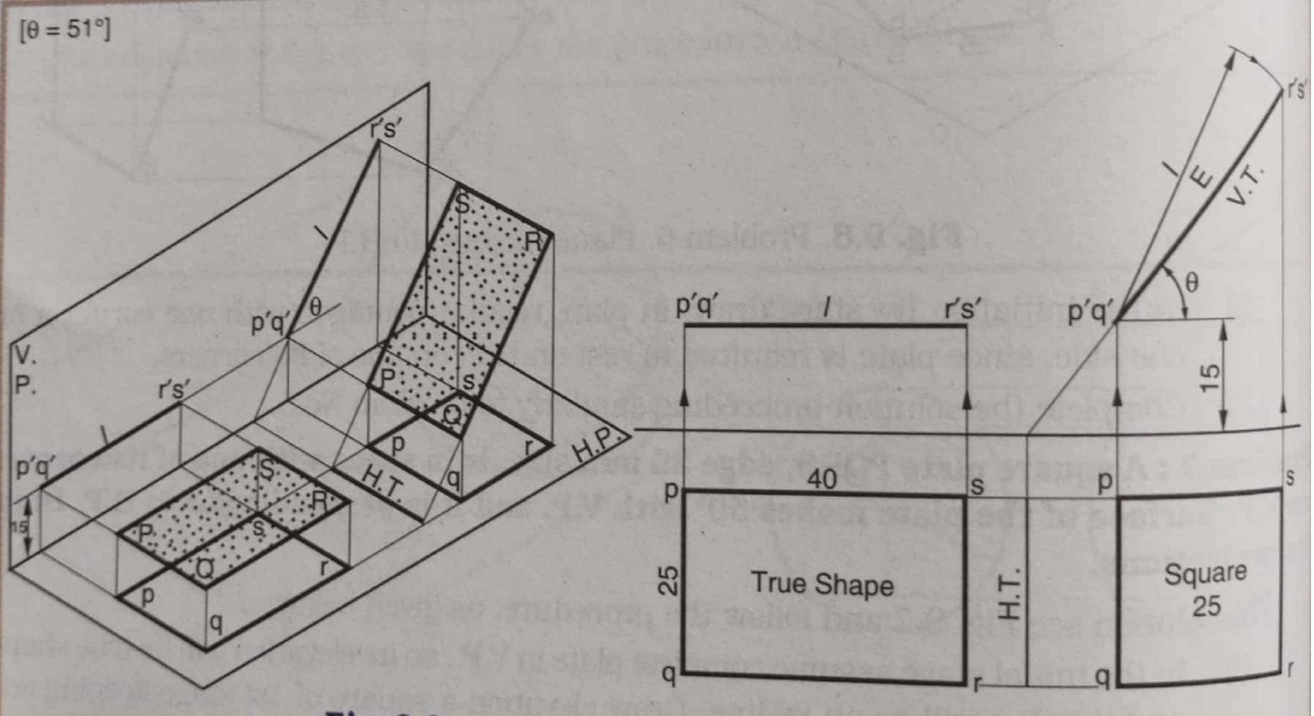


Fig. 9.8. Problem 8. Plane Inclined to H.P.

(i) In the initial stage take plate P Q R S parallel to H.P. and 15 mm above it as shown in figure. Its plan will be T.S. and its elevation will be straight line parallel to and 15 mm above xy line.

(ii) As the plan of 2nd and final stage is to be square draw plan (sq. of 25 mm size) pqrs as shown in the figure. Project all points vertically upward.

(iii) Take p'q' 15 mm above xy line on a projector through p and q and then draw arc of circle of R.40 to cut projector through (r) and (s) at r's'. Join (p'q' - r's') to get elevation. Measure angle θ made by elevation with xy line. Find traces as usual.

Conclusions :

(i) PROJECTIONS OF A PLANE ON A PRINCIPAL PLANE TO WHICH IT IS PERPENDICULAR IS A STRAIGHT LINE. THIS STRAIGHT LINE PROJECTION WILL MAKE AN ANGLE WITH XY LINE EQUAL TO THE ANGLE OF PLANE WITH OTHER PRINCIPAL PLANE. THIS STRAIGHT LINE IS ALSO GOING TO BE THE TRACE OF PLANE ON THAT PRINCIPAL PLANE.

(ii) TRACE ON OTHER PRINCIPAL PLANE WILL BE PERPENDICULAR LINE TO xy LINE.

(iii) TWO TRACES PRODUCED IF REQUIRED WILL MEET ON xy LINE.

(iv) PROJECTION OF PLANE ON A PRINCIPAL PLANE TO WHICH IT IS INCLINED IS FORESHORTENED. OUT OF THE TWO DIMENSIONS, DIMENSION IN THE DIRECTION PARALLEL TO xy IS SHORTENED AND PERPENDICULAR TO xy REMAINS SAME OR TRUE. SEE FIG. 9.8 DIMENSION 25 mm REMAINS TRUE OR SAME AND DIMENSION 40 mm IS REDUCED TO 25 mm.

5. OBLIQUE PLANES :

General :

When a plane is inclined to both the principal planes neither elevation nor plan will give straight line view nor will show the true shape i.e. views will be smaller than the true shape and cannot be drawn straight way.

Problems of projections of oblique planes are solved by taking two stages prior to the final required stage.

Initial stage mostly consists of one view as the true shape and other view as straight line view. In other words initially plane is assumed parallel to one and perpendicular to other principal plane.

If a plane is resting on ground or on H.P. on a corner or on an edge, plan or top view is drawn as the true shape and elevation is taken as straight line view. If a plane is resting on V.P. on an edge or on a corner, elevation is drawn as true shape and plan is taken as straight line view. In other words plane is assumed initially on H.P. or on ground or on V.P. or parallel to one of the above.

In the second stage straight line view is rearranged, as to settle the angle of plate surface with principal plane or to settle the distance of particular corner or edge from principal plane or to settle the required shape of subsequent projection.

Further if a plane is required to rest on corner then that corner is arranged on one side while drawing true shape. Similarly, if a plane is required to rest on an edge, then that edge is drawn perpendicular to xy line on one side while drawing true shape in initial stage.

Further solution of problems can be done by 1st angle system or by 3rd angle system. Shape of projection or views will not change due to change in system. Only their relative positions will change.

Problems can be solved also by taking auxiliary planes e.g. problem No 9, 11, 14, etc.

In auxiliary plane method required number of projections to be drawn are reduced. In fact repetition of same view is eliminated. This method is also known as rotation of ground line method.

Now we shall study the method of solution to this type of problems by solving a few concrete problems.

Problem 9 : A regular pentagonal plate, of 50 mm sides, has one of its corners on H.P. The plane of the pentagon is inclined at 30° to the H.P. The side of the pentagon, which is opposite to the corner, which is on H.P., is inclined at 45° to the V.P. Draw the projections of the plate.

For solution see Fig. 9.9 and follow the procedure as given below:

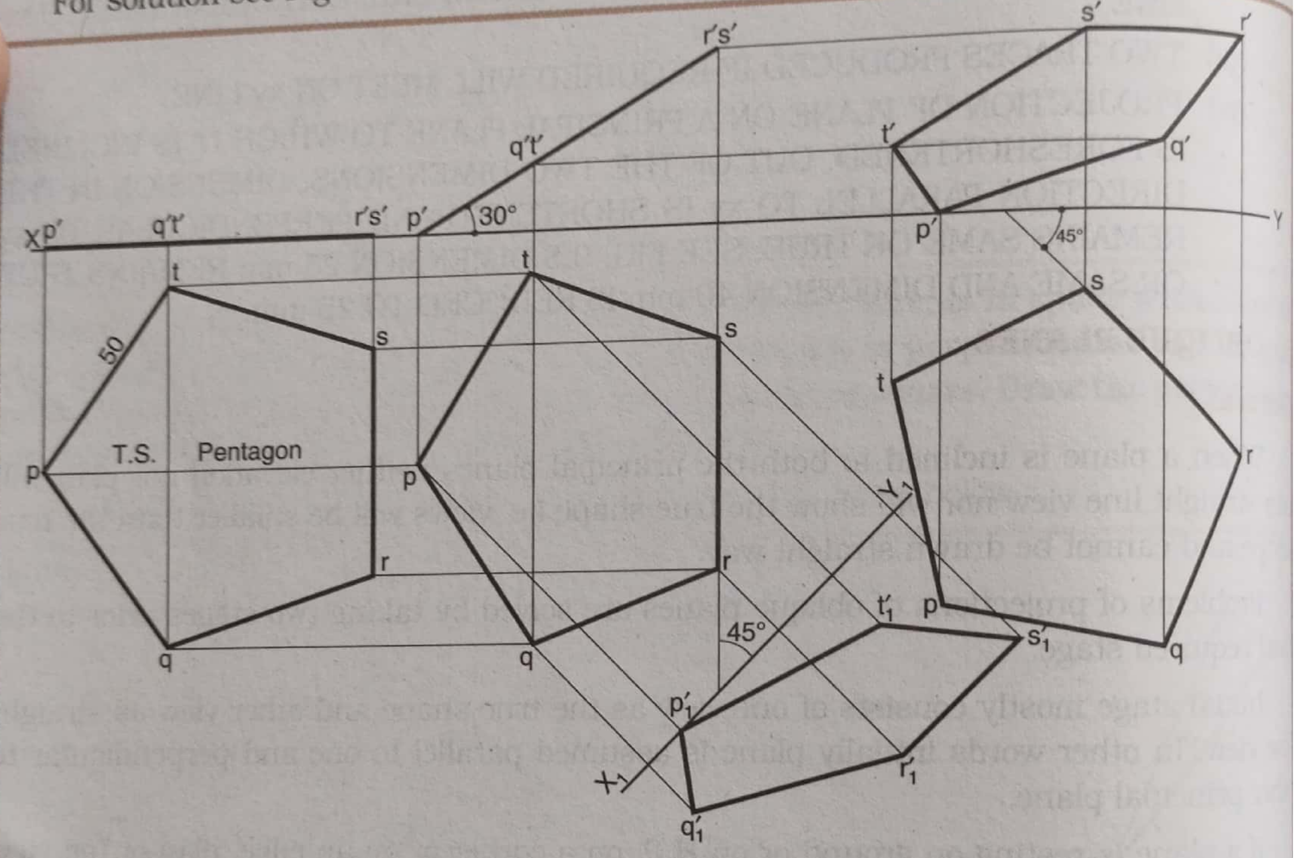


Fig. 9.9. Problem 9. Oblique Plane

The pentagonal plate must be in first quadrant, since it is given that corner is on H.P. And hence the solution is done by first angle projection system.

(i) In the initial stage pentagonal plate is assumed on H.P. and hence plan is drawn of true shape with one of the corners on one side. And elevation is drawn straight line on xy line.

(ii) As the plane of the plate is inclined to the H.P. by 30° and as one corner is on H.P., rearrange straight line view of 1st stage to the position given in Fig. 9.9 i.e. line view must make 30° with xy line and point p' should be xy line.

(iii) Draw projectors downward from the rearranged line view and draw horizontal lines from projection showing T.S. to get plan points by intersections.

(iv) Now line RS, opposite to corner P, on which it rests on H.P. is making 45° with

V.P. and hence draw new x_1y_1 line at 45° to rs line and project all points on x_1y_1 line. On these projectors mark points by taking distances of elevation points from xy line.

Problem 10 : Draw the projections of a circle, of 70 mm diameter, resting on the H.P. on a point A of the circumference. Plane is inclined to the H.P. such that the plan of it is an ellipse of minor axis 40 mm. The plan of the diameter, through the point A, is making an angle of 45° with the V.P. Measure the angle of the plane with the H.P.

For solution see Fig. 9.10 and follow the procedure, as given below :

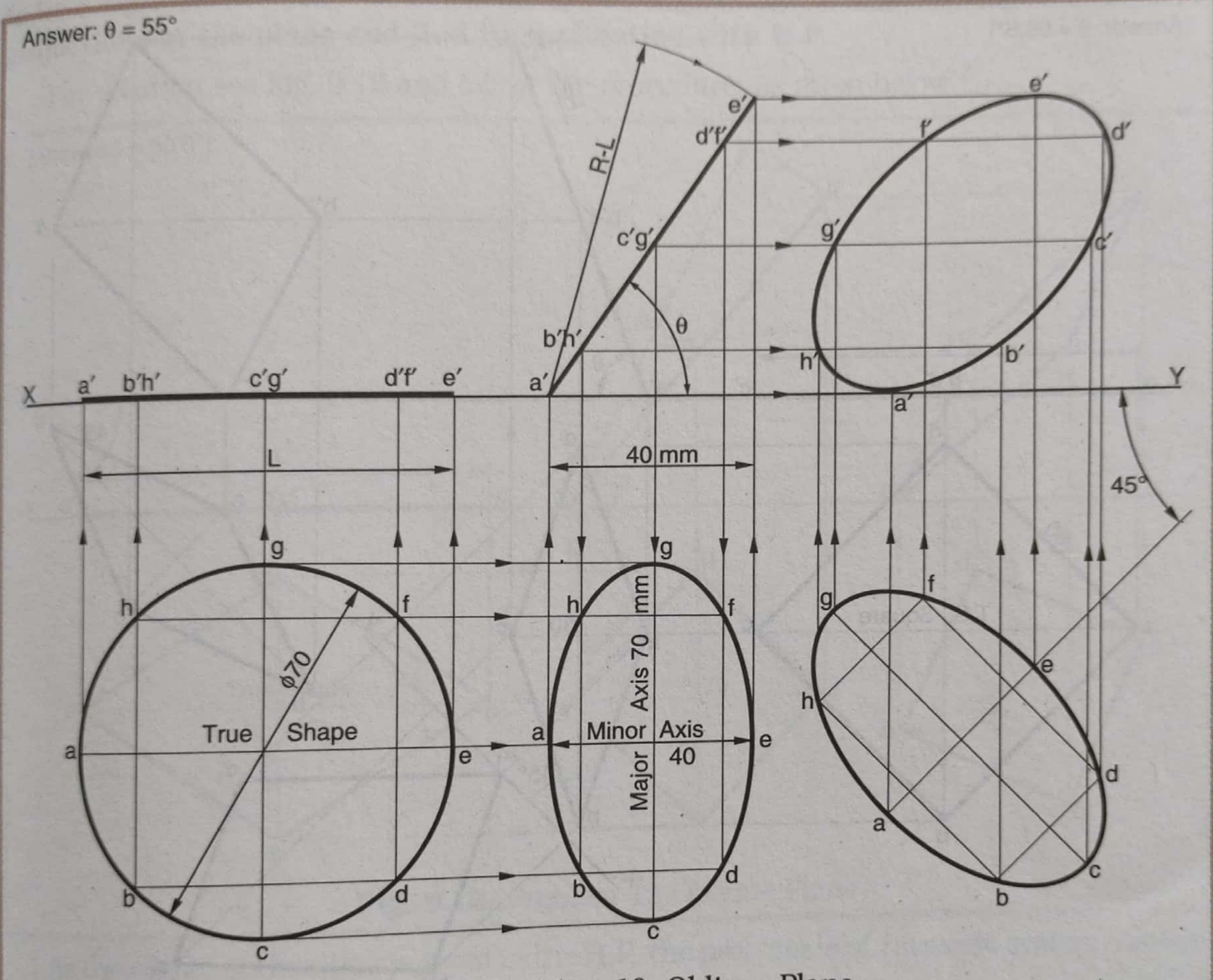


Fig. 9.10. Problem 10. Oblique Plane

Given that circle rests on H.P. and hence the solution is done according to 1st angle system. Further assume initially circle on H.P. and hence.

(i) Draw plan as circle of 70 mm diameter and draw its elevation, straight line on xy line.

(ii) Now rearrange this straight line view of elevation to such an inclination so that the distance between two extreme projectors becomes 40 mm. This is done to get minor axis of 40 mm of ellipse in subsequent projection. Measure this inclination θ. In fact in this position the plane of circle makes an angle θ with H.P.

(iii) By projection as usual, get ellipse in plan.

(iv) Since the plan of the diameter through A is making 45° with V.P., rearrange the ellipse to new position such that plan of the diameter AE i.e. ae makes 45° with xy line. Project all points and as usual get the final elevation.

Problem 11 : A square plate, of side 60 mm, is held on a corner on H.P. with a diagonal horizontal and inclined at 45° to V.P. (FRP). The plate is seen as a rhombus in plan with one of its diagonals measuring 30 mm. Draw the principal views of the plate and determine the angle it makes with H.P.

For solution see Fig. 9.11 and follow the procedure, as given below :

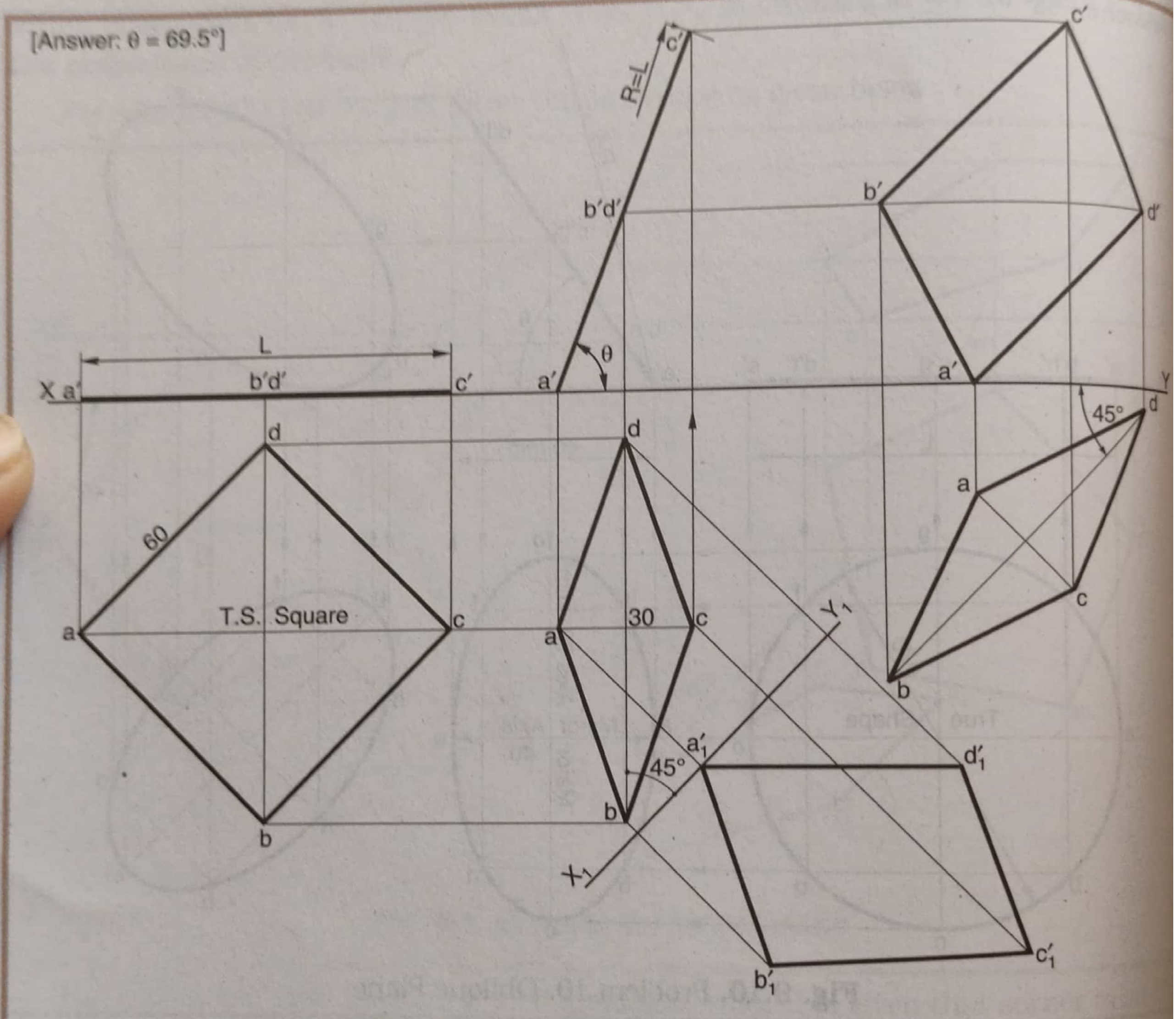


Fig. 9.11. Problem 11. Oblique Plane

As the corner is on H.P. use 1st angle system and keep the plate initially on H.P.

(i) Draw plan T.S. (square of 60 mm) below xy line keeping a corner on one side and draw elevation as straight line on xy line.

(ii) As the required plan is rhombus of 30 mm diagonal, rearrange this straight line elevation to such an inclination such that projectors through extreme points become 30 mm apart. Keep a' on xy line to keep corner A on H.P. Measure the inclination θ, which is the inclination of plate with H.P.

(iii) By projection as usual, get plan as rhombus with one diagonal as 30 mm and another diagonal as true length.

(iv) Since it is given that the horizontal diagonal is inclined at 45° to V.P., either draw new X_1Y_1 at 45° to bd and project elevation on X_1Y_1 or rearrange (rhombus) plan with bd line making 45° with xy line and project elevation from it on xy. Follow any one method. Here both methods are used to show that the shape of elevations achieved by different methods are the same. Further their positions w.r.t. corresponding XY lines is also the same.

Problem 12 : ABCD is a rhombus of diagonals AC = 110 mm and BD = 70 mm. Its corner A is in the H.P. and the plane is inclined to the H.P. such that the plan appears to be a square. The plan of diagonal AC makes an angle of 20° to the V.P. Draw the projections of the plane and find its inclination with H.P.

For solution see Fig. 9.12 and follow the procedure, as given below :

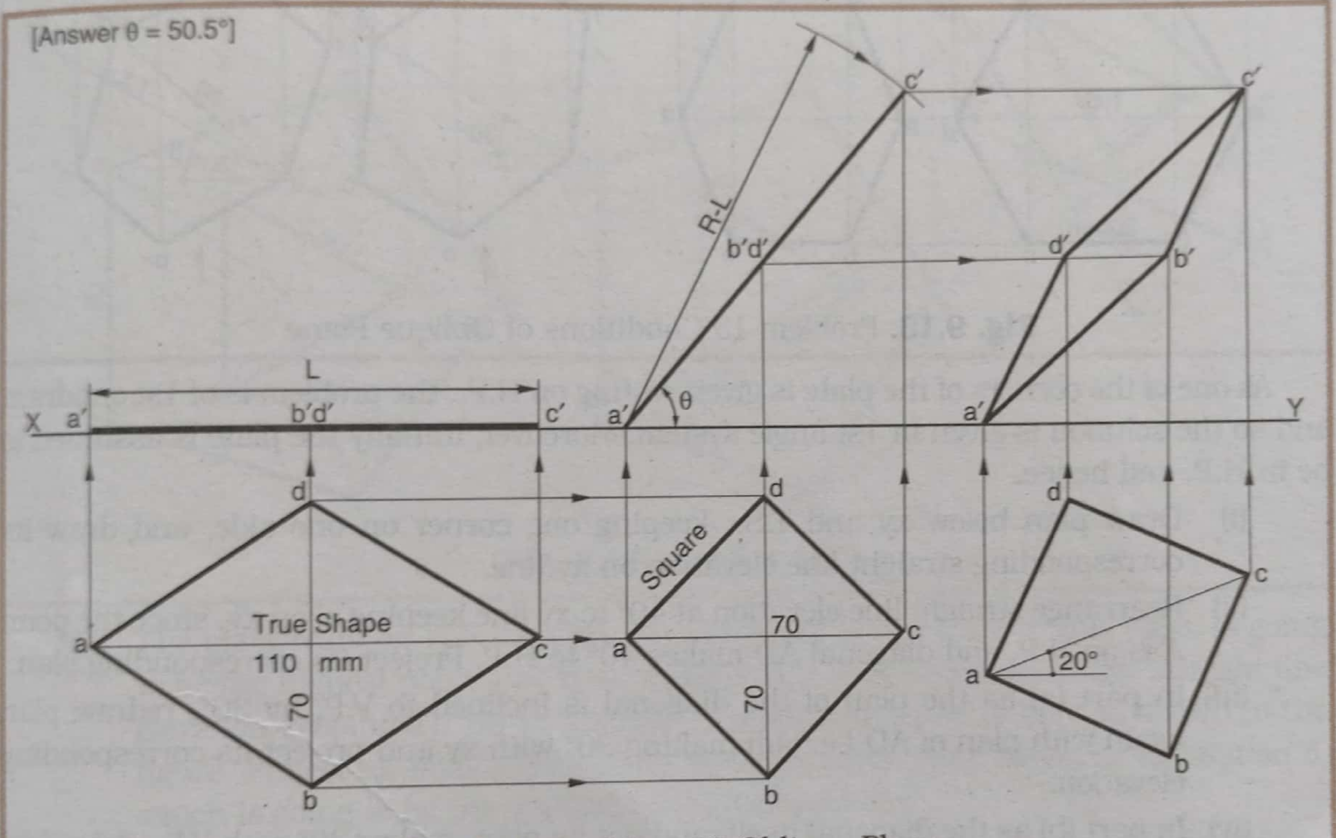


Fig. 9.12. Problem 12 Oblique Plane

As the corner of rhombus is given in the H.P. the problem is of 1st angle system. Assume initial position of rhombus in H.P. and hence.

(i) Draw (T.S.) rhombus of diagonals 110 mm and 70 mm as plan below xy with diagonal of 110 mm parallel to xy line and draw its corresponding elevation, straight line on xy line itself.

(ii) Rearrange this straight line elevation to new position as shown in the figure, keeping a' on xy line and projectors through end points 70 mm apart. Purpose of keeping 70 mm distance between projectors is to get a square in plan of diagonals 70 mm. Measure inclination θ, which is the inclination of rhombus with H.P.

(iii) Rearrange (square) plan, with plan of AC i.e ac making 20° with xy and project corresponding elevation.

Problem 13 : A regular hexagonal plate, 50 mm side, is resting on one of its corne in H.P. The diagonal through that corner is inclined at 40° to H.P. and (a) the plan that diagonal inclined to V.P. by 30° and (b) diagonal is inclined at 30° to V.P.

For solution see Fig. 9.13 and follow the procedure, as given below :

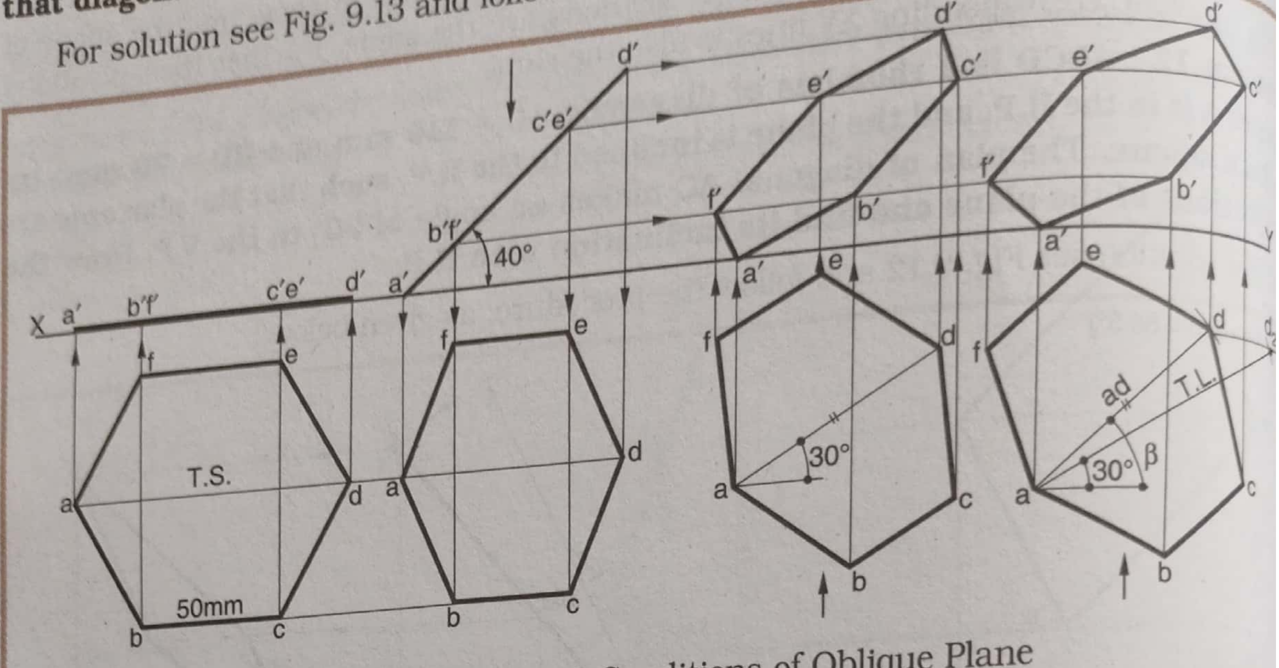


Fig. 9.13. Problem 13 Conditions of Oblique Plane

As one of the corners of the plate is given resting on H.P., the problem is of 1st quadra and so the solution is given in 1st angle system. Moreover, initially the plate is assumed be in H.P. and hence.

(i) Draw plan below xy and T.S., keeping one corner on one side, and draw i corresponding straight line elevation on xy line.

(ii) Rearrange straight line elevation at 40° to xy line keeping a' on xy, since the poi A is on H.P. and diagonal AD makes 40° to H.P. Project its corresponding plar

(iii) In part (a) as the plan of the diagonal is inclined to V.P. by 30°, redraw pla again with plan of AD i.e. (ad) making 30° with xy and project its correspondir elevation.

(iv) In part (b) as the diagonal itself, and not its plan, makes 30° with V.P., draw fir T.L. (ad_2) of diagonal AD at 30° and adjust the plan (ad) length, as shown in th figure at β angle. Redraw the complete plan with ad making an angle β with and project its corresponding elevation.

Problem 14 : A pentagonal plate, of sides 50 mm, has a central equilateral triangul hole of 40 mm sides, with a side of plate and that of triangle parallel to each othe The plate is kept on H.P. on this side, the side being inclined at 30° to FRP (V.P.). the highest point of the plate is 40 mm above the H.P., Determine the angle the plat makes with H.P. Project the triangular hole in all the views.

For solution see Fig. 9.14 and follow the procedure, as given below :

As the plate is to rest on H.P. on an edge, use 1st angle system and assume initia position of plate completely on H.P. with one of the edges of the plate perpendicular to x and on one side.

(i) Draw regular pentagon abcde of 50 mm side as plan below xy with edge ab perpendicular to xy. Draw its corresponding elevation on xy line. Draw equilateral triangle of 40 mm side pqr in pentagon keeping the centre same and one edge of triangle pq parallel to ab. Project pqr also in elevation.

[Answer: θ = 31.5°]

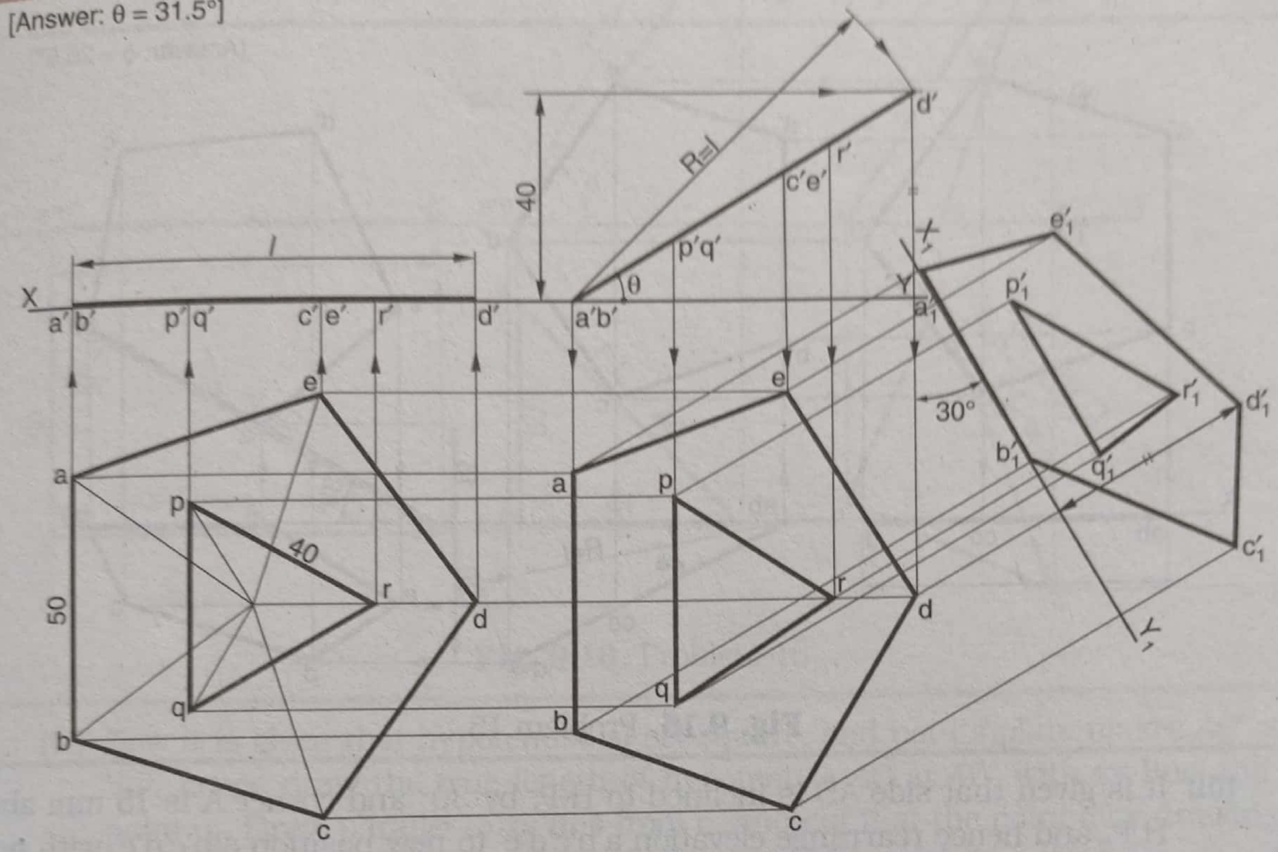


Fig. 9.14. Problem 14

(ii) The plate rests on H.P. on edge AB and point D, opposite to this edge, is going to be the highest point and 40 mm above H.P. And hence rearrange straight line elevation keeping a'b' on xy line and d' at 40 mm above xy line, as shown in the figure. Project all points and get its corresponding plan. Measure inclination θ, which is going to be the inclination of plate with H.P.

(iii) Now as the edge, on which it rests on H.P., makes 30°with V.P, draw new x_1y_1 line at 30° to ab line and project plan to get elevation $a¢_1b¢_1c¢_1d¢_1e¢_1$ and $p¢_1q¢_1r¢_1$, as shown in the figure.

Problem 15 : A regular pentagon ABCDE, of 30 mm sides, has its side AB in the V.P. and inclined at an angle of 30° to the H.P. The corner A is 15 mm above H.P. and the corner D is 20 mm in front of V.P. Draw the projections of the plane and find its inclination with the V.P.

For solution see Fig. 9.15 and follow the procedure, as given below :

As the side AB is required to be in V.P., assume initially pentagon ABCDE in V.P. with side AB perpendicular to xy or H.P. First angle system is used.

(i) Draw regular pentagon a'b'c'd'e' as elevation above xy line with edge or side a'b' perpendicular to xy line. Draw its corresponding plan on xy line.

(ii) It is given that the corner D is 20 mm in front of V.P. and the side AB is in V.P. and hence rearrange straight line plan view (plan) in such a way that ab remains on xy line and d remains 20 mm below xy. Measure angle Ø between xy line and rearranged straight line plan view. This angle Ø is the inclination of plane with V.P. Project corresponding elevation a'b'c'd'e'.

[Answer: $\phi = 25.5°$]

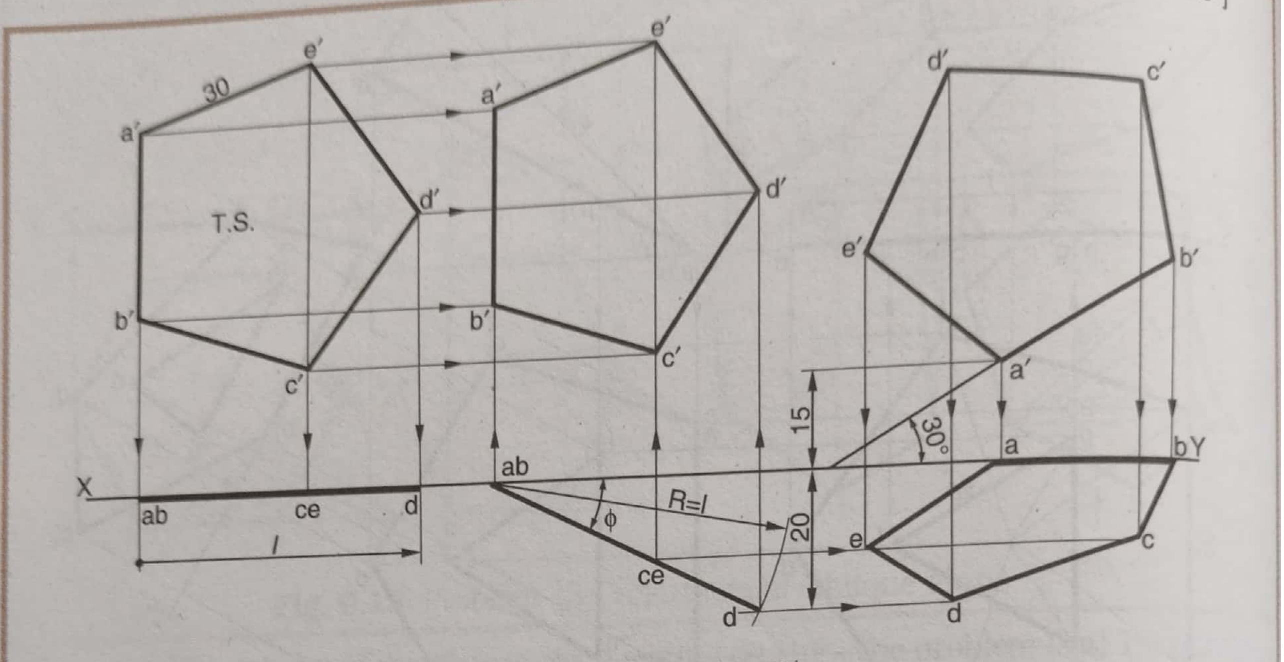


Fig. 9.15. Problem 15

(iii) It is given that side AB is inclined to H.P. by 30° and corner A is 15 mm above H.P. and hence rearrange elevation a'b'c'd'e' to new position a'b'c'd'e' with point a' 15 mm above xy and line a'b' making 30° with xy. Project its corresponding plan abcde.

Problem 16 : A 30° - 60° set square has its shortest side 50 mm long and is in the H.P. The top view of the set square is an isosceles triangle and the hypotenuse of the set square is inclined at an angle of 40° with the V.P. Draw the projections of the set square and find its inclination with the H.P.

For solution see Fig. 9.16 and follow the procedure, as given below :

It is given that shortest side of 50 mm length rests on H.P., assume initial position of set square in H.P. with the shortest side perpendicular to V.P. As it rests on H.P., use 1st angle system.

(i) First draw Δabc T.S. of set square. Project its corresponding straight line elevation a'b'c' on xy line.

(ii) It is given that the top view of the set square is an isosceles triangle and hence first draw isosceles triangle of 50 x 50 x hypotenuse, as shown in the figure and rearrange straight line view a'b'-c' such that it suits or fits to its corresponding plan already drawn. Measure the angle θ that rearranged straight line elevation a'b'-c' makes with xy line. This angle θ is the inclination of plane of set square with H.P.

[Answer: θ = 55°]

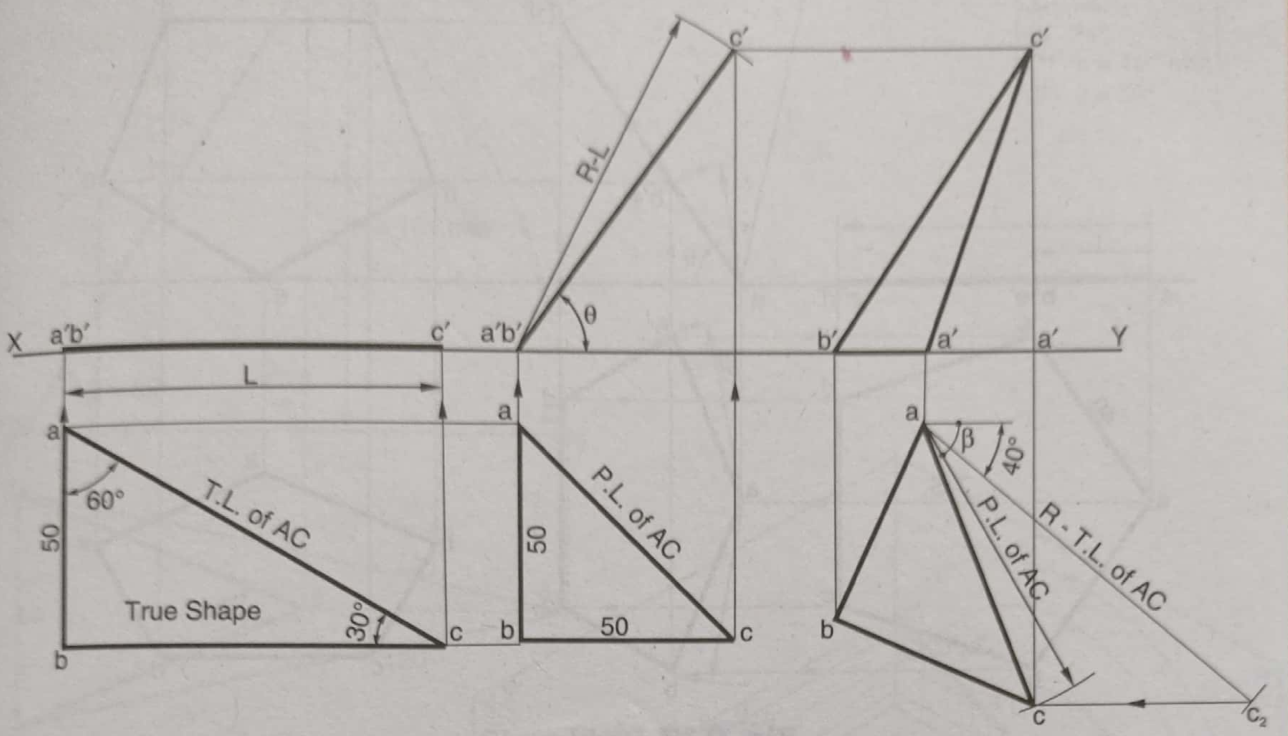


Fig. 9.16. Problem 16

(iii) Now it is given that hypotenuse of set square, and not its plan, makes 40° with V.P. hence draw the true length of hypotenuse AC at 40° with xy line and get point c_2. Draw parallel to xy line from c_2 and cut it at the point c by drawing an arc with (a) as the centre and radius equal to plan length (ac) of hypotenuse. Join ac and rearrange plan on this line ac. Project its corresponding elevation a'b'c', as shown in figure.

Problem 17 : ABCDE is a regular pentagonal plate, of 40 mm sides, has its corner A on the H.P. The plate is inclined to the H.P. such that the plan length of the edges AB and AE is each 35 mm. The side CD is parallel to both the reference planes. Draw the projections of the plate and find its inclination with the H.P.

For solution see Fig. 9.17 and follow the procedure, as given below :

As the corner A is given on H.P. the problem is required to be treated in 1st angle system.

Further assume initially the pentagonal plate on H.P. with corner A on one side.

(i) Draw regular pentagon abcde of 40 mm side as plan below xy line with corner a on one side and project its corresponding straight line elevation on xy line.

(ii) It is given that plan of AE and AB are of 35 mm length so first arrange 35 mm length lines ae and ab on parallel lines to xy line from a, e and b, as shown in figure. Draw projector through a to get a' on xy line.

(iii) Draw common projector through b and e and draw arc with a' as the centre and radius equal to r to intersect at b'e' as shown in the figure. Join a' and b'e' and extend it upto c'd'. This straight line a' - b'e' - c'd' is the rearranged position of straight line elevation a' - b'e' - c'd'. Project points downward to complete plan abcde.

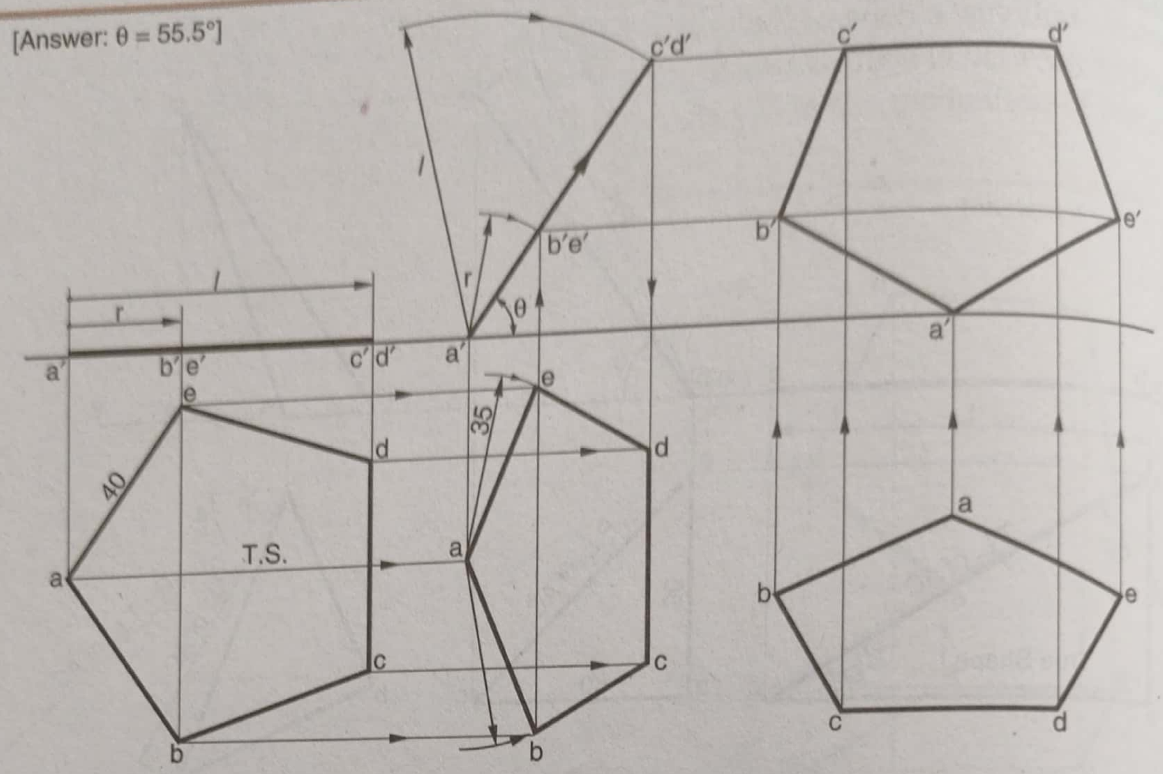


Fig. 9.17. Problem 17

(iv) Rearrange plan of CD i.e. cd parallel to xy and redraw the plan and project its corresponding elevation.

Angle θ is the inclination of the plate with H.P. Measure it.

Problem 18 : The top plan, of a pair of equal legs AB and AC of compass, appears as an isosceles triangle having base bc 50 mm and vertex angle at A 45°. Actual lengths of compass legs AB and AC are 120mm. Assume points B and C on H.P. and line connecting B and C perpendicular to V.P. Draw the projections and find

(1) The actual angle between two legs.

(2) The height of point A above the H.P. and

(3) Angle the plane, containing compass, makes with H.P.

For solution see Fig. 9.18. It is self explanatory.

Problem 19 : Draw the three projections of a circular lamina, of 50 mm diameter, having one end of the diameter resting on H.P. and the other end of the diameter on V.P. and the surface of the lamina inclined at 30° to the H.P. and at 60° to the V.P.

For solution see Fig. 9.19. It is self explanatory.

Answer:
(i) 24°
(ii) h = 101 mm
(iii) θ = 59°

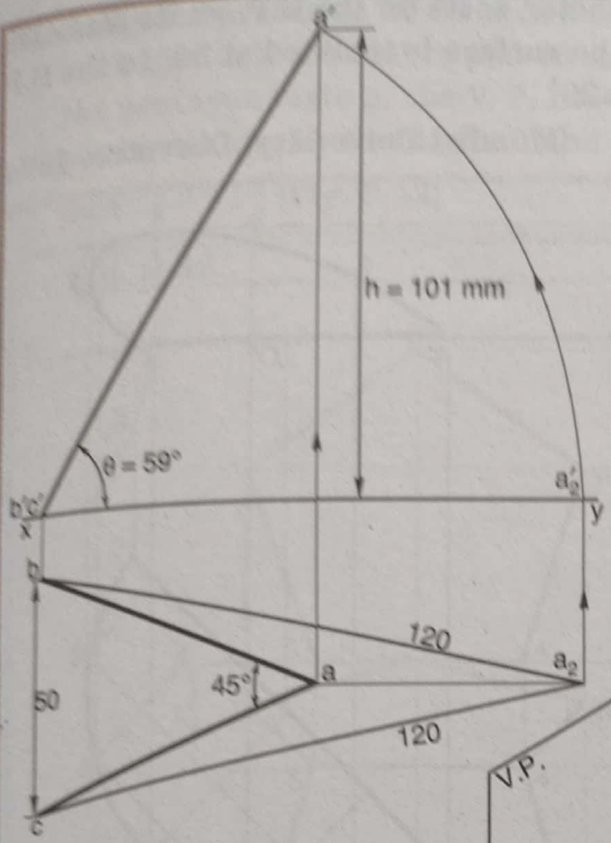
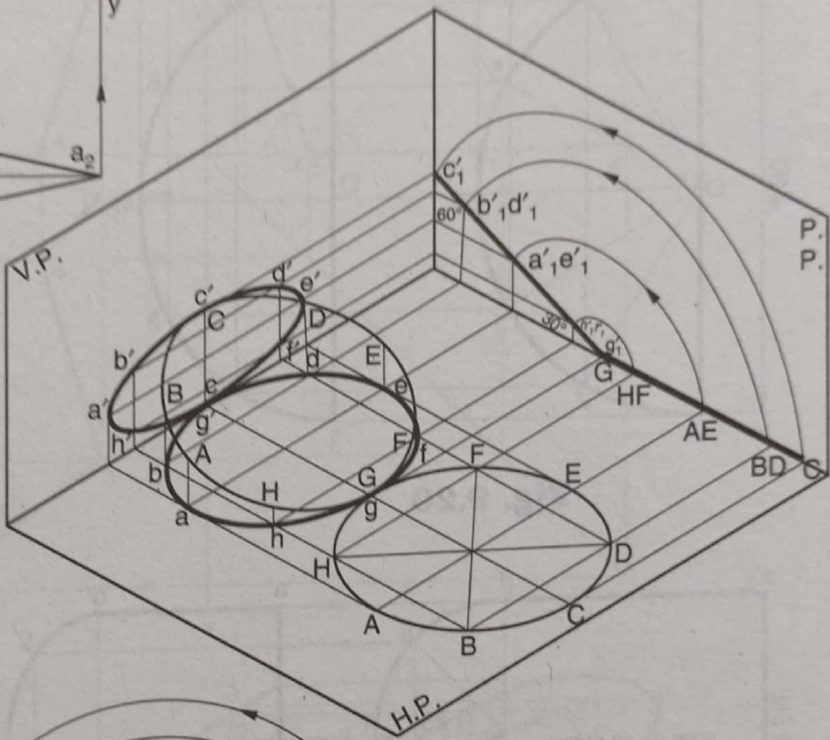


Fig. 9.18 Problem. 18



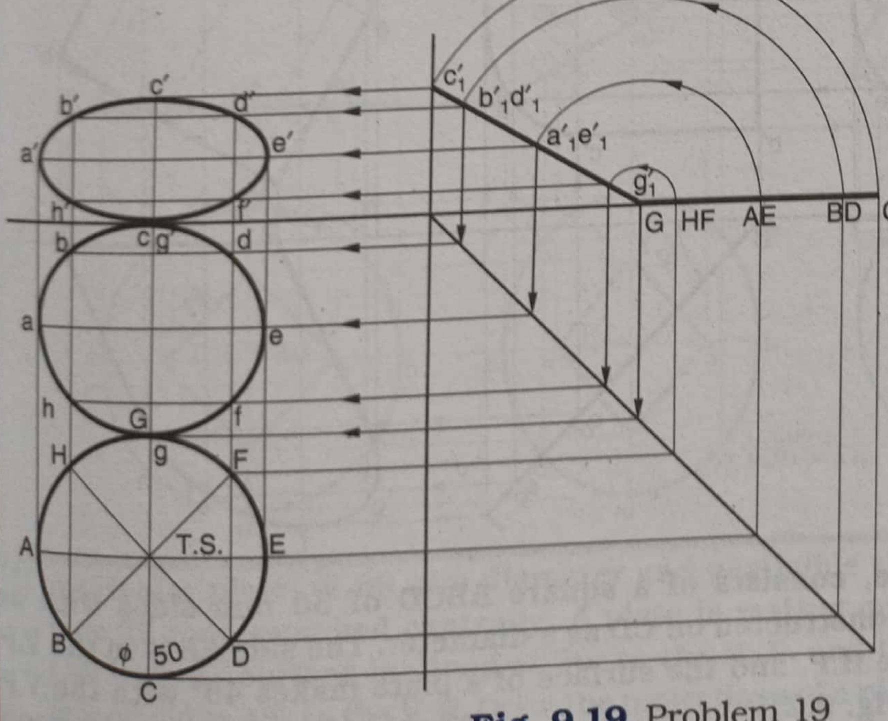


Fig. 9.19. Problem 19

(1) A semi-circular thin plate, of 60 mm diameter, rests on the H.P. on its diameter which is inclined at 45° to the V.P. and the surface is inclined at 30° to the H.P. Draw the projections of the plate. [Fig. 9.20]

[Mumbai University, December 1994]

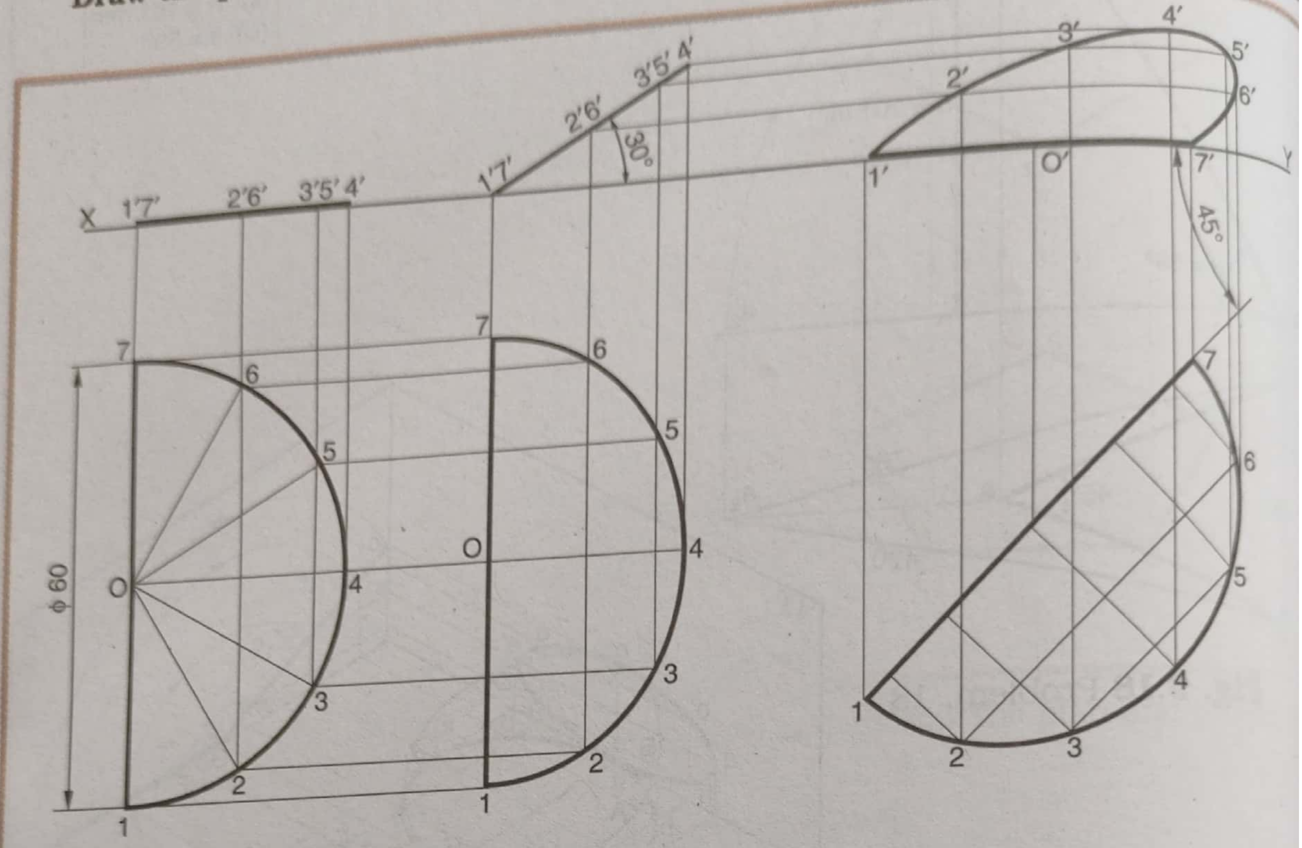


Fig. 9.20

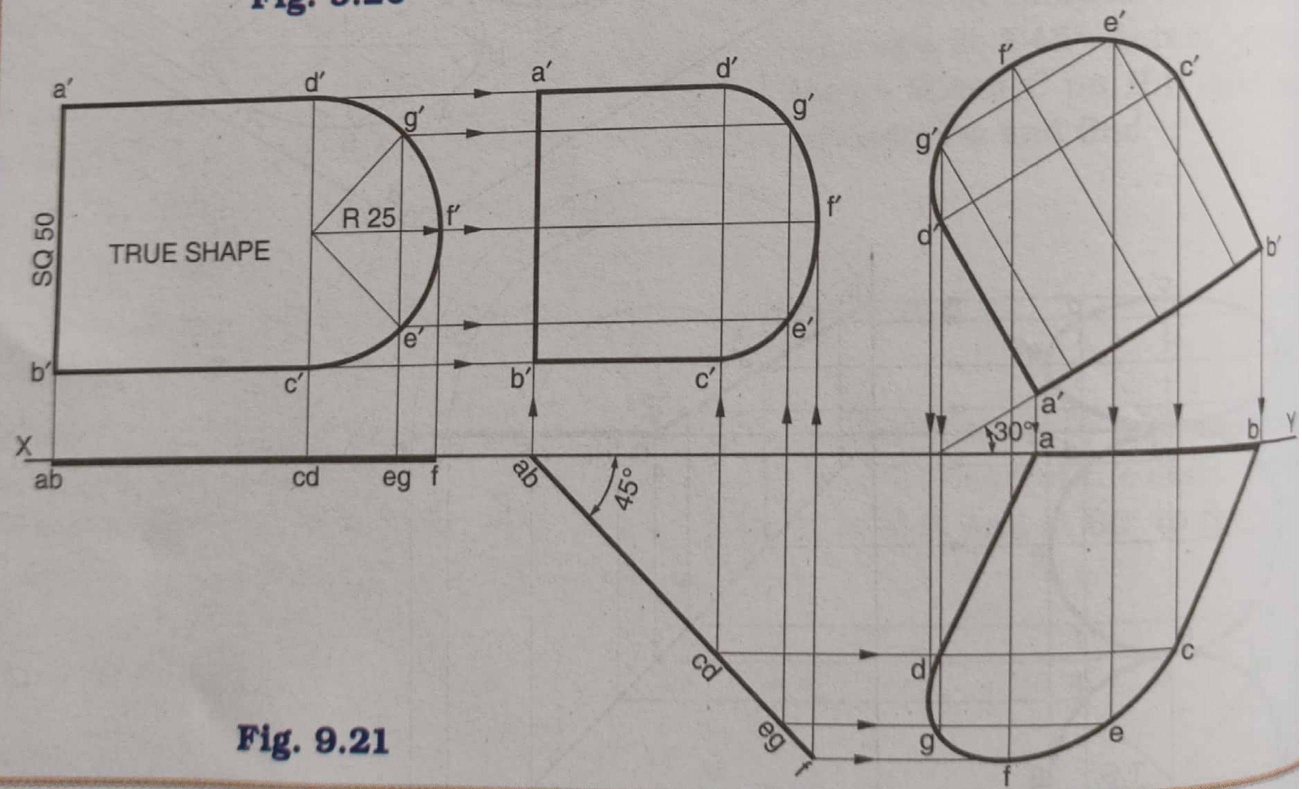


Fig. 9.21

(2) A thin composite plate, consists of a square ABCD of 50 mm sides with a additional semi-circle constructed on CD as a diameter. The side AB is in the V.P. and makes 30° with the H.P. and the surface of a plate makes 45° with the V.P. Draw its projections. [Fig. 9.21]

[Mumbai University, June 1996]

(3) **A regular pentagon, of 50 mm sides, is resting on one of its sides on the H. P. such that it is parallel to and 25 mm infront of the V. P. If the highest corner of the pentagon rests in the V. P. Draw its projections and find the angle made by a plane with the H. P., the H. T. and the V. T. of the plane.**

[Ans : θ = 71°] [Fig. 9.22] **[Mumbai University, December 1997]**

Fig. 9.22

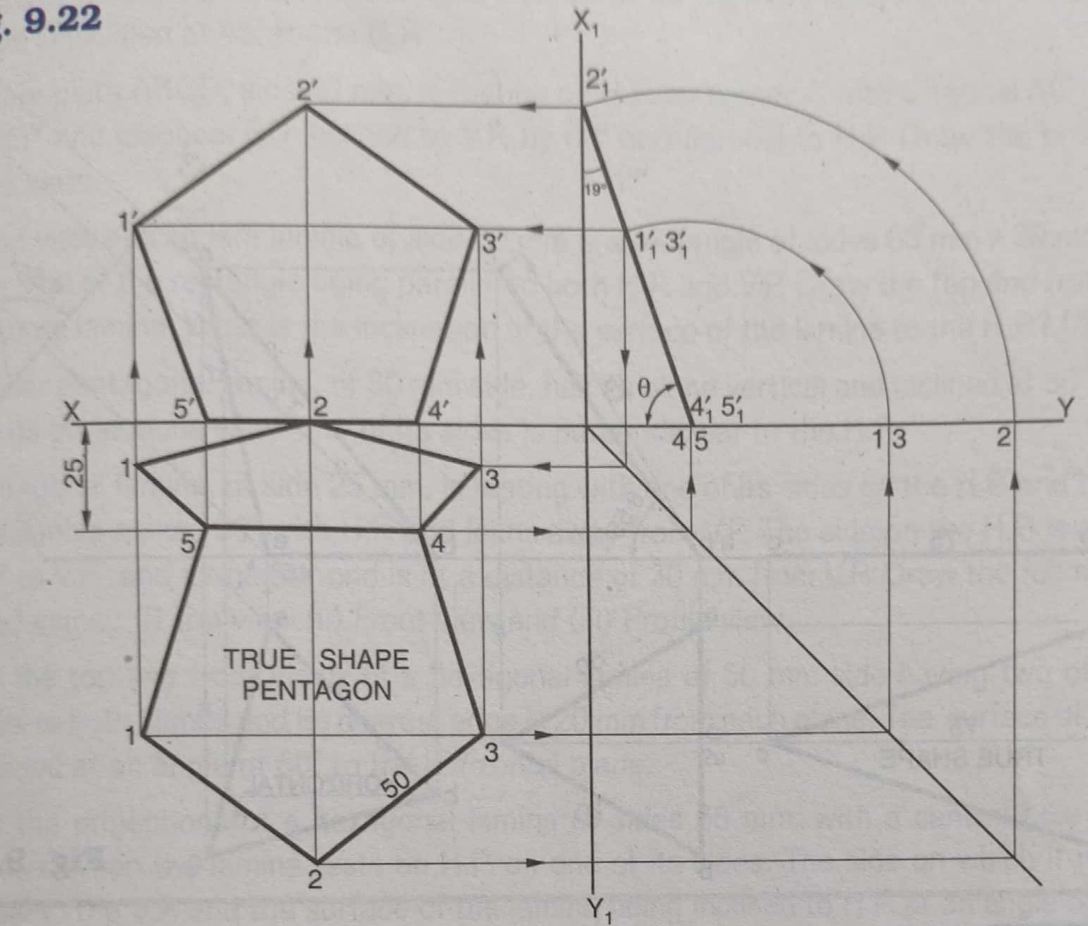
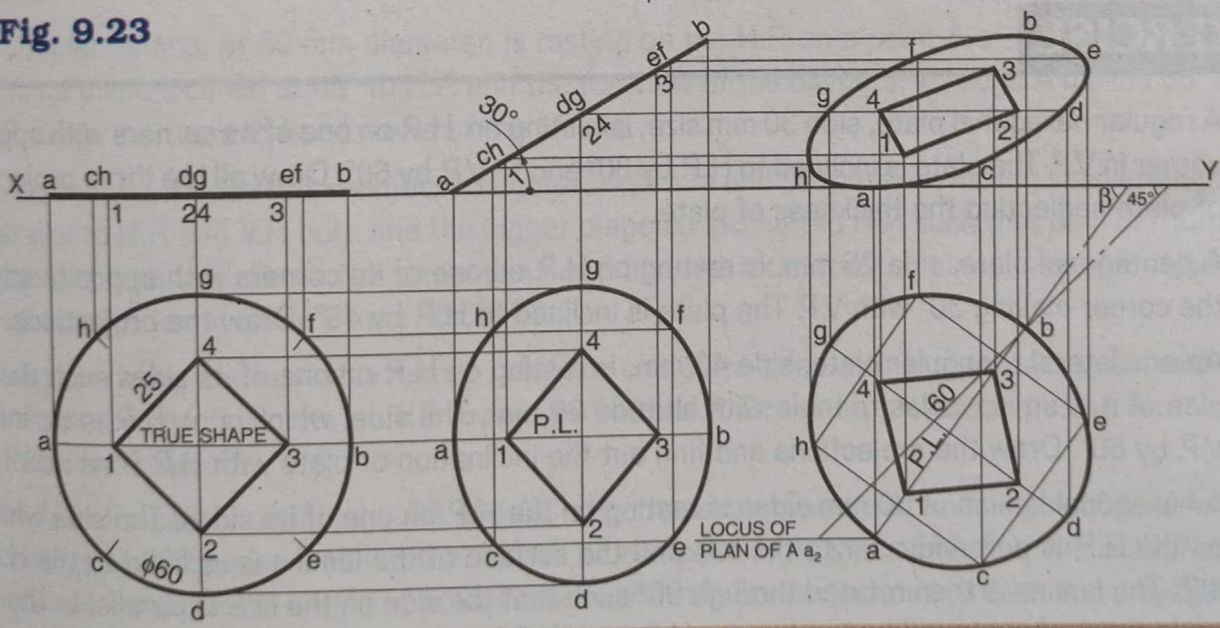


Fig. 9.23



(4) **A circular plate, of 60 mm diameter and negligible thickness, has a square hole side 25 mm, punched centrally. A plate is resting on the H. P. on point A of its rim with its surface inclined at 30° to the H. P. and the diameter AB, through A, is inclined at 45° to the V. P. Draw the projections of a plate with the hole. [Fig. 9.23]**

[Mumbai University, December 1996]

(5) **An isosceles triangular plate, of 50 mm base and 75 mm altitude, appears as an equilateral triangle of 50 mm in top view. Draw the projections of a plate if its 50 mm long edge is on the H. P. and inclined at 45° to the V. P. What are the inclinations of a plate with the H. P. and the V. P.? [Fig. 9.24]**

[Mumbai University, December 1995]

[Ans : (1) θ = 55° (2) θ = 55°]

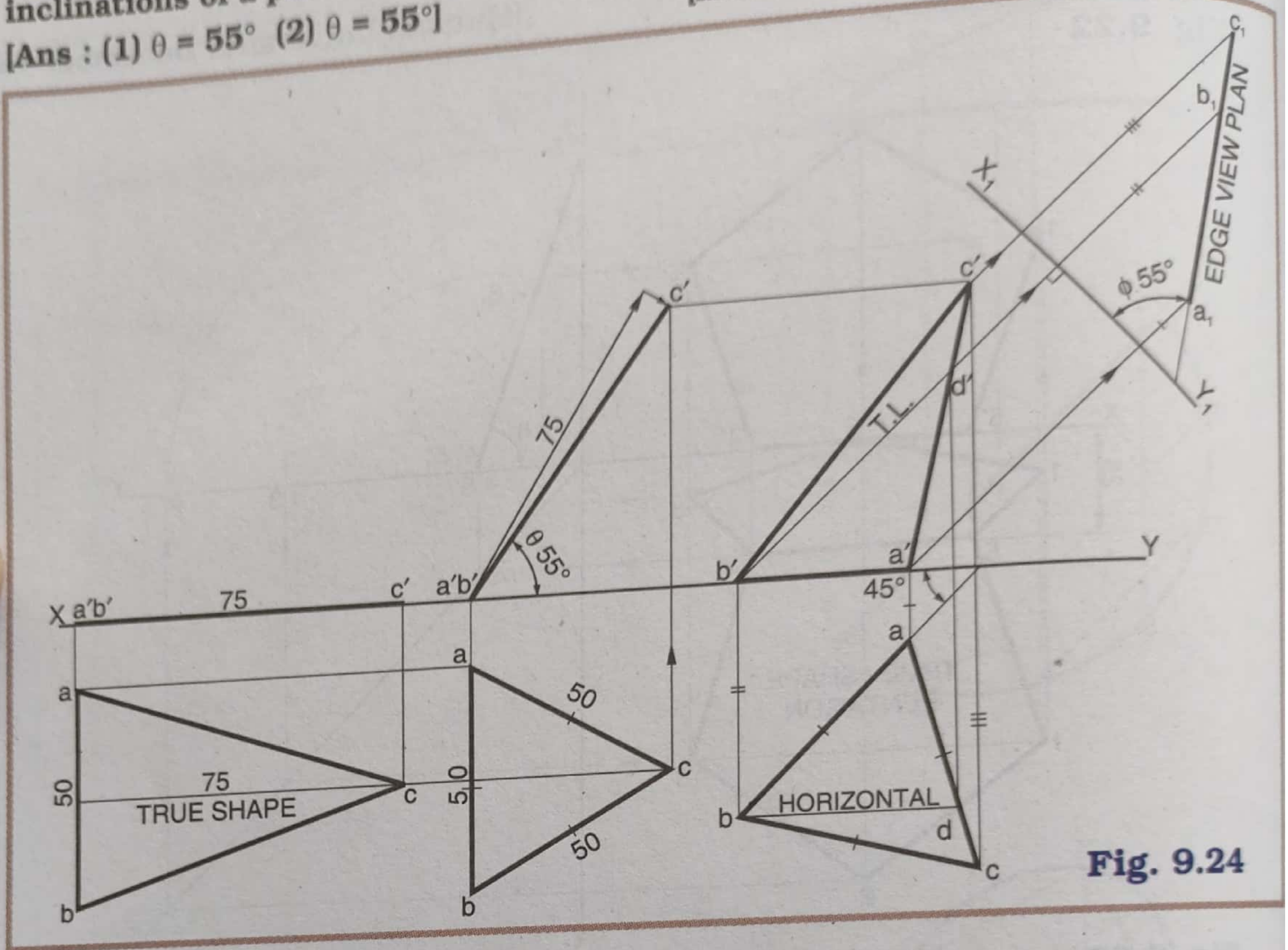


Fig. 9.24

EXERCISE

1. A regular hexagonal plate, side 30 mm size, is resting on H.P. on one of its corners with opposite corner in V.P. The plate is inclined to H.P. by 30° and to V.P. by 60°. Draw all the three projections of plate neglecting the thickness of plate.

2. A pentagonal plate, side 25 mm, is resting on H.P. on one of its corners with opposite edge to the corner making 30° with V.P. The plate is inclined to H.P. by 45°. Draw the projections.

3. An equilateral triangular plate, side 40 mm, is resting on H.P. on one of its sides such that the plan of it is an isosceles triangle with altitude 20 mm. The side, which is on H.P. is inclined to V.P. by 60°. Draw the projections and find out the inclination of plate with H.P. [Ans : 55°]

4. A hexagonal lamina of 30 mm sides is resting on the H.P. on one of its sides. The side which is on the H.P. is perpendicular to the V.P. and the surface of the lamina is inclined to the H.P. at 45°. The lamina is then rotated through 90° such that the side on the H.P. is parallel to the V.P., while the surface is still inclined to the H.P. at 45°. Draw the front view and the top view of the lamina in its final position.

5. ABC is a triangle of sides AB = 75 mm, BC = 60 mm and CA = 45 mm. Its longest side AB is in V.P. and inclined at 30° to H.P. Its surface makes an angle of 45° with the V.P. Draw its projections.

6. The top view of a 45° set square, with the side BC in H.P. and the side AB in the V.P., is a triangle abc. The side bc = 200 mm, being perpendicular to the XY line and angle bca = 25°. Draw the top and front views and measure the inclination of the set square to the H.P. Draw the auxiliary view on X_1Y_1 line perpendicular to xy line. [Ans : 62°]

7. Draw an equilateral triangle of 75 mm side and inscribe a circle in it. Draw the projections of the figure, when its plane is vertical and inclined at 30° to the V.P. and one of the sides of the triangle is inclined at 45° to the H.P.

8. A square plate ABCD, side 40 mm, is resting on H.P. on corner A with diagonal AC making 30° with H.P. and diagonal BD inclined to V.P. by 60° and parallel to H.P. Draw the projections of square plate.

9. The top view of a square lamina of side 60 mm is a rectangle of sides 60 mm x 20 mm with the longer side of the rectangle being parallel to both H.P. and V.P. Draw the top and front views of the square lamina. What is the inclination of the surface of the lamina to the H.P.? [Ans : 70.5°]

10. A regular pentagonal lamina, of 30 mm side, has its plane vertical and inclined at 30° to the V.P. Draw its projections when one of its sides is perpendicular to the H.P.

11. A pentagonal lamina, of side 25 mm, is resting with one of its sides on the H.P. and the surface of the lamina makes 30° with H.P. and leans away from V.P. The side on the H.P. is at an angle of 40° to V.P. and its nearer end is at a distance of 30 mm from V.P. Draw the following views of the lamina : (i) Top view (ii) Front view and (iii) Profile view.

12. Draw the top and front views of a hexagonal lamina of 50 mm side having two of its edges parallel to both planes and its nearest edge is 20 mm from each plane. The surface of the lamina is inclined at an angle of 60° to the horizontal plane.

13. Draw the projections of a hexagonal lamina of sides 35 mm, with a central hole of 40 mm diameter, when the lamina rests on H.P. on one of its sides. The side on which it rests being parallel to the V.P. and the surface of the lamina being inclined to H.P. at an angle of 40°.

14. A circular lamina, of 60 mm diameter, is resting on the H.P. on a point A of the circumference, with its plane inclined at 45° to H.P. and the top view of the diameter through A makes 30° with V.P. Draw the top and front views of the lamina.

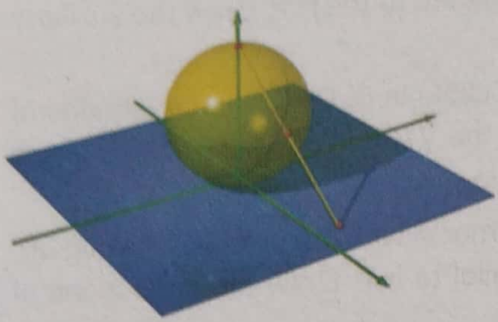
15. Draw the projections of a rhombus, diagonals 125 mm and 50 mm size, having smaller diagonal parallel to H.P. and V.P. both and the bigger diagonal inclined to H.P. such that plan of rhombus becomes a square. Draw the projections and find the inclination of plane with H.P. Use 1st angle projection system. [Ans : 66.5°]

16. A regular hexagonal lamina is resting in V.P. on one of its sides with lamina making 45° with V.P. The side on which it rests on V.P. makes 60° with H.P. Draw the projections using rotation of ground line method.

17. The T.V. of a pair of equal legs AB and AC of compass, appears as two lines of 10 cms length meeting at 30° angle. If the actual length of compass legs is 15 cms, find the actual angle between two legs and height of point A above the H.P. For projections use 1st angle system of projections. Assume ends B and C on H.P. [Ans : 20°, 11.2cms.]

CHAPTER - 10

Projections of Point, Line and Plane on Auxiliary Planes



1. Introduction :

We have studied two types of auxiliary planes :

(1) Auxiliary Inclined Plane (A. I. P.)

(2) Auxiliary Vertical Plane (A. V. P.)

We also know that projections on A. I. P. are known as auxiliary plans and projecti on A. V. P. are known as auxiliary elevations. When plan and elevation of a point, lin plane are given, we can get auxiliary plan and auxiliary elevation on A. I. P. and A. V respectively by the method studied in the chapter of projection of a point. These auxil views play most important role in determining the following :

(a) The true length of a line

(b) Point view of a straight line

(c) Distance between two skew lines

(d) Edge view or line view of a plane

(e) True shape of a plane

(f) Angle between two intersecting planes

(g) Distance between two parallel lines

(h) Distance of A point from plane surface

(i) In orthographic projections of objects real or true shapes of inclined surfaces seen in auxiliary views taken on auxiliary planes parallel to inclined surfaces of objec

We shall now take some illustrative practical problems and find the solutions of t by the use of auxiliary views.

2. Illustrative Problems :

Problem 1 : A point A is 45 mm above H. P. and 20 mm in front of V. P. Draw and elevation of the point A.

Project auxiliary plan of the point A on A. I. P. which is perpendicular to V. P. inclined to H. P. by θ (60°).

For solution see Fig. 10.1 and follow the procedure, as given below :

(i) First draw xy line and locate a' and a.

(ii) Draw now new ground line either X_1Y_1 or X_2Y_2 or X_3Y_3 or X_4Y_4 at θ (60°) to xy line around elevation a'. Draw perpendicular line from a' to either X_1Y_1 or X_2Y_2 or X_3Y_3 or X_4Y_4 and mark on it a_1 or a_2 or a_3 or a_4 respectively by taking distance of plan a from xy line i.e. 20 mm beyond X_1Y_1 or X_2Y_2 or X_3Y_3 or X_4Y_4, as shown in the figure.

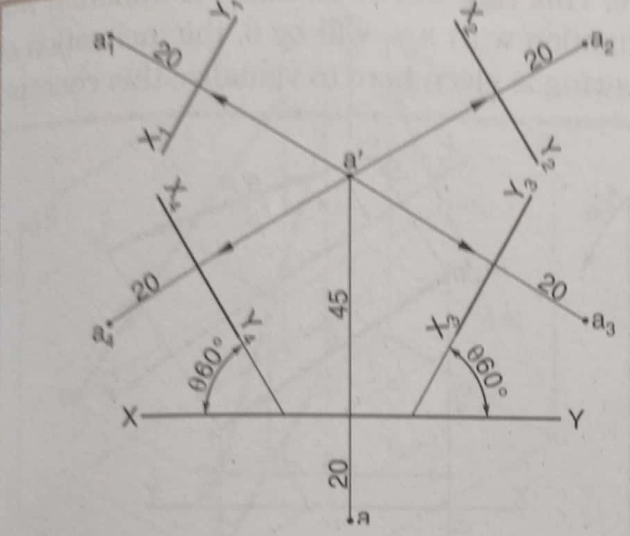


Fig. 10.1. Auxiliary plan of a point on A.I.P.

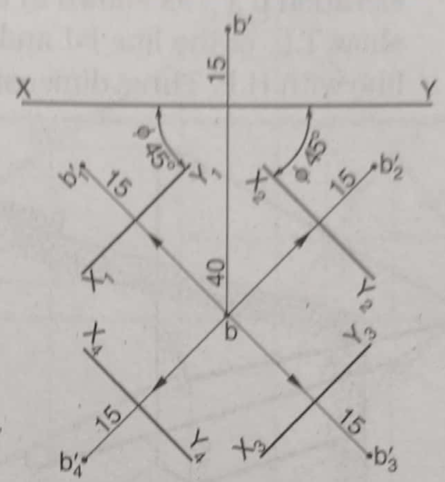


Fig. 10.2. Auxiliary elevation of a point on A.V.P.

Problem 2 : A point B is 15 mm above H.P. and 40 mm in front of V.P. Draw the projections and find its auxiliary elevation on A.V.P. perpendicular to H.P. and inclined to V.P. by an angle Ø (45°).

For solution see Fig. 10.2 and follow the procedure, as given below :

(i) First draw b' and b.

(ii) Draw now new G.L. either X_1Y_1 or X_2Y_2 or X_3Y_3 or X_4Y_4 at Ø (45°) to xyline around plan b. Draw perpendicular line from b to either X_1Y_1 or X_2Y_2 or X_3Y_3 or X_4Y_4 and mark on it b'_1 or b'_2 or b'_3 or b'_4 respectively by taking distance of elevation from xy line i.e. 15 mm beyond X_1Y_1 or X_2Y_2 or X_3Y_3 or X_4Y_4, as shown in the figure.

1. (a) True Length of A Straight Line :

We have studied in the chapter on straight lines that when a plan of a straight line is parallel to ground line, its corresponding elevation will show T.L. and true inclination with H.P.

Similarly, when elevation of a straight line is parallel to ground line, its corresponding plan will show T.L. and true inclination with V.P. This concept is used to get T.L., θ and Ø when plan and elevation are given.

Problem 3 : The distance between the end projectors of a straight line PJ is 45 mm. Point P is 45 mm above H.P. and 10 mm in front of V.P. Point J is 15 mm above H.P. and 25 mm in front of V.P. Draw the projections and find its T. L, θ and Ø by auxiliary plane method.

For solution see Fig. 10.3 (a) and (b) and follow the procedure, as given below :

(i) First draw elevation p'j' and plan pj.

(ii) Draw X_2Y_2 ground line parallel to elevation p'j' and project on it auxiliary plan p_2j_2, as shown in the figure. This view will be auxiliary plan. It will show T.L. of the line PJ and its inclination with X_2Y_2 will be Ø, the inclination of the line with V.P.

(iii) Similarly, draw X_1Y_1 ground line parallel to plan pj and project on it auxiliary elevation $p'_1j'_1$, as shown in the figure. This view will be auxiliary elevation. It will show T.L. of the line PJ and its inclination with x_1y_1 will be θ, the inclination of line with H.P. Three dimensional drawing is given here to visualize this concept.

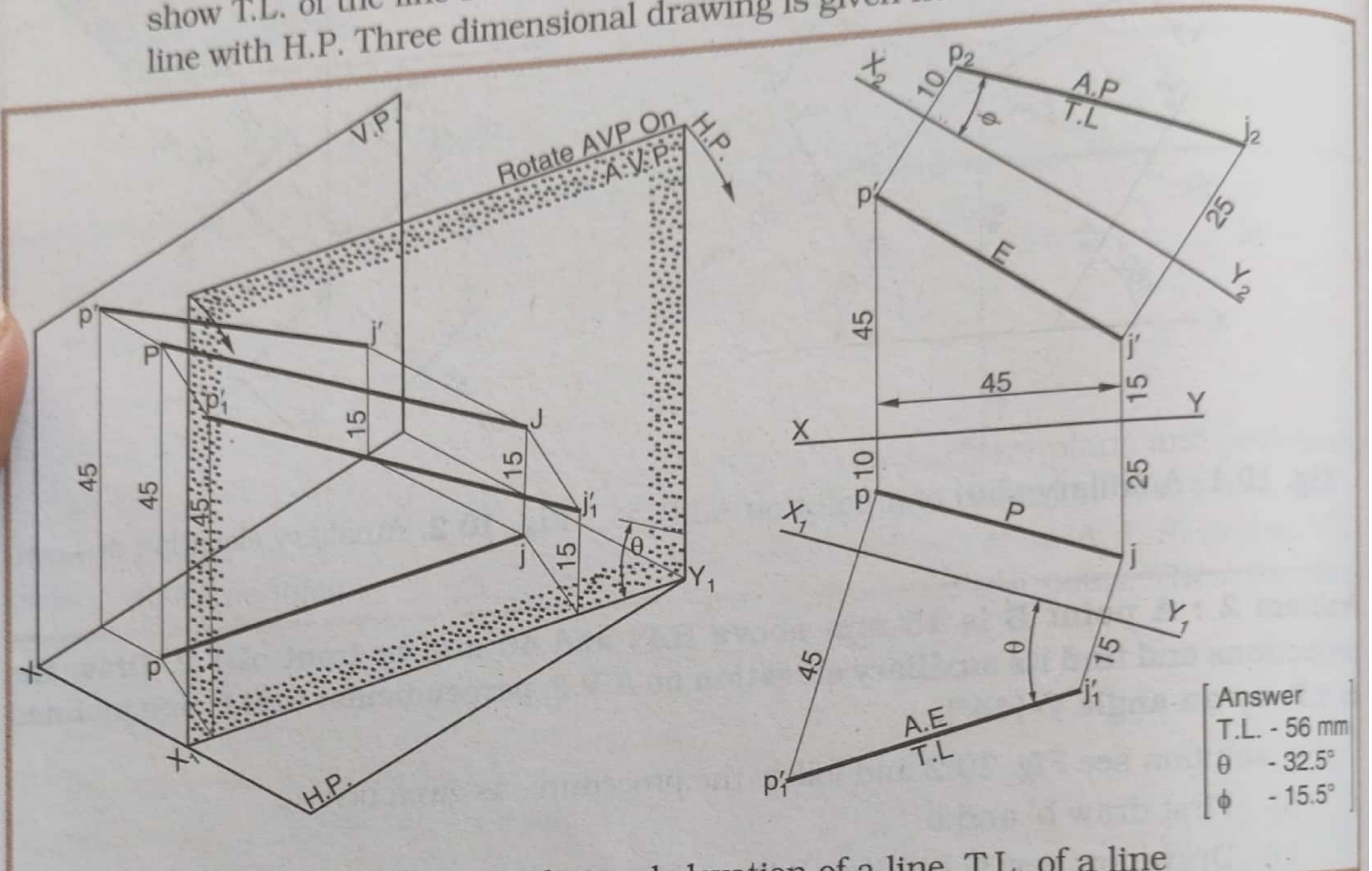


Fig. 10.3. Auxiliary plan and elevation of a line. T.L. of a line

1. (b) Point View of A Straight Line and

1. (c) Distance Between Two Skew Lines :

Whenever point view of a line is required from the given elevation and plan of a line, one must first find out T.L. view of a line by the procedure similar to problem 3, and then take new ground line perpendicular to T.L. view of a line already achieved.

xy line is always seen as a point view in an end view or side view. So whenever distance from xy line is required, it can be achieved from the side view.

Problems 4, 5 and 6 are solved by this concept. Whenever two skew pipelines are required to be connected by shortest possible pipeline, then problem can be solved by this concept.

Problem 4 : The distance between the end projectors of a line AB is 50 mm. Point A is 15 mm above H.P. and 35 mm in front of V.P. Point B is 50 mm above H.P. and 10 mm in front of V.P. Draw the projections of AB and find out (a) distance of mid point M of AB from xy and (b) the shortest distance between AB and xy lines.

For solution see Fig. 10.4 (a) and (b) and follow the procedure, as given below :

(i) Draw plan ab and elevation a'b', with the given data. Mark the mid points m and

(ii) Draw X_1Y_1 ground line perpendicular to xy and project on it end view $a'_1b'_1$ by the usual method. Project m'_1 also on it. xy line will be seen in end view as a point view O.

(iii) Join Om'_1 and measure it. This is the distance of mid point M from xy line.

(iv) Draw perpendicular On'_1 from O on $a'_1b'_1$ and measure it. It will be the shortest distance between AB and xy lines.

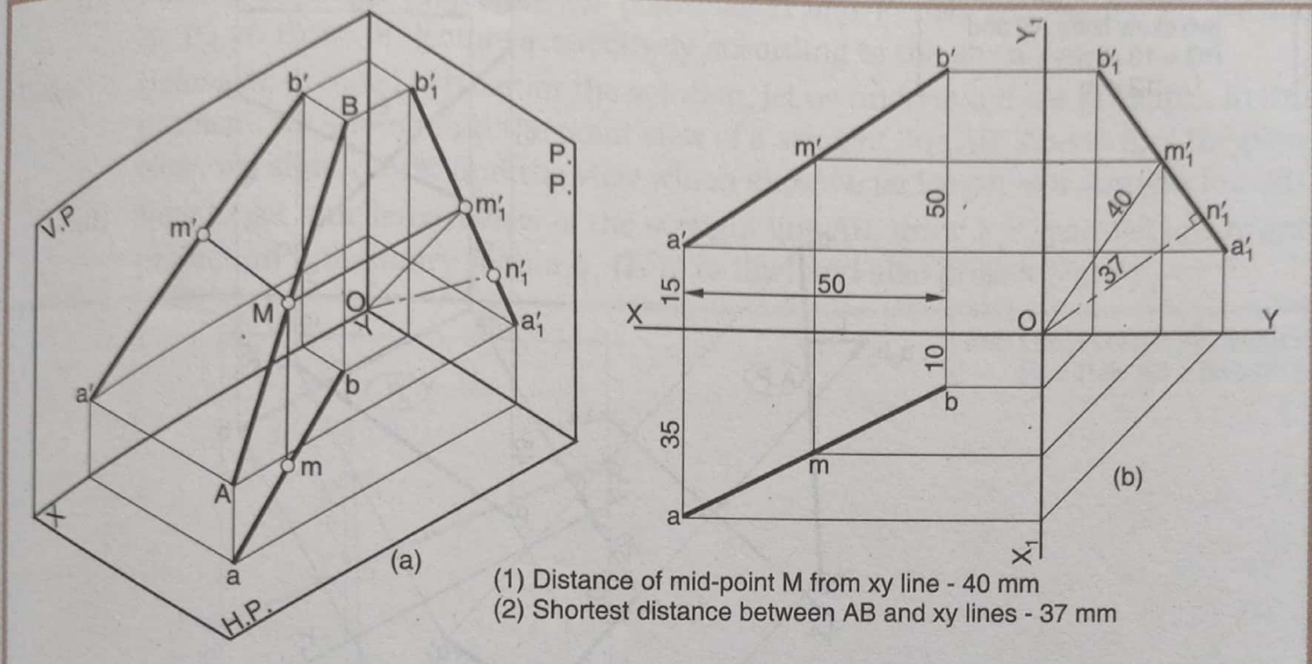


Fig. 10.4. Distance between xy line and given line.

Three dimensional drawing is given to visualise the same.

1. (c) To Find The Shortest Distance Between Two Non-Parallel, Non-Intersecting Skew Lines :

How to find the shortest distance between two skew lines, is explained in the problem given below :

Problem 5 : AB and PQ are two skew lines. Distance between end projectors of AB and PQ are 48 mm and 45 mm respectively. Projector of the point Q is 12 mm to the left of projector of B. Point A is 20 mm above H.P. and 1 mm in front of V.P. Point B is 40 mm above H.P. and 28 mm in front of V.P. Point P is 55 mm above H.P. and 45 mm in front of V.P. Point Q is 10 mm above H.P. and 15 mm in front of V.P. Draw the projections of the lines AB and PQ and find the shortest distance between the lines AB and PQ.

For solution see Fig. 10.5 and follow the procedure, as given below :

First of all draw projectors of points P, A, Q and B keeping distances between projectors of AB = 48 mm, of PQ = 45 mm and of QB = 12 mm. Knowing the distances of the points P, A, Q and B from V.P. and H.P., draw a' and a on projector of A, p' and p on projector of P, q' and q on projector of Q and b' and b on projector of B. Join a'b', ab, p'q' and pq.

Important discussion :

Out of the two lines AB and PQ, we must get point view of any one line. Further to get point view of a line, we must get the true length of the same line in the previous view. Here point view of line AB is obtained.

Now to get the true length of the line AB, select and draw X_1Y_1 ground line at any convenient distance parallel to plan of AB i.e. ab.

On this new ground line X_1Y_1, draw perpendicular projectors from a, b, p and q and on them, plot distances of a', b', p' and q' respectively from XY line on the other side of X_1Y_1 as shown in the figure for the point A.

[Ans: Shortest distance between two skew lines AB and PQ = 13 mm
L = 13 mm]

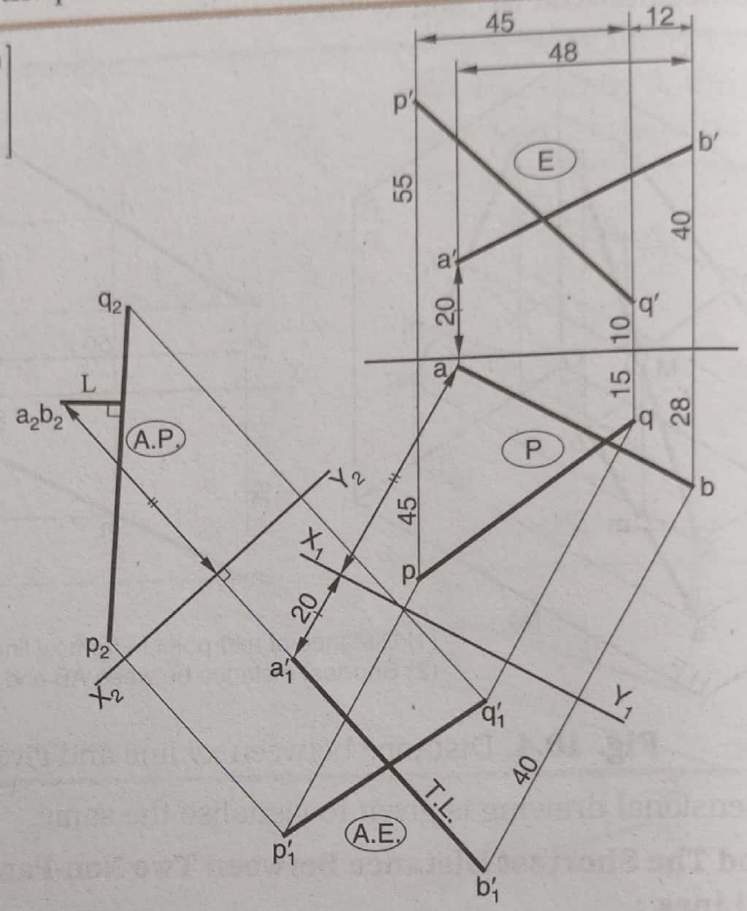


Fig. 10.5. Shortest distance between two skew lines

New projections of AB and PQ i.e. $a'_1b'_1$ and $p'_1q'_1$ are auxiliary elevations since the projectors are from plan points and distances are of elevation points. In this view, we have a'_1 b'_1 showing the true length since its plan ab is parallel to X_1Y_1. In fact X_1Y_1 is taken parallel to ab with this purpose only.

Now select and draw at any convenient distance, new ground line X_2Y_2 perpendicular to a'_1 b'_1 (or line showing T.L.). Draw perpendicular projectors from a'_1, b'_1, p'_1 and q'_1 on this new ground line X_2Y_2 and on them plot distances of a, b, p and q from X_1Y_1 line on the other side of X_2Y_2, as shown in the figure.

New projections of AB and PQ i.e. a_2b_2 and p_2q_2 are auxiliary plans since the projectors are from elevation (auxiliary) points and distances are of plan points. In these new projections we have a_2b_2 auxiliary plan of AB as point view.

From this point view a_2b_2, draw perpendicular to p_2q_2 and measure it. This distance (L) as shown in the figure is the shortest distance between skew lines AB and PQ. It comes out as 13 mm.

Problem 6 : The distance between the end projectors of a straight line AB is 80 mm. The line lies in the third quadrant. End A is 15 mm from V.P. and 60 mm from H.P.

while B is 50 mm from V.P. and 10 mm from H.P. A point P, situated on the projector at a distance of 25 mm from the projector through A, (the distance being measured towards the projector of B), is also in the third quadrant and is 70 mm from each plane. A perpendicular is drawn from P on AB. Locate it in plan and elevation and measure it.

For solution see Fig. 10.6 and follow the procedure, as given below :

(i) First draw three projectors for points A, B and P, and plot (a, a'), (b, b') and (p, p') on three projectors respectively according to the given data.

Hints : Before we proceed further for the solution, let us understand the principle. In this problem, we should find the point view of a straight line AB. And to find the point view, we should first find the view which shows true length of a straight line AB.

(ii) Now to get true length view of the straight line AB, draw X_1Y_1 parallel to a'b' and project on it auxiliary plan a_1b_1 (T. L. of line) and also project point p_1.

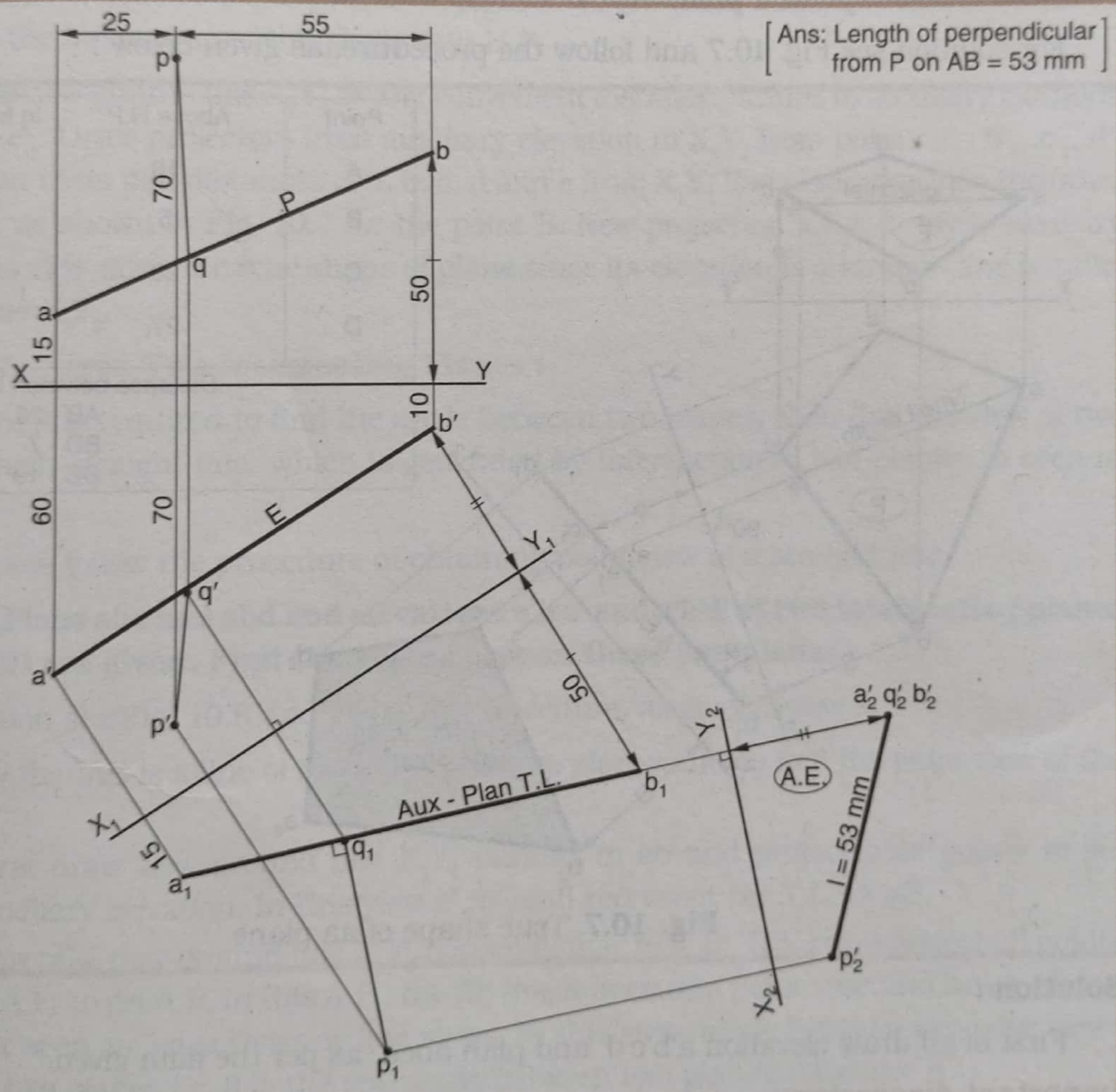


Fig. 10.6. Perpendicular from A Point to A Line

(iii) Now take new ground line X_2Y_2 perpendicular to a_1b_1 (perpendicular to T.L. of a line) and project on it straight line as well as p_1. We get a'_2 b'_2, a point view of the straight line AB, and view of point P as p'_2. Join these two points to get the true length of a perpendicular line from P on AB.

(iv) In the previous view i.e. auxiliary plan, draw perpendicular p_1q_1 from p_1 to a_1b_1 and transfer the point Q to all views to get views of perpendicular in all views, as shown in Fig. 10.6 Length of perpendicular measures 53 mm.

1. (d) & (e) Edge View of A Plane and True Shape of A Plane :

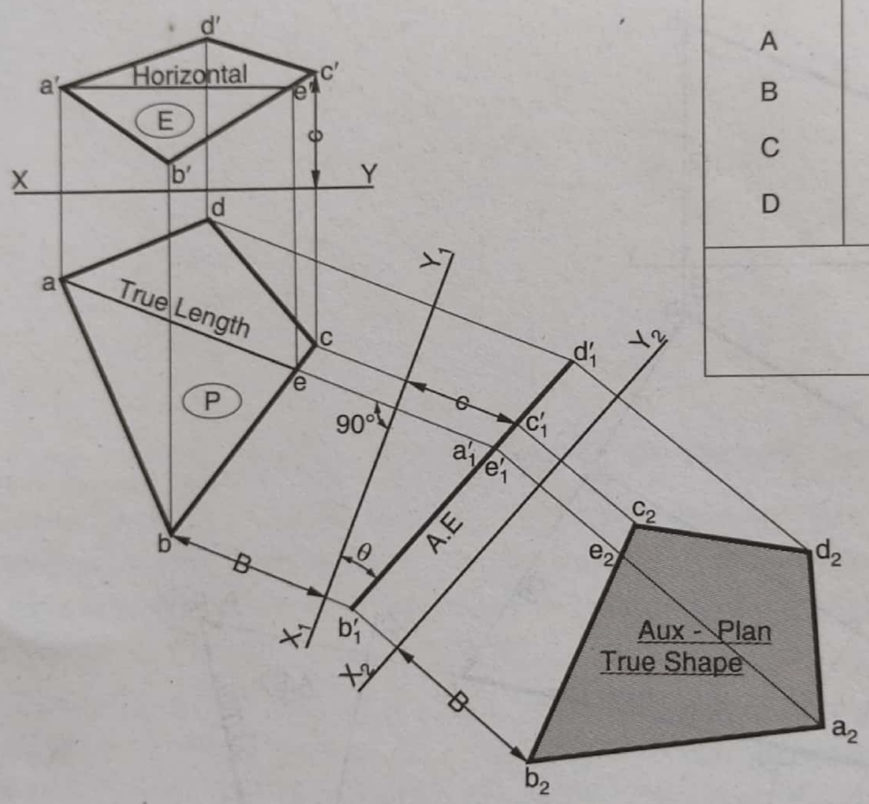
It is known to us that when a plan of a plane is a straight line and parallel to ground line, its corresponding elevation will show true shape. Similarly, when elevation of a plane is a straight line and parallel to ground line, its corresponding plan will show T.S.

Further, if a ground line is taken perpendicular to the true length of any straight line of plane, then subsequent view of the plane will give a straight line.

How to find the true shape of a plane when plan and elevation are given, is explained in problem 7, given below :

Problem 7 : a'b'c'd' and abcd are the elevation and plan of a plane ABCD, respectively. Find the true shape of a plane ABCD and its inclination with H.P.

For solution see Fig. 10.7 and follow the procedure, as given below :



Point	Above H.P.	In front of V.P.
A	18	16
B	5	60
C	20	28
D	27	5

Distance between Projectors
AB - 26
BD - 7
DC - 19

Fig. 10.7. True shape of aa plane [Ans θ = 21

Solution :

First of all draw elevation a'b'c'd' and plan abcd, as per the data given.

Important discussion :

To get the true shape of a plane, we must get line view of a plane in the previous view or projection.

Again to get line view of a plane, we must get true length of any straight line of the plane in previous projection.

Again to get true length of any straight line in elevation or plan of a plane, the plan or elevation respectively of that straight line must be parallel to x-y line.

So as per the above discussion, we draw a'e' parallel to XY in elevation of a plane. Point e' falls on b'c' and so transfer it to plan on bc and get the plan of E as e. ae will give the true length.

Now to this true length line ae draw perpendicular line X_1Y_1 (new G.L.) at any convenient distance.

Draw perpendicular projectors to X_1Y_1 from points a, b, c, d and e and on them plot distances of a', b', c', d' and e' from XY line respectively on other side of X_1Y_1, as shown in Fig. 10.7 for the point C.

New projection a'_1, b'_1, c'_1, d'_1, e'_1 is an auxiliary elevation since the projectors are from plan points and distances are from elevation points. In this view, we have achieved straight line view of a plane. Whatever angle (θ) this straight line view (here auxiliary elevation) makes with X_1Y_1 is the inclination of the plane with H.P.

Now take new ground line X_2Y_2 at any convenient distance parallel to auxiliary elevation a'_1 b'_1 c'_1 d'_1 e'_1. Draw projectors from auxiliary elevation to X_2Y_2 from points a'_1, b'_1, c'_1, d'_1 and e'_1 and on them plot distances of a, b, c, d and e from X_1Y_1 line respectively on the other side of X_2Y_2, as shown in Fig. 10.7 for the point B. New projection $a_2b_2c_2d_2$ is an auxiliary plan and this view gives the true shape of plane since its elevation is a straight line parallel to ground line X_2Y_2.

1 (f) Angle Between Two Intersecting Planes :

Whenever it is required to find the angle between two planes, then find the view of two planes in which straight line, which is generated by intersection of two planes, is seen in point view.

That means follow the procedure of obtaining point view of a straight line.

Problem 8 : Plans abc and abd and elevations a'b'c' and a'b'd' of two intersecting planes ABC and ABD are given. Find the angle between these two planes.

For solution see Fig. 10.8 and follow the procedure, as given below :

We know that AB is a line of intersection of two planes and so find the point view of the line AB.

(i) First draw new ground line X_1Y_1 parallel to ab and project plan points to get auxiliary elevation. In this view a'_1 b'_1 will represent the T.L. of AB.

(ii) Now take new ground line X_2Y_2 perpendicular to $a'_1b'_1$ (T.L.) and project all points of A.E. to get A.P. In this A.P., the AB line is seen as a point view and hence planes are seen as edge views or line views. In this view, angle between two edge views of two planes i.e. θ is the real angle between two planes. Measure it.

(iii) If one wants to get θ in A. E., initially draw X_1Y_1 parallel to a'b' instead of ab and proceed in the same way.

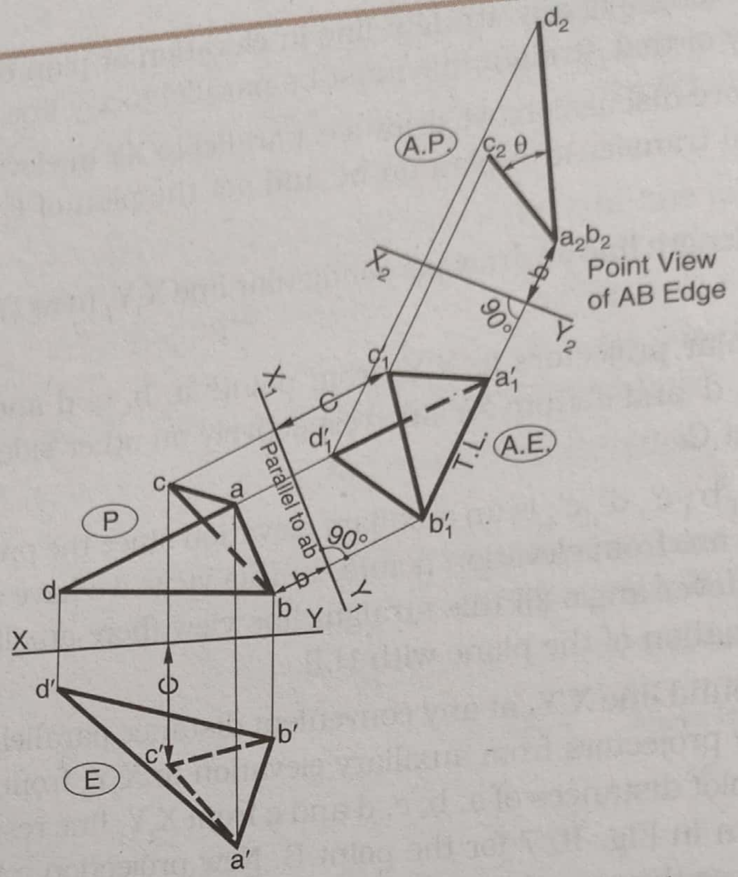


Fig. 10.8. Angle between two intersecting planes

1 (g) Distance Between Two Parallel Straight Lines :

The solution of finding the distance between two parallel straight lines, whose plan an elevation are given, can be done by following any one method given below :

(a) By finding the point views of both the straight lines and measuring distanc between these point views,

[OR]

(b) By constructing a plane in each view out of two views of two given straight line and then finding the true shape of that constructed plane. In the true shape plane, distance between the two lines is the distance between two parallel straig lines.

3. Miscellaneous Problems :

Problem 9 : An isosceles triangle PQR having the base PQ 60 mm and altitude 8 mm has its base in the V.P. making an angle of 30° with the H.P. The corner P is 1 mm above the H.P. and the corner R is in the H.P. Draw the projections of the plan

For solution see Fig. 10.9 and follow the procedure, as given below :

(i) Initially, assume the triangle to be lying completely in the V.P. with edge P making 30° with H.P. and the point P 10 mm above H.P., and hence draw Δp'q' with base p'q' of 60 mm making 30° with xy and with point p' 10 mm above and the altitude of triangle 85 mm.

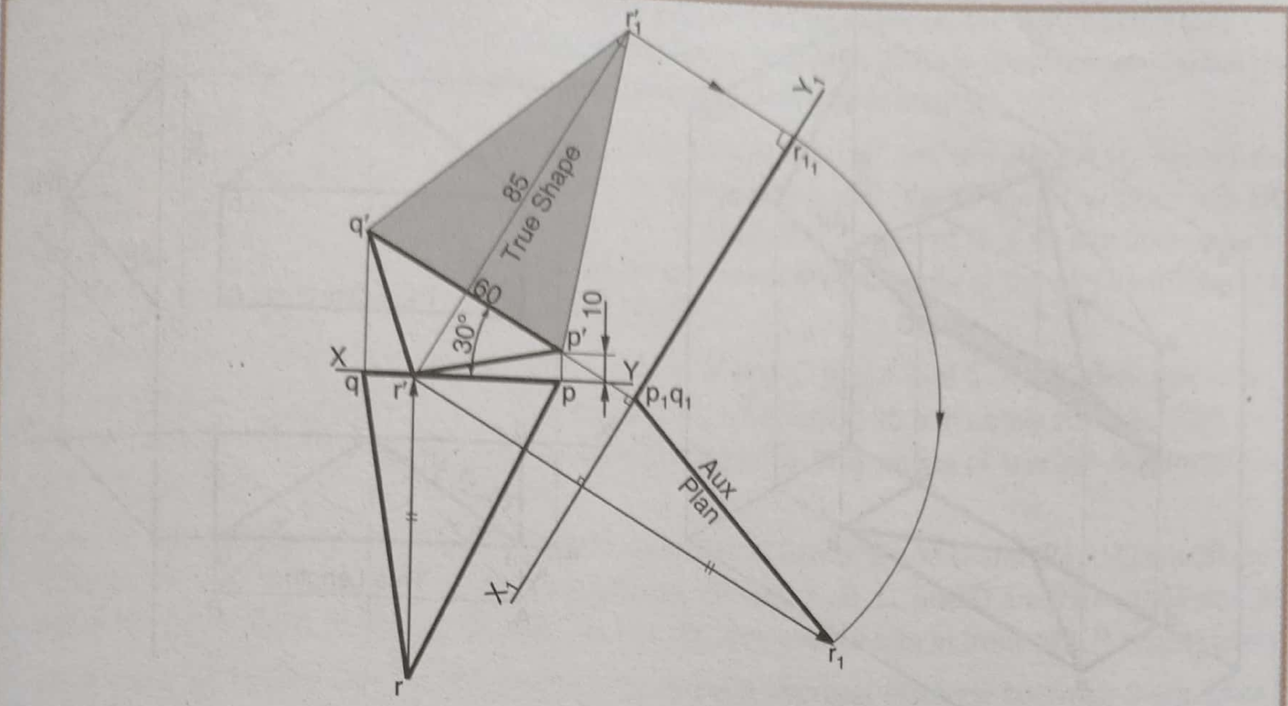


Fig. 10.9. Problem - 9

(ii) Now rotate the triangle about p'q' till the point R gets its position in H.P. During this rotation, the elevation of point R will move along a line, from r'_1, perpendicular to p'q'. The point at which this line cuts xy line, mark that point as r', since point R is in H.P. During this rotation, the auxiliary plan of point R on A.I.P, whose x_1y_1 line is perpendicular to p'q', is an arc of circle with point p_1q_1 as the centre and radius equal to p_1q_1 - r_1 (85 mm). From r', draw projector on x_1y_1 to cut the above arc at r_1. Measure the distance of r_1 from x_1y_1 and take the same distance to mark r on projector to xy from r'.

(iii) Join q'r' and p'r' to complete elevation and join qr and pr to complete plan.

Problem 10 : A picture frame, 2 m wide and 1 m high, is to be fixed on a horizontal wall railing by two straight wires attached to the top corners. The frame is to make an angle of 40° with the wall and the wires are to be fixed to a hook on the wall on the centre line of the frame and 1.5 m above the railing. Find the length of the wires and the angle between them.

For solution see Fig. 10.10 and follow the procedure, as given below :

Discussion :

To visualise the position of the frame and wire with respect to the wall, three dimensional drawing is given. The true width of the frame will be seen in elevation and plan and the true height of the frame will be seen in the side view or the end view. True angle (40°) of the frame plane with wall will be seen in the side view. So here we shall have to draw all views simultaneously.

(i) Draw plan, elevation and end view of the frame along with wires, as shown in Fig. 10.10

(ii) Find the true length of one wire. Second wire will be of the same length. Procedure for T.L. is shown in the figure for wire OA.

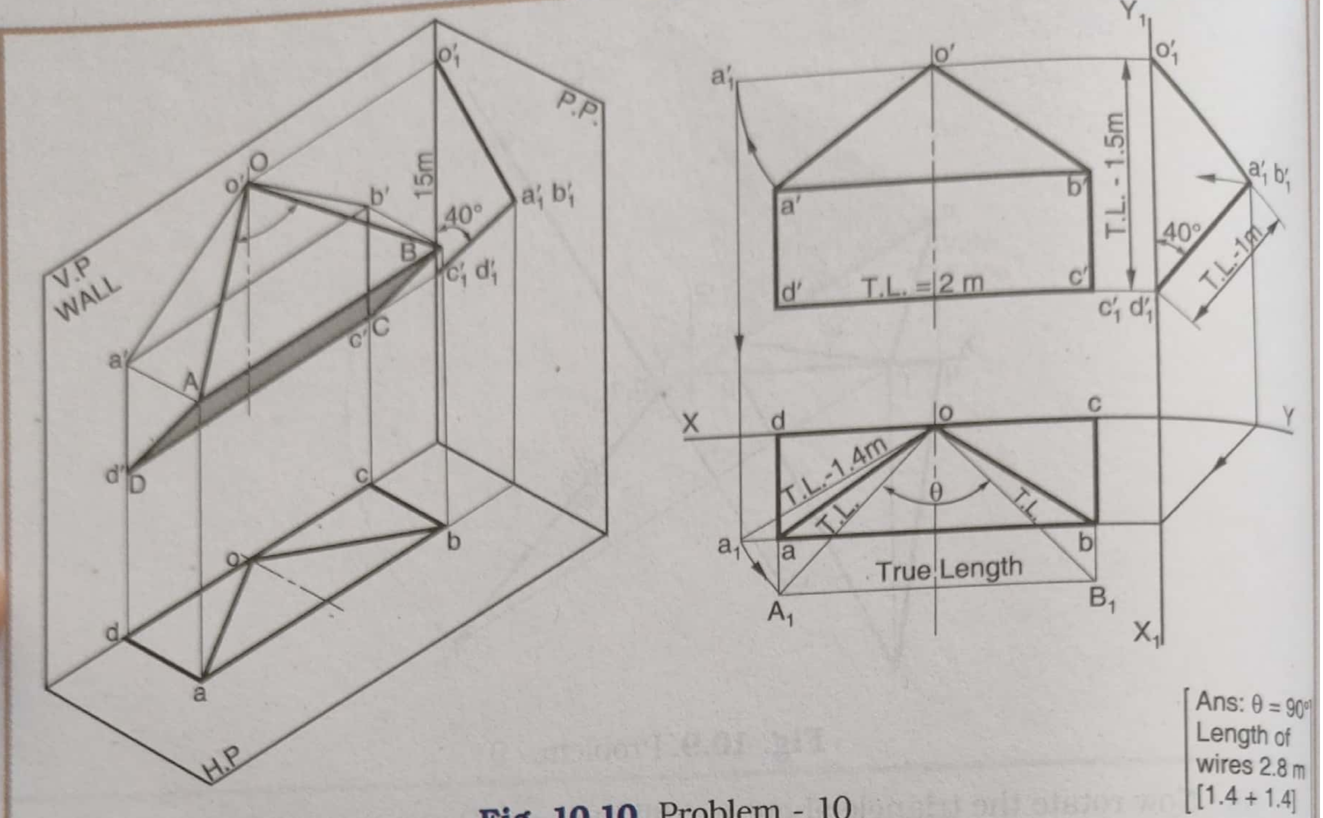


Fig. 10.10. Problem - 10

(iii) Draw triangle with two sides as T.L. of the wire and one side as 2 m width of the frame. In this triangle θ will be the true angle, so measure it. Measured true length of wire is 1.4 m and hence total length of two wires is 2.8 m and the measured true angle θ is 90° between wires.

EXERCISE

1. The distance between the end projectors of a straight line PQ is 70 mm. The end P is 45 mm above H.P. and 25 mm in front of V.P. The end Q is 35 mm above H.P. and 55 mm. in front of V.P. Find out the true length of a line and its inclination with H.P. and V.P. both, by auxiliary plane method. [Ans : 74 mm, θ =7.5° ø=23°]

2. A hexagonal plate, of 30 mm long sides, has an edge on the H.P. and inclined at 45° to the V.P. Hexagonal plate makes an angle of 30° with the H.P. Draw the projections using auxiliary plane method.

3. The distance between the end projectors of a line AB is 75 mm. The end A is 40 mm below H.P. and 25 mm behind V.P. The end B is 60 mm below H.P. and 10 mm behind V.P. Find (i) the shortest distance between two lines AB and xy and (ii) the distance of mid point M of AB from xy line. [Ans : 45 mm ; 53 mm]

4. A divider POQ has two equal legs OP and OQ of 100 mm length. It is resting on ground on two leg's ends P and Q. The distance between P and Q is 35 mm and the line PQ is inclined to the V.P. by 60°. The hinge point O is 50 mm above the ground.

   Draw the projections of the divider by auxiliary plane method and find out the inclination of the plane, containing the divider with ground or H.P. [Answer 30.5°]

5. A regular pentagon, of 50 mm side, is resting on one of its sides on the H.P., having that side parallel to and 25 mm in front of V.P. It is tilted about that side till its highest corner rests in the V.P. Draw the projections of the pentagon using auxiliary plane method.

6. ABC is a thin triangular plate with the edge AB lying in the H.P. and the point A 10 mm in front of the V.P. The distance between the projectors through A and B is 45 mm. The sides AB, BC and CA measures 70 mm, 80 mm and 60 mm respectively. The point C is 45 mm above the H.P. Draw the projections of the triangular plate and measure the angle of the plate with the H.P. Solve the problem by auxiliary plane. [Ans : 52°]

7. The distance between the projectors of A and B, B and C and A and C are 50 mm, 25mm and 75 mm respectively. Points A, B and C are 50 mm, 65 mm and 25 mm above H.P. and 12.5 mm, 50 mm and 25 mm in front of V.P. respectively. Draw the projections of triangle ABC and find out the true shape of the triangle ABC.

8. The distance between projectors of A and C, A and B, C and B, B and D and C and D are 30 mm, 100mm, 70 mm, 30 mm and 100 mm respectively. Points A, B, C, and D are 21 mm, 44mm, 80 mm and 54 mm above H.P. and 30 mm, 70 mm, 20 mm and 10 mm in front of V.P. respectively.

   Draw the projections of lines AB and CD and find out shortest distance between them. [Ans : 50 mm]

9. Draw, first, projections of two intersecting planes PQR and QRS from the data given below and then find the angle between two planes PQR and QRS by auxiliary plane method.

   The distance between projectors of R and P, P and Q, R and S, R and Q and S and P are 62 mm, 28 mm, 44 mm, 90 mm and 18 mm respectively. Points R, S, P and Q are 60 mm, 24 mm, 84 mm and 44 mm above H.P. and 44 mm, 80 mm, 70 mm and 20 mm in front of V.P. respectively. [Ans. 72°]

10. ABC is a triangular plate in the first quadrant, the projections of which appear as follows :

    Elevation triangle : a'b' = 80 mm, b'c' = 45 mm and c'a' = 80 mm a'b' making an angle of 45° with xy line and the corner A of the triangle is 5 mm above the H.P.

    Plan triangle : ab makes 30° with the xy line, bc = 95 mm, corner B is 5 mm in front of the V.P.

    Draw the projections of the plane and determine the true shape with the help of auxiliary views. Also find the inclinations of the plane with the H.P. and V.P. both. [Ans. : 42°, 68°]

11. In the triangular plate ABC of problem 10, draw a perpendicular from corner B to the opposite side AC of the triangle and measure its length. Also show the perpendicular line in the elevation and plan of the triangle. [Ans. : 80 mm]